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FOR ENGINEERS

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THE GLASGOW TEXT BOOKS
EDITED BY G. MONCUR

PLANE AND GEODETIC SURVEYING

FOR ENGINEERS

VOL II

HIGHER SURVEYING

BY

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PUBLISHERS' PREFACE

Volume I of this book, dealing with Plane Surveying, was revised by the author in 1931, shortly before his death It was his intention to revise Volume II, more particularly as changes in the method of reckoning astronomical time and extensive changes in the *Nautical Almanac* had rendered certain portions out of date The publishers were therefore faced with the task of turning elsewhere for the necessary revision Fortunately they secured the co-operation of Dr L J. Comrie, Superintendent of H M Nautical Almanac Office, and Lieut -Col J. E. E Craster, formerly of the Ordnance Survey Chapters I and II of Volume II as now presented have been rewritten and reprinted, but it was decided to retain unchanged Chapters III to VII, of which a sufficient supply of printed sheets was held. Recent developments have been covered by a series of short appendices. In the 1931 revision of Volume I, to which references are given in Volume II, the page numbers were changed For this reason principally, the list of corrigenda for Volume II that follows this preface has been provided.

THE PUBLISHERS

PREFACE TO THE FIRST EDITION

The entire field of Plane Surveying having been dealt with in the first volume, the subject-matter of this volume covers the remainder of a degree course, and includes Field Astronomy, Geodetic Surveying and Levelling, Topographical and Reconnaissance Surveying, and Mapping

As in Volume I, an endeavour has been made to meet the requirements of private students and practising surveyors, particularly with regard to the subject of Field Astronomy In this branch of surveying it is essential that the student should acquire a sound grasp of the fundamental conceptions before proceeding to a consideration of the actual observations The author has emphasised this necessity by devoting Chapter I entirely to explanations regarding the quantities dealt with in astronomical determinations It is hoped that the arrangement of this chapter and the inclusion of typical worked examples on the use of the *Nautical Almanac* will facilitate an understanding of the fundamental principles of astronomical measurements

Chapter II deals with the instruments and practice of Field Astronomy as applied to the determination of Time, Azimuth, Latitude and Longitude For each determination the primary method is described as well as the less refined observations by ordinary field instruments

Chapter III covers the field work of Geodetic Surveying The catenary method of base measurement by invar wires or tapes, having superseded all others, is treated at length, but a few typical base bars have been described as a matter of interest

The subjects of Theory of Errors, Survey Adjustment and Geodetic Computations are dealt with in Chapter IV In a general textbook, considerations of space preclude detailed treatment of these subjects, but it is thought that the matter given will prove amply sufficient for the needs of the majority of readers

Chapter V deals with Geodetic Levelling, including Precise Spirit and Trigonometrical Levelling

Topographical, Geographical and Reconnaissance Surveys are treated in Chapter VI Several of the survey methods have already been given in detail in Volume I, and these are referred to only to the extent of indicating special features applicable to small scale work The subjects of Barometric Levelling and Photographic

Surveying, including Stereo-Photographic and Aerial Surveying, are given at greater length.

Chapter VII deals with Map Construction, and includes a *résumé* of the principal map projections employed.

Sets of illustrative numerical examples and answers are given, and lists of references on the various subjects treated are inserted after the appropriate chapters. As in the case of Volume I, no attempt has been made to prepare exhaustive bibliographies, but the author has selected textbooks and papers which are commonly accessible and many of which he has himself found useful. He would take this opportunity of expressing his indebtedness to various official publications, particularly those of the Survey of India and the United States Coast and Geodetic Survey

This volume is illustrated by 114 diagrams, which are original with the exception of Figs 23, 25, 31 and 70 by Messrs Bausch and Lomb, 88 by Messrs C F Casella and Co, 29 by M A. Jobin, 26 by Messrs C H Steward, 22 by Messrs Troughton and Simms, 45, 57 and 58 by Messrs E R Watts and Son, and 72, 96, 97 and 99 by Messrs. Carl Zeiss The author gratefully acknowledges the kindness of those firms in placing their illustrations at his disposal, and would also express his thanks to Messrs The Cambridge and Paul Scientific Instrument Co for information relative to their invar precise staff.

D C

PREFACE TO THE SECOND EDITION

In revising Chapters I and II, we have merely followed closely the author's original text, introducing modifications that were rendered necessary by the recent changes in the mode of reckoning astronomical time and in the *Nautical Almanac*, and revising the paragraphs on the prismatic astrolabe, wireless signals and the determination of longitude, as well as the examples In addition a number of minor alterations have been made, but there has been no attempt to make a general revision, as our desire was to retain the features of Prof. Clark's presentation.

L. J. Comrie
J. E. E Craster

1934 *March.*

CONTENTS

CHAPTER I

FIELD ASTRONOMY—INTRODUCTORY

CHAPTER II

FIELD ASTRONOMY—OBSERVATIONS

CHAPTER VI

TOPOGRAPHICAL AND RECONNAISSANCE SURVEYING

CHAPTER VII

MAP CONSTRUCTION

CORRIGENDA (Vol. II)

(See Publishers' Preface)

Page	Line	*For*	*Read*
130	10	sunshiere	sunshine
130	11	diffn	differ
138	– 14	27	33
140	– 7	69	85
142	21	72	88
142	– 5	56	66
143	– 17	58	69
146	4	58	64
154	7	67	72
174	10	$2a^2$	a^2
186	6	The logarithms of P, Q, R, S and T are not included in the last edition of the *Text Book of Topographical and Geographical Surveying*	
195	– 3	28	30
198	14	95	101
198	– 5	87	101
199	– 7	83	101
200	– 18	98	105
204	– 19	199	232
224	7	161	187
228	14	389	248
240	10	219	255
240	– 10	151	177
268	10	**In future the transverse Mercator projection will be used for the one-inch and smaller scale maps of Great Britain.**	
271	31		

PLANE AND GEODETIC SURVEYING FOR ENGINEERS

HIGHER SURVEYING

CHAPTER I

FIELD ASTRONOMY—INTRODUCTORY

SURVEY methods in general are directed to obtaining only the relative positions of points, but when their absolute positions on the earth's surface are required, recourse must be had to astronomical determinations The branch of mathematical astronomy with which the surveyor has to deal relates to the determination of time, true meridian, latitude and longitude Apart from their applications in pure geodesy, observations for these quantities are required for the following purposes

(1) In the execution of the framework of a large survey it is necessary to obtain the true bearing of at least one of the lines and the latitude and longitude of at least one of the stations, so that the survey may be located as a portion of the surface of the earth.

(2) The direction of true meridian may be determined in order to provide a reference meridian for traverse bearings Its superiority over magnetic meridian, by virtue of its permanence, is particularly evident if survey lines have to be retraced at a later date On other than small surveys, reference to true meridian is essential because of the facility afforded of checking the angular work at intervals by astronomical observations

(3) Rapid surveys of an exploratory character are sometimes entirely controlled either by means of the latitudes and longitudes of a few points at wide intervals or by the latitudes of points and the directions, or azimuths, of the lines joining them.

(4) The field work of establishing interstate boundaries, involving the setting out of a considerable length of a meridian, a

parallel of latitude, or an oblique line, is dependent upon astronomical determinations

The Celestial Sphere.—The celestial bodies with which the surveyor is concerned are the so-called fixed stars and the sun, the moon and planets being of minor importance

On viewing the stars, one sees a number of bodies of very varied distance from the earth, but all of which are so remote that straight lines from a star to different points on the earth, or even to different points on the earth's orbit round the sun, may for all practical purposes be considered parallel In other words, the orbit of the earth may be treated as a point in comparison

Since the surveyor has to deal only with the angular positions of the stars, and not with their linear distances from the earth, it is very convenient to consider them studded upon the surface of an imaginary sphere, called the celestial sphere, at the centre of which the observer is stationed This conception is quite legitimate, since the relative angular positions of the stars as they are seen from the earth would remain unchanged if they were projected along the straight lines joining them to the observer until they were situated on the spherical surface The radius of the celestial sphere is to be imagined indefinitely great, so that the diameter of the earth's orbit may be regarded relatively as a point

On watching the stars for some time, it is seen that, although they maintain the same situations with respect to each other, their positions relative to the horizon are continually changing, some stars apparently moving more than others in a given time Indeed, the observed motion resembles that which might be expected if the stars were actually fixed upon a sphere rotating from east to west This motion is an apparent one, and is due to the real daily rotation of the earth from west to east about the axis joining its north and south poles The apparent rotation of the celestial sphere is, therefore, equal in rate and opposite in direction to that of the earth, and is about the axis of the earth produced The points in which the prolongation of the earth's axis meets the celestial sphere are called respectively the north and south celestial poles About those points the stars appear to travel along concentric circular paths with perfectly uniform angular velocity

It will be found convenient to disregard the fact that the diurnal motion of the celestial sphere is only apparent and to assume that it is real, and that the earth's rotation is annulled

Geometrical Properties of the Sphere.—Many of the quantities involved in astronomical determinations are parts of the celestial sphere, and the surveyor should have some knowledge of spherical geometry and trigonometry. The more important definitions relating to parts of the sphere are here collected for convenience of reference.

(1) A *great circle* of a sphere is the circle in which it is intersected by any plane passing through the centre The diameters of all great circles are also diameters of the sphere. The circles EAQ, PCAP′, and EBDQ (Fig 1) are great circles

A *small circle* is the circle in which the sphere is intersected by any plane not passing through the centre. Thus BCD is a small circle.

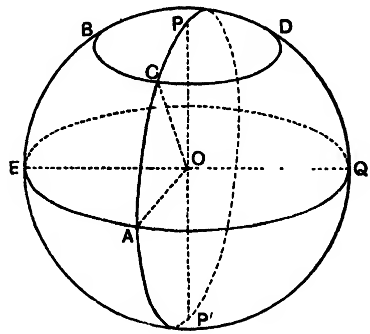


FIG 1

(2) The *axis* of any circle of a sphere is that diameter of the sphere which is perpendicular to the plane of the circle, the ends of the axis being called the *poles* of the circle Thus P and P′ are the poles of the great circle EAQ and also of the small circle BCD, if the planes of those two circles are parallel

In the case of great circles, the poles are equidistant from the plane of the circle, but are not so with regard to small circles, the nearer being referred to as *the* pole

Great circles passing through the poles of a great or small circle are termed *secondaries* to that circle Thus the great circle PCAP′ is a secondary to both EAQ and BCD

(3) The distance between two points on the surface of a sphere is measured by

(*a*) The arc of the great circle on which both points lie, or

(*b*) The angle which this arc subtends at the centre of the sphere

Thus the distance between the points A and C may be expressed either as the arc AC, since PCAP′ is a great circle, or as the angle AOC

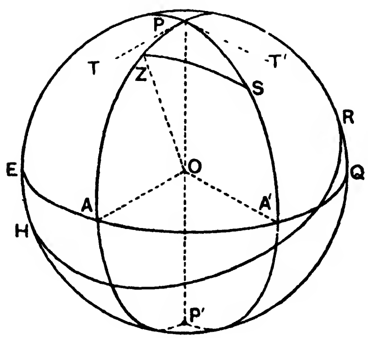


FIG 2

The distance between a point on the spherical surface and a great or small circle is the distance from the given point to the point of intersection of the circle by that one of its secondaries which passes through the given point The arc AC, or the angle COA, therefore expresses the distance of C from the circle EAQ, since PCAP′ is the secondary to that circle which passes through C.

(4) The angle between two great circles is measured by :

(*a*) The spherical angle at either of their points of intersection. Thus in Fig. 2, the angle between the great circles PAP′ and PA′P′ is the spherical angle APA′

(*b*) The angle between the planes in which they lie, as angle AOA′.

(*c*) The plane angle between the tangents to the circles at either intersection, as angle TPT′

(*d*) The arc intercepted by them on the great circle to which they are both secondaries, as arc AA′ of the great circle EAA′Q

(*e*) The distance between their poles Thus the angle between the great circles EAA′Q and HR is the distance between their poles P and Z, namely the arc PZ of the great circle PAP′ or angle POZ

(5) A *spherical triangle* is a figure on the surface of the sphere bounded by three arcs of *great* circles. Thus if the points Z and S are joined by an arc of a great circle, PZS is a spherical triangle Since an arc of a great circle is proportional to the angle at the centre of the sphere subtended by it, the sides of a spherical triangle are measured in terms of the central angles which they subtend

(6) If three of the six parts of a spherical triangle are known, the methods of spherical trigonometry enable the triangle to be solved.

Let A, B and C denote the angles of a spherical triangle, and a, b and c the sides opposite these angles, *i e* the angles at the centre of the sphere subtended by the sides Let $s = \frac{1}{2}(a+b+c)$

The General Triangle —(1) Given the three sides, a, b and c

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$$

$$\text{or } \cos\frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}$$

$$\text{or } \tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}}$$

the other angles being obtained from the corresponding formulæ.

(2) Given the three angles, A, B and C

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}$$

(3) Given two sides and the contained angle, as b, c and A

$$\tan\tfrac{1}{2}(B+C) = \frac{\cos\frac{1}{2}(b-c)}{\cos\frac{1}{2}(b+c)}\cot\tfrac{1}{2}A$$

$$\tan\tfrac{1}{2}(B-C) = \frac{\sin\frac{1}{2}(b-c)}{\sin\frac{1}{2}(b+c)}\cot\tfrac{1}{2}A$$

whence B and C, a being obtained as in case (2), or from

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

(4) Given two angles and the side between them, as A, B and c.

$$\tan \tfrac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \tfrac{1}{2}c$$

$$\tan \tfrac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \tfrac{1}{2}c$$

whence a and b, C being obtained as in case (1).

(5) Given two sides and the angle opposite one of them, as a, b and A.

$$\sin B = \frac{\sin A \sin b}{\sin a}$$

$$\tan \tfrac{1}{2}C = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2}(A+B)$$

$$\tan \tfrac{1}{2}c = \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} \tan \tfrac{1}{2}(a+b)$$

This case, as in plane trigonometry, is ambiguous, since there are two possible values for the angle B corresponding to the calculated value of $\sin B$, one being the supplement of the other The fact that the greater side subtends the greater angle removes this ambiguity in certain cases, and in practice it is usually possible to select the correct value for the unknown angle.

(6) Given two angles and the side opposite one of them, as A, B and a

$$\sin b = \frac{\sin a \sin B}{\sin A}$$

C and c being obtained as in case (5). There is here the same ambiguity as in the last case

The Right-angled Triangle —If one of the angles of the spherical triangle is a right angle, the formulæ are considerably simplified. Since only two parts, other than the right angle, need be known for solution of the triangle, the following formulæ meet all cases.

Let C be the right angle

$$\begin{aligned}
\sin a &= \sin A \sin c &&= \cot B \tan b \\
\sin b &= \sin B \sin c &&= \cot A \tan a \\
\cos c &= \cos a \cos b &&= \cot A \cot B \\
\cos A &= \cos a \sin B &&= \tan b \cot c \\
\cos B &= \cos b \sin A &&= \tan a \cot c
\end{aligned}$$

DEFINITIONS OF ASTRONOMICAL TERMS

The quantities dealt with in field astronomy may be classified as relating to : (1) the earth, the celestial sphere, and the observer ; (2) the position of a celestial body , (3) the classification of stars ; (4) the sun ; (5) corrections ; (6) time.

The Earth, the Celestial Sphere, and the Observer (Fig. 3)

The Terrestrial Equator, $E_1A_1Q_1$, is the great circle of the earth, the plane of which is at right angles to the axis of rotation The north and south terrestrial poles, P_1 and P_1', are its geometrical poles, and are consequently equidistant from it

The Celestial Equator, EAQ, is the great circle of the celestial sphere in which it is intersected by the plane of the terrestrial equator It is therefore midway between the celestial poles P and P′.

The Horizon.—(*a*) The *Sensible Horizon* is a plane passing through the observer at right angles to the direction of gravity at the place of observation It is the plane in which the line of sight of a level telescope lies (*b*) The *True, Rational* or *Geocentric Horizon* is a parallel plane through the centre of the earth

The distance between these planes being negligible in comparison with the radius of the celestial sphere, they are to be regarded as coincident and intersecting the celestial sphere in a great circle called the *Celestial Horizon*, or simply the *Horizon*. In Fig. 3, HR represents the horizon of the place X In the case of the relatively near bodies forming the solar system, the radius of the earth subtends an appreciable angle at the celestial body, and results of observations from the sensible horizon must be corrected to the values they would have if taken at the centre of the earth from the true horizon (see Parallax, page 17)

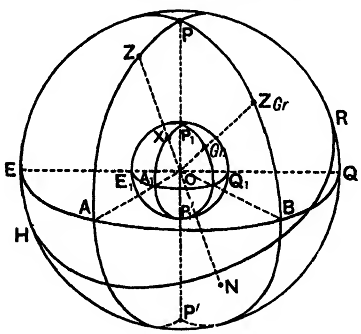


FIG 3

The Zenith and Nadir are the poles of the celestial horizon, the zenith Z being the point overhead in which the direction of a plumb line at the observer would meet the celestial sphere, the nadir N being the corresponding point vertically below him

Terrestrial Meridians are lines of intersection of the surface of the earth by planes passing through its axis, as $P_1A_1P_1'$ If the earth were perfectly spherical, they would be great circles through the poles and secondaries to the equator.

Celestial Meridians are corresponding great circles of the celestial sphere passing through the celestial poles The celestial meridian of a place or an observer, referred to at the place as *the* meridian, is that great circle which passes through the poles and the zenith and, necessarily, through the nadir also It is therefore a secondary

of the celestial horizon as well as of the equator, and cuts both the horizon and the equator at right angles PZP′ is the meridian of the observer X.

Vertical Circles are great circles of the celestial sphere through the zenith and nadir They are all secondaries to the celestial horizon. The observer's meridian is one such circle.

The Prime Vertical is that particular vertical circle which is at right angles to the meridian, and which therefore passes through the east and west points of the horizon

The Latitude (ϕ) of a place is the angle between the direction of a plumb line at the place and the plane of the equator It is marked + or − (or N or S) according as the place is north or south of the equator It will be seen from the definitions of the zenith and the celestial equator that latitude may also be defined as the angle between the zenith and the celestial equator The latitude of X is therefore represented by ZOA

The Co-latitude of a place is the complement of the latitude, and is therefore the angle ZOP between the zenith and the celestial pole.

The Longitude (L) of a place is the angle between a fixed reference meridian, called the prime or first meridian, and the meridian of the place Longitude is measured from 0° to 180° eastwards or westwards from the prime meridian, and must therefore be marked E or W The prime meridian universally adopted for astronomical and geodetic work is that of Greenwich $PZ_{Gr}BP'$ being Greenwich meridian, the longitude of X is angle BOA west, measured on the plane of the equator

The Position of a Celestial Body

The fact that the situation of a point on the surface of the earth is completely specified by the co-ordinates latitude and longitude suggests the employment of similar means to designate the position of a body on the celestial sphere In any system of co-ordinates for this purpose, it is necessary to adopt a plane of reference, one of the two angular co-ordinates being measured at right angles to this plane, and the other upon it This plane should pass through the centre of the sphere, and the first co-ordinate is consequently measured upon a secondary to the great circle formed by the intersection of the reference plane with the sphere. The second is expressed as the angular distance between

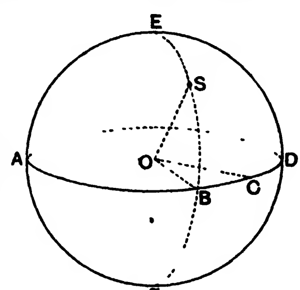


FIG. 4.

a certain specified point on this great circle and the secondary through the body. The radius to the specified point may be regarded as a fixed direction of reference Thus in Fig. 4, let ABCD be the great circle of reference, or its plane, C being the fixed point thereon. The position of the heavenly body S is designated by (1) the angle SOB, or arc SB, of ESBF, the secondary to ABCD through S, (2) the angle COB, on the plane of reference, or the arc CB of the reference circle, between the fixed direction OC and the plane of the secondary through S. To avoid all ambiguity, it is necessary in designating the first co-ordinate to state on which side of the reference plane the body is situated, and to have an established method of measuring the second angle as regards direction.

Co-ordinate Systems.—Several systems of co-ordinates are available according to the particular reference plane and reference point adopted. Three systems are of service in field astronomy: (1) altitude and azimuth; (2) declination and right ascension, (3) declination and hour angle. The horizon is the reference plane in (1), and the equator in (2) and (3).

The Altitude (h) of a celestial body is its angular distance above the horizon, and is measured on the vertical circle passing through the body In Fig 5 angle AOS, or arc AS, represents the altitude of the star S to an observer whose horizon is HAR

Note —It follows from the definition of latitude that the altitude of the pole P equals the latitude of the place of observation

Co-altitude or Zenith Distance (z) is the complement of the altitude, and is therefore the angular distance between the body and the zenith, *i e* angle SOZ, or arc SZ.

The Azimuth (A) of a celestial body is the angle between the observer's meridian and the vertical circle through the body There is no universally accepted manner of reckoning azimuth, both quadrantal and whole circle methods being used, and in the latter the south point of the horizon is most often taken as the origin ZPR being the meridian, and R the north point of the horizon in Fig. 5, the azimuth of S may be expressed as angle ROA, arc RA, or the spherical angle PZS, measured from north towards east, or by angle HOA = 180° + ROA, or the corresponding quantities measured clockwise from south.

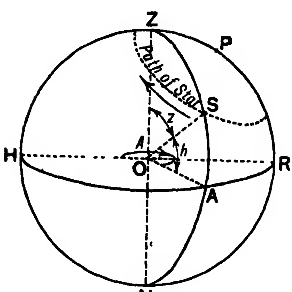


Fig. 5.

The Altitude and Azimuth System.—The co-ordinates altitude and azimuth are much used in field astronomy. The observation of altitude is that most frequently required, and since it is measured, subject to correction, by the angle of elevation as registered on the vertical circle of a theodolite, the observation is a simple one.

Since these co-ordinates not only depend upon the position of the observer, but are constantly changing owing to the diurnal apparent motion of the celestial body about the pole (Fig 5), they are useless as a means of permanently recording the positions of heavenly bodies. For this purpose it is necessary to employ a system of invariable co-ordinates, and the equator must be the reference plane, since a star in its daily motion maintains a practically constant angular distance from the equator The point of reference on the equator must partake of the apparent movement for constancy of the second co-ordinate This is secured in the declination and right ascension system

The Declination (δ) of a celestial body is its angular distance from the plane of the equator, and is measured on the meridian—sometimes called the declination circle—through the body Declination is measured from 0° to 90°, and is marked + or − according as the body is north or south of the equator Thus in Fig 6 the declination of the star S is measured by the angle BOS or the arc BS

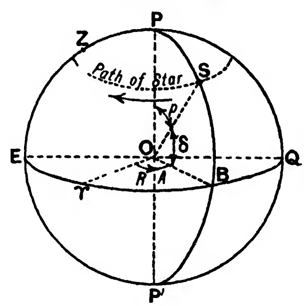


FIG. 6.

Co-declination or Polar Distance (p) is the angular distance between the body and the pole It is termed north polar distance when measured from the north pole, its value being 90° − the declination or 90° + the declination according as the declination is + or − When reference is made to the south pole, the quantity is called south polar distance, and equals 90°—the declination or 90° + the declination according as the latter is − or + The angle SOP, or arc SP, is the north polar distance of S, and angle SOP′, or arc SP′, the south polar distance

The Right Ascension (*R A*) of a celestial body is the angle between the meridian through a fixed point on the equator called the *First Point of Aries*, ♈ (page 15), and the meridian or declination circle of the body It is measured eastwards from ♈, the right ascension of S in Fig 6 being angle ♈OB or arc ♈B of the equator. Right ascension is intimately related to time measurement, and it is found convenient to reckon it in time units from 0 to 24 hours instead of from 0° to 360° (see page 20).

The Declination and Right Ascension System.—The observations of field astronomy do not involve the measurement of declination and right ascension, but these co-ordinates are constantly required in the reduction of observations, and are employed for recording the positions of celestial bodies Their values for a fixed point in the heavens, although nearly constant, are not absolutely so. Owing to the fact that the earth is not quite spherical, the attractions of the sun, moon and planets exert a disturbing couple which causes a slow secular movement of the plane of the equator, called precession In addition, due to variation in the value of this couple, a small periodic disturbance, called nutation, is produced. These phenomena cause a variation in the position not only of the plane of the equator but also of the first point of *Aries*, and in consequence, although the positions of the stars relatively to each other are unaffected, both their declinations and right ascensions undergo slow alteration, increasing in some cases, decreasing in others The fact that the earth's velocity in its orbit is an appreciable fraction of the velocity of light causes a displacement, known as aberration, of star positions from their mean position A further source of variation in the magnitude of the co-ordinates arises from the fact that the stars do not occupy quite fixed situations on the celestial sphere This motion, called proper motion, is partly real, due to an actual change of position of the body in the heavens, and partly apparent, due to the movement of the solar system in space. The correction of star positions is described and illustrated in the *Nautical Almanac* The variation of the declination and right ascension of the sun is very much greater than for the stars, and is treated on page 14

The Declination and Hour Angle System.—The plane of the equator is also used as the reference plane in this system of co-ordinates, the first co-ordinate being declination as defined above The reference direction is that of the observer's meridian, the reference point being that in which the celestial equator is intersected by the meridian.

The Hour Angle (t) of a celestial body is the angle between the meridian of the observer and the meridian or declination circle of the body It is measured westwards from the part of the observer's meridian situated above the pole, and is reckoned in time from 0 to 24 hours. In Fig 6 the hour angle of S is about 16^h, and is represented by angle $EOB = 180° + QOB$, arc EQB, or the corresponding spherical angle ZPS. Owing to the apparent motion of a celestial body, its hour angle is constantly changing, but at a uniform rate.

Circumpolar Stars are those having polar distances less than the latitude of the place of observation Such stars are, at any point

in their diurnal path round the pole, always above the observer's horizon, and therefore do not set

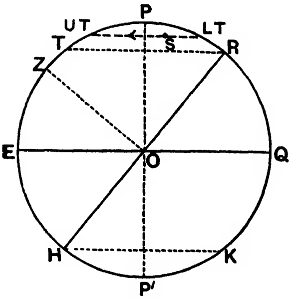


FIG 7

In Fig. 7, the latitude of the observer whose horizon is HOR is angle ZOE =POR Any star such as S, the polar distance of which is less than POR, and which therefore lies within the segment PTR of the celestial sphere, is continually above the horizon, and is circumpolar to the observer Stars in the segment P'HK are never visible to him, while all the others appear to rise and set No stars are circumpolar to a person on the equator, his horizon being POP', while to an observer at the north pole all the stars in the northern celestial hemisphere are circumpolar, since his horizon is EOQ

Transit or Culmination.—When a celestial body crosses the observer's meridian, it is said to transit or culminate In one revolution round the pole every body crosses the meridian twice, the two transits being distinguished as the upper and lower respectively (U T and L T in Figs 7 and 8), the former occurring above, and the latter below the pole Fig 8, which represents the polar region from the point of view of the observer, shows that lower transits can be observed only in the case of circumpolar stars, and that a star attains its greatest altitude at upper transit and its least altitude (or greatest distance below the horizon, if non-circumpolar) at lower transit

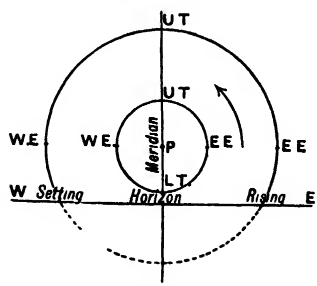


FIG 8

Elongation.—A celestial body is said to elongate or to be at elongation when it appears to attain its maximum distance from the meridian The two elongations, E E. and W E in Fig 8, are distinguished as the eastern and western elongations respectively.

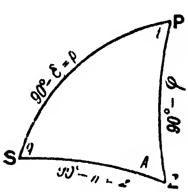


FIG 9

The Astronomical Triangle.—Many of the observations of field astronomy involve the evaluation of such parts of the spherical triangle formed by the pole, the zenith and a celestial body as will enable it to be solved for the quantity under

determination. This triangle PZS (Fig. 9) is called the astronomical triangle. Its three sides are :

PZ (an arc of the observer's meridian) = the co-latitude of the observer
PS (an arc of the declination circle of the body) = the co-declination or polar distance of the body
ZS (an arc of the vertical circle through the body) = the co-altitude or zenith distance of the body.

The angles are :

at P = the hour angle of the body
at Z = its azimuth
at S = the parallactic angle.

Solution of the triangle requires that three of the parts should be known. The side PS is obtained from the declination of the body tabulated in the *Nautical Almanac* Two of the quantities PZ, ZS and t commonly make up the data. Of these PZ can be obtained by independent observations, ZS may be observed, and t is obtained from the readings of a chronometer The unknowns, usually t, A or ZS, are then calculated by means of the formulæ given on page 4 or their equivalents

The Right-Angled Astronomical Triangle.—Two important cases occur according as the parallactic angle or the azimuth is a right angle.

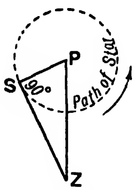


FIG. 10.

In the former case (Fig 10) the celestial body appears to the observer to attain its maximum distance east or west of the meridian, i e the body is at elongation Solution of the triangle for the altitude, azimuth and hour angle of the body in terms of its declination and the latitude of the place gives

$$\sin h = \frac{\sin \phi}{\sin \delta}$$

$$\sin A = \frac{\cos \delta}{\cos \phi}$$

$$\cos t = \frac{\tan \phi}{\tan \delta}$$

When the azimuth is a right angle, the body is on the observer's prime vertical (Fig 11), and

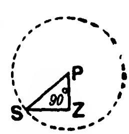


FIG. 11.

$$\sin h = \frac{\sin \delta}{\sin \phi}$$

$$\cos t = \frac{\tan \delta}{\tan \phi}$$

The Classification of the Stars

Nomenclature.—For convenience in distinguishing particular stars, they have been arranged in named groups, called constellations This system of classification originated with the ancients, and the number of constellations has been somewhat extended in modern times, but several of the groupings which have been proposed are not in common use

The stars of each constellation are named by assigning to each a letter of the Greek alphabet, followed by the name of the constellation in the genitive case The letters are assigned roughly in order of brightness Thus α *Tauri* is the most brilliant star belonging to the constellation *Taurus*, the Bull, β *Leonis* is the second brightest in the constellation *Leo*, the Lion, and so on Letters indicating the relative brightness of stars refer only to the constellation to which they belong, and do not serve to compare the brightness of stars in different constellations. Thus η *Ursæ Majoris* is a much brighter star than α *Draconis*. The fainter stars in a constellation are distinguished by numerals increasing in the order of their right ascensions.

A second way of referring to a particular star is to quote its index number in one of the various star catalogues that have been published. Thus Boss 1234 is the star so numbered in the Preliminary General Catalogue of Lewis Boss

Several of the more important stars have been given special names, e g α *Ursæ Minoris* or *Polaris*, the Pole Star, α *Lyræ* or *Vega*; β *Orionis* or *Rigel*, etc A full list of these names will be found in the *Nautical Almanac*, or in Norton's *Star Atlas*

Magnitude.—The quantity termed the magnitude of a star is a measure of its brightness Magnitudes are noted in the *Nautical Almanac* for all the stars of which particulars are given, and are indicated by numbers, the smaller the number, the brighter being the star The brightness of a star of magnitude m is about $2\frac{1}{2}$ times that of a star of magnitude $m+1$ There are 10 stars brighter than magnitude 1·0, and their magnitudes are given as fractions. Thus the magnitude of *Vega* is 0·1, indicating a brightness 0·9 of a magnitude greater than the unit Of those stars, *Sirius* and *Canopus* have their magnitudes represented by the negative quantities $-1{\cdot}6$ and $-0{\cdot}9$, showing that their brilliancy is 2·6 and 1·9 magnitudes greater than a star of unit magnitude.

Since a bright star is easily discovered in the field of view of a telescope, and observations of it can consequently be made with greater convenience than in the case of a faint star, a table of magnitudes is of service in enabling the surveyor to select suitable stars for observation. Under favourable conditions, stars down to

about the sixth magnitude are visible to the naked eye, but, if possible, theodolite observations should be restricted to those that are brighter than the fourth magnitude.

The Sun

The preceding explanations of the apparent positions of celestial bodies have been given more particularly with reference to the stars, which are so remote that the values of their astronomical co-ordinates are unaffected by the orbital motion of the earth. In considering the apparent motion of the sun, however, it must be kept in view that (*a*) its mean distance from the earth is only about 1/250,000 that of the nearest star, (*b*) the earth performs a circuit of the sun yearly The apparent path of the sun in the heavens is therefore the result of both the diurnal and annual real motions of the earth

Effect of Earth's Annual Motion.—The principal facts relating to the earth's motion about the sun and their effects upon the apparent position of the sun are as follows

(1) The earth's orbit lies in a plane The apparent path of the sun is necessarily in the same plane, and, since this plane passes through the centre of the celestial sphere, it intersects the latter in a great circle, called the *Ecliptic* The apparent motion of the sun is along this great circle

(2) The earth's orbit is an ellipse having the sun in one of the foci The earth is thus at varying distances from the sun, being nearest about January 2, when it is said to be in perihelion, the sun being then in perigee, and farthest about July 4, when the earth is said to be in aphelion and the sun in apogee The points of perihelion and aphelion in the earth's orbit, being situated at the ends of the major axis of the ellipse, are termed the apses of the orbit, the line joining them being called the apse line

(3) The plane of the ecliptic is not coincident with that of the equator The angle between them is called the *Obliquity of the Ecliptic*, the value of which at 1935·0 is 23° 26′ 52″ with a mean annual diminution of 0″·47 The axis of the earth is therefore inclined to the plane of the ecliptic at an angle of 90°—the obliquity of the ecliptic, or about 66½°, and remains practically parallel to itself throughout the year

The effect of the obliquity is shown by the diagrammatic plan and sections of the earth's orbit, Figs 12, 13 and 14 On or about March 21 the axis of the earth is perpendicular to the line joining the earth and the sun, and the sun is in the plane of the equator. The instant at which this occurs is called the *Vernal Equinox*, day and night being of equal duration throughout the earth About

June 21 the earth is so situated that its axis is in the same plane as the line joining the earth and the sun, this line now making an angle with the plane of the equator equal to the obliquity of the ecliptic The sun is therefore vertically over a point on the parallel of latitude 23° 27′ N , called the Tropic of Cancer, this instant being termed the *Summer Solstice* The *Autumnal Equinox* occurs about September 23, and the *Winter Solstice* about December 22, the sun being then over the Tropic of Capricorn, or 23° 27′ S. The line of equinoxes is the line of intersection of the planes of the ecliptic and the equator, and is at right angles to the line of solstices.

Apparent Motion of the Sun.—The apparent motion of the sun produced by the annual circuit of the earth may now be traced.

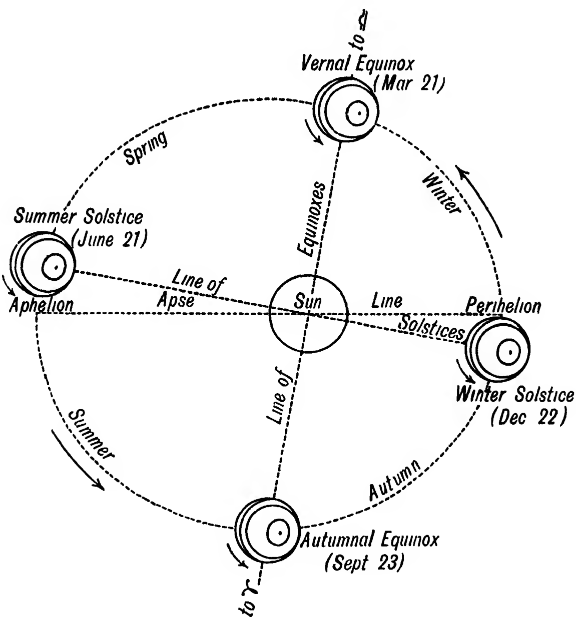


FIG 12 —PLAN OF THE EARTH'S ORBIT

At the vernal equinox the sun is seen from the earth as if projected along the line of equinoxes upon a certain point on the celestial equator. This equinoctial point was at one time in the constellation *Aries*, but, owing to the precession of the equinoxes (page 10) towards the west at the rate of about 50″·3 a year, it is now situated in the constellation *Pisces* It is called the *First Point of Aries* (♈), and has already been referred to as the point from which right

ascensions are reckoned The sun's right ascension at the vernal equinox is therefore zero. The changing position of the earth in its orbit causes the sun to travel apparently eastward along the ecliptic, *i e.* in the opposite direction to the apparent diurnal rotation of the celestial sphere. The latter motion being super-

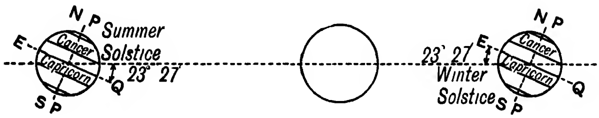


FIG 13 —SECTION ON LINE OF SOLSTICES

imposed, the sun appears to move westward across the heavens daily at a slightly slower rate than the stars Since right ascensions are measured eastwards from ♈, the right ascension of the sun increases, at a variable rate (page 21), by about 1° or 4^m daily, having the values 6^h at the summer solstice, 12^h at the autumnal

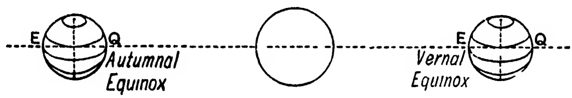


FIG 14 —SECTION ON LINE OF EQUINOXES

equinox, and 18^h at the winter solstice, the return to the vernal equinox completing the annual circuit of the ecliptic

Since the sun crosses the equator at the vernal equinox, its declination is then zero From March 21 until June 21 the sun travels northwards from the equator, attaining its maximum north declination of about 23° 27′ at the summer solstice From June 22 to September 23 the north declination decreases until it is again zero at the autumnal equinox The sun then crosses the equator at the other equinoctial point, called the *First Point of Libra* (♎), and proceeds southwards, its south declination increasing to a maximum of 23° 27′ at the winter solstice on December 22. The south declination thereafter decreases until the equator is again crossed at ♈.

Corrections

In general, the immediate results of astronomical measurements require correction before they can be utilised Excluding corrections which depend entirely upon the instrument and method of observation employed, the more important corrections required in ordinary field astronomy are here described.

Refraction.—Rays of light proceeding from a celestial body to an observer, on entering and passing through the earth's atmosphere, undergo a gradual bending due to the increasing density of the atmosphere as the surface of the earth is approached. In consequence of the observer's being conscious only of the direction in which the rays reach him, the body is seen displaced from its actual position in altitude Thus in Fig 15, the celestial body S appears to the observer A to be situated at S′, and a refraction correction, R=S′AS, must be applied to reduce the observed or apparent altitude h' to the true altitude h The correction is always negative to observed altitudes

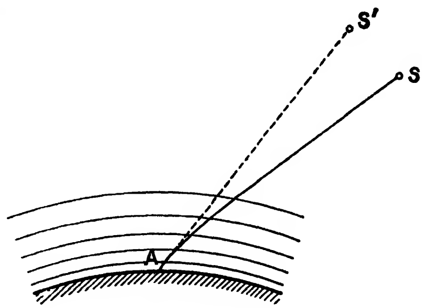


FIG. 15.

Except for low altitudes, the magnitude of the correction is nearly proportional to the cotangent of the apparent altitude, being zero when the body is in the zenith It also depends upon the density of the air through which the rays pass, and this varies with changing barometric pressure and temperature To obtain the value of the correction for a particular observed altitude, barometric pressure and temperature, reference is made to refraction tables, those of Bessel* being most commonly used

In the absence of tables, and neglecting pressure and temperature corrections, refraction is approximately given by

$$R = 58'' \cot h'$$

provided h' is large If the body is very near the zenith, R is approximately 1″ for each degree of zenith distance.

The value of the refraction correction is always somewhat uncertain, since the assumptions made in the calculation of tabulated values are not applicable to all states of the atmosphere With a view to minimising the uncertainty, observations should not be made of stars of low altitude, and should be duplicated in such a way that errors of refraction are more or less balanced out by taking the mean of the resulting values

Parallax.—Parallax is the apparent change in the direction of a body when viewed from different points. The particular case with which we are concerned may be distinguished as parallax in altitude, or diurnal parallax, and relates to the difference between the altitude of a celestial body as observed from the sensible horizon at the sur-

* Given in mathematical tables, such as Chambers's.

face of the earth and its altitude from the true horizon at the earth's centre. Thus in Fig. 16, if A is an observer, and O the centre of the earth, the altitudes of a body S at A and O being respectively h' and h, the parallax of S is $h-h' = \text{ASO} = P$.

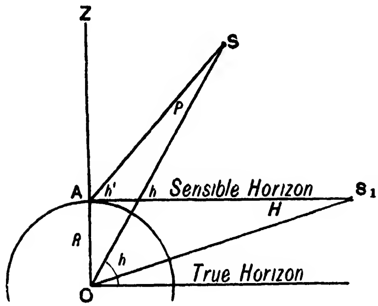


FIG 16

The co-ordinates of celestial bodies are given in the *Nautical Almanac* with reference to the centre of the earth, and altitudes to be used in reductions must also be referred to the centre. Observed altitudes are converted to geocentric altitudes by application of the parallax correction, which is represented by the angle P subtended at the body by the earth's radius, and which is always positive. The value of the correction becomes negligible in the case of the stars because of their remoteness and the comparative insignificance of the length of the earth's radius. In the case of observations of the sun or other bodies of the solar system, however, parallax becomes appreciable, and an appropriate value for the correction must be applied.

Parallax vanishes when the body is in the zenith. It attains its greatest value when the body is on the horizon, as at S_1, and the quantity is then termed the horizontal parallax, H, of the body. On the assumption that the earth is spherical, the amount of the correction may be investigated as follows.

Let D = distance of the body from the earth's centre

R = mean radius of the earth,

then $\sin H = \frac{R}{D}$, whence H.

From triangle OAS, $\frac{\sin P}{\sin(90°+h')} = \frac{R}{D}$

$\therefore \sin P = \sin H \cos h'$

Except in the case of the moon, P and H are sufficiently small to allow of the approximation

$$P = H \cos h'$$

The mean value of the sun's horizontal parallax is $8''\cdot80$. It varies from $8''\cdot95$ early in January to $8''\cdot66$ early in July on account of the sun's varying distance from the earth, and is given in the *Nautical Almanac* for every tenth day of the year.

Semi-diameter.—In taking a single measurement of the sun's altitude, the observation is made upon either the lower or the upper edge or limb. To reduce the measured altitude to that of the

sun's centre, a correction for semi-diameter is applied, positively in the one case and negatively in the other. The amount of the correction is the value of the angle subtended by the sun's radius. It varies throughout the year between limits of about 16′ 18″ and 15′ 45″ owing to the varying distance of the earth from the sun Its value is tabulated in the *Nautical Almanac* for every day of the year

The necessity for applying the correction is obviated if, as is frequently the case, observations are made on both limbs The mean of the two measured altitudes then represents the apparent altitude of the centre at the instant corresponding to the average of the times of the two observations, provided that the interval of time between the observations is small.

Time

A knowledge of some of the aspects of time measurement is required by the surveyor because of its intimate relationship with hour angle, right ascension and longitude.

The rotation of the earth on its axis and its complete revolution in its orbit, being each performed with absolute regularity, afford suitable standards by which to divide up what is known as time or duration, the former interval being the day, the latter the year It will be sufficient to consider the division of time into days, as the measurement of long periods does not enter into field astronomy.

The Day.—The day may be defined as the interval between successive transits of a celestial body in the same direction across the meridian according as the body is a star, the sun or the moon, we have a sidereal, solar or lunar day

Although the rate of rotation of the earth is perfectly uniform, yet the day as measured by transits of the sun or moon is of variable length, since the apparent diurnal motion of those bodies is not simply due to the earth's axial rotation The sidereal day may, however, be regarded as constant It is not strictly so, owing to the existence of precession and nutation, which, by affecting the position in space of the earth's axis, have a minute effect upon the times of transit of the stars. By adopting the interval between the transits of ♈ as the day, we have a unit which for every practical surveying purpose is invariable

Sidereal Time.—The sidereal day is the interval of time between two successive upper transits of the first point of *Aries.*

The sidereal time at upper transit of ♈ is 0^h. A clock rated to keep sidereal time is called a sidereal or astronomical clock. It is set to register 0^h at upper transit of ♈ and 24^h at the succeeding upper transit, and at any instant the reading of the clock is the

sidereal time which has elapsed since ♈ was on the meridian. It therefore follows from the definition of hour angle (page 10), and with the convention that hour angle is measured westwards from the meridian of the observer, that

The sidereal time at any instant is the hour angle of the first point of Aries,

and further, *The hour angle of a star is the sidereal time that has elapsed since its transit.*

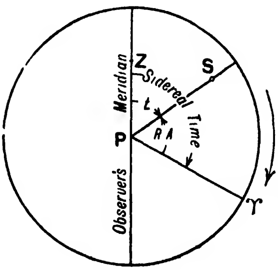


Fig. 17.

Thus in Fig. 17, which represents a plan of the celestial sphere, the hour angle of ♈ is ZP♈ = the sidereal time at the instant represented The hour angle of the star S is ZPS, but the right ascension of this star is ♈PS, right ascension being measured eastwards, and we therefore have the further relationship,

Star's Hour Angle + Star's Right Ascension = the Sidereal Time,

24^h being subtracted when the sum exceeds 24^h.

Note —The reader will find it instructive to make several sketches similar to Fig. 17 with ♈ and S in various positions, and to verify the relationship in each case

At the instant when the star makes its upper transit its hour angle is zero, and the above equation reduces to

Star's Right Ascension = the Sidereal Time of its Transit.

A sidereal clock therefore records the right ascensions of stars as they make their upper transits

It will now be evident why hour angle and right ascension are measured in time in preference to angular units. In the case of the stars, those angles are reckoned in sidereal time units Since a rotation of the celestial sphere through 360° occupies 24^h, one hour is equivalent to 15°, and hours, minutes and seconds are converted to degrees, minutes and seconds by multiplying throughout by 15 Thus, $5^h\ 12^m\ 43^s$ is equivalent to 78° 10′ 45″. Conversely, division by 15 reduces angular measure to time. Conversion tables are given in the *Nautical Almanac.*

The sidereal division of time, although of great importance in astronomy, is not suited to the needs of everyday life, for the purposes of which the sun is the most convenient time measurer. On account of the increase in the sun's right ascension from 0^h to 24^h in the course of the year, the noon transit of the sun is recorded by the sidereal clock as occurring at 0^h on March 21, 6^h on June 21, 12^h on September 23, and 18^h on December 22. The adoption of sidereal time for ordinary purposes would thus lead to endless inconvenience, which is obviated by the use of solar time.

Solar Time.—The interval between successive noon transits of the sun is not constant, two causes operating to produce a variation in the length of the solar day

(1) The apparent diurnal path of the sun differs from those of the stars because it lies in the ecliptic In consequence, even although the eastward progress of the sun in the ecliptic were uniform, the time elapsing between the departure of a meridian from the sun and its return thereto would vary because of the obliquity of the ecliptic

(2) The sun does not advance at a constant rate along the ecliptic, because the motion of the earth in its orbit is not uniform on account of the ellipticity of the orbit

The variation in the solar day may be examined with reference to the sun's right ascension The obliquity of the ecliptic and the sun's unequal motion both cause a variable rate of increase of the sun's right ascension If the yearly change in right ascension from 0^h to 24^h were at a uniform rate, then, since right ascensions are measured on the plane of the equator, the solar day, although different from the sidereal day, would be of constant length throughout the year

Time as reckoned from the transits of the sun is called *Apparent Solar Time* It is the time given by a sun-dial, but, owing to the variable length of the day, cannot be recorded by a clock having a constant rate

To obviate this disadvantage, there has been adopted for common use a system of measurement called *Mean Solar Time*, or simply mean time, in which the day is of invariable length, and which is at the same time free from the disadvantages of sidereal time The mean solar day is the average of all the apparent solar days of the year Mean solar time may be conceived as being determined by an imaginary point, called the mean sun, which moves at a uniform rate along the *equator*, its constant rate of increase of right ascension being the average rate of increase of the sun's right ascension The mean solar day may therefore be defined as the interval between two successive transits of the mean sun.

Civil and Astronomical Reckoning.—In the ordinary reckoning of time the day commences at midnight, and is divided into two periods, *a m* and *p m* Astronomers, however, with a view to avoiding ambiguity, count the mean solar day as beginning at midnight, but divide it continuously from 0^h to 24^h. The former system is called civil time, and the latter astronomical time..

To Convert Civil Time into Astronomical.—If the civil time is *a m.*, make no change ; if *p.m.*, add 12^h.

Thus, June 30, 2 a m is expressed astronomically as June $30^d\ 02^h$.
June 30, 2 p m. ,, ,, June $30^d\ 14^h$.

To Convert Astronomical Time into Civil.—If the astronomical

time is less than 12^h, make no change and mark *a m* , if greater than 12^h, subtract 12^h, marking *p m*

Thus, June $30^d\ 10^h$ is equivalent to June 30, 10 a.m.

June $30^d\ 20^h$,, ,, June 30, 8 p m

It is to be observed that what are called civil and astronomical times are simply systems of reckoning mean solar time *

The Hour Angle and Right Ascension of the Sun.—The relationships between hour angle, right ascension and time which have been given on page 20 for sidereal time are similar in the case of solar time We have, therefore,

At any instant the apparent solar time is the hour angle of the sun $+12^h$, *and the mean solar time is the hour angle of the mean sun* $+12^h$

Sun's Hour Angle + Sun's Right Ascension = the Sidereal Time, and similarly,

Mean Sun's Hour Angle + Mean Sun's Right Ascension = the Sidereal Time

As before, since the hour angle is zero at upper transit,

Sun's Right Ascension = the Sidereal Time of Apparent Noon, and

Mean Sun's Right Ascension = the Sidereal Time of Mean Noon

Interconversion of Time Systems.—The aim of observations for time is usually to determine mean time at any instant, but since the mean sun is merely a convenient hypothesis, mean time cannot be found directly Apparent solar time is obtained from sun observations, while star observations give sidereal time, and it is therefore necessary to be able to deduce the mean time equivalent of the computed result

Relationship between Mean and Apparent Solar Time.—The difference at any instant between apparent and mean time is known as the *Equation of Time* (*E*) This quantity is usually defined as the amount which must be added to or subtracted from apparent time to give mean time It may be expressed as the angle SPM (Fig 18) between the meridian of the true sun and that of the mean sun, and is therefore the difference at any instant between the respective hour angles or right ascensions of the true and mean suns. In particular, at the instant of apparent noon, the sun's hour angle

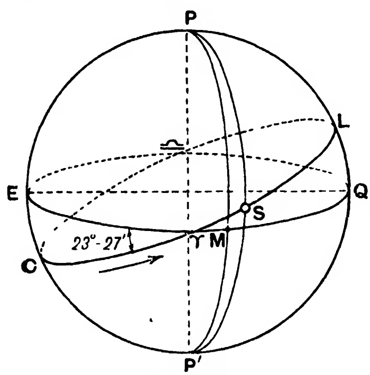


Fig 18.

* Before 1925 January 1, the astronomical day was reckoned from noon, instead of from midnight. For further explanation see the *Nautical Almanac.*

being then zero, the mean time equals the equation of time $+12^h$. Thus if the value of the equation of time at apparent noon on a certain day is $+8^m$, apparent noon occurs at 12 8 p m , the mean sun being ahead of the true sun If the value is -8^m, the sun transits at 11 52 a m.

The amount of the equation of time and its variation are due to the two sources of variation in the length of the apparent solar day, namely, the obliquity of the ecliptic and the sun's unequal motion therein. The difference due to obliquity has a maximum value of about $\pm 10^m$, that due to unequal motion about $\pm 7^m$ The periods of these two components are different, and on combining them we have the equation of time as represented in Fig 19 The true sun and the mean sun are on the same meridian four times

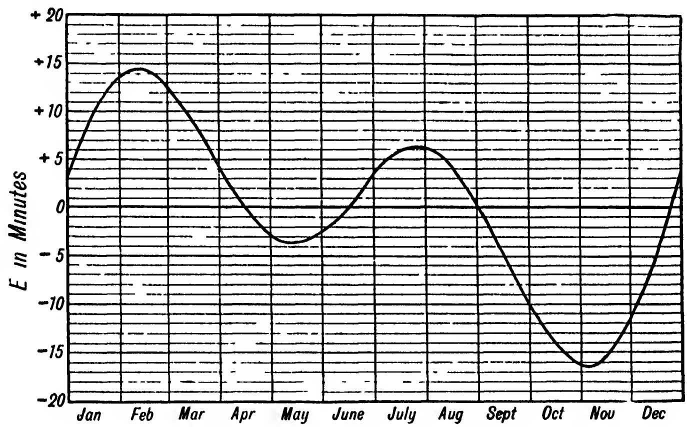


FIG 19 —EQUATION OF TIME

during the year, the equation of time then vanishing This occurs about April 16, June 14, September 1 and December 25 The maximum values attained are roughly $+14^m$ about February 11, -4^m about May 14, $+6^m$ about July 27, and -16^m about November 3

The equation of time is tabulated in the *Nautical Almanac* for every midnight, but in the sense apparent – mean, so that it must be added algebraically to mean time to give apparent time, and *vice versa* The G M T of apparent noon, *i e* the moment when the sun transits at Greenwich, is also given

Relationship between Mean and Sidereal Time Intervals.—The difference between the lengths of the mean and sidereal days is a constant amount, since both are defined by points (the mean sun and ♈) which move at constant rates along the equator The

motion of the mean sun being in an eastward direction, the mean solar day is evidently longer than the sidereal by the daily increase in the right ascension of the mean sun. Now the total increase of 24^h in this quantity is accomplished in the interval from one vernal equinox to the next. This period is called the tropical year, and is equivalent to about 365·2422 mean solar days The daily increase in right ascension of the mean sun, or the amount by which the mean solar day exceeds the sidereal, is therefore

$$\frac{24^h}{365 \cdot 2422} = 3^m\ 56^s \cdot 56 \text{ sidereal.}$$

The matter may be considered in another way In a tropical year the earth actually makes 366·2422 revolutions on its axis, but since it has in that period travelled round the sun, the latter has made one upper transit less than ♈, so that we have the relationships

$$365 \cdot 2422 \text{ mean solar days} = 366 \cdot 2422 \text{ sidereal days}$$

$$\frac{\text{Mean time units in any interval}}{\text{Sidereal time units in the same interval}} = \frac{365 \cdot 2422}{366 \cdot 2422} = \frac{1}{1 \cdot 002738}$$

$$= 0 \cdot 997270$$

$$1 \text{ mean solar day} = 24^h\ 03^m\ 56^s \cdot 56 \text{ sidereal time}$$

$$1 \text{ sidereal day} = 23^h\ 56^m\ 04^s \cdot 09 \text{ mean time}$$

The conversion of mean time intervals to sidereal units, or *vice versa*, is facilitated by the use of the tables of equivalents published in the *Nautical Almanac* as Tables III and IV

It is to be understood that the above relationships refer only to the conversion of time *intervals* from one unit to another, and do not enable one to deduce the mean time at a particular instant from the sidereal time of that instant, or *vice versa* (for which see pages 30 *et seq*).

Connection between Time and Longitude.—Let PA and PB (Fig. 20) be the meridians of two places separated by δL of longitude,

t_A and t_B = the respective hour angles at those places of a time measurer T (♈, the apparent sun, or the mean sun) at any instant

Then $t_A - t_B = \text{APB} = \delta L$.

Difference of longitude, expressed in time units, is therefore equal to difference of local time, whether the time system is sidereal, apparent solar or mean solar.

In particular, in Fig. 21, let PEP′Q be the meridian of Greenwich, S the sun, and M the mean sun It is evidently afternoon at a place X west of Greenwich, the sun having crossed the meridian PAP′ some little time before The local apparent time is AOS′ (the hour angle of the sun)$+12^h$ and AOM$+12^h$ represents local mean

time. Now at Greenwich the sun transited some hours before, Greenwich apparent time being represented by QOS′ + 12^h, and Greenwich

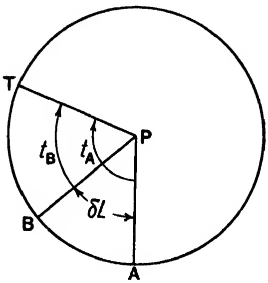


Fig 20

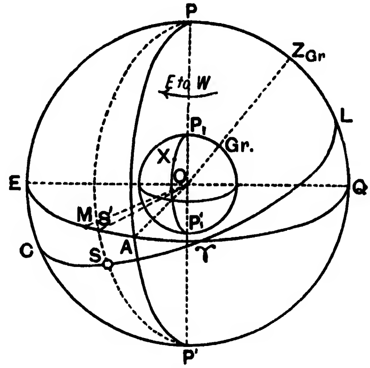


Fig. 21

mean time by QOM + 12^h Therefore, with the convention that west longitudes are positive and east longitudes negative, we have

Greenwich Time − Local Time = Longitude in time

At the instant of local midnight, local time is 0^h, so that

Greenwich Time of Local Midnight = Longitude in time

These relationships are applicable to sidereal, apparent solar or mean solar time The local time referred to, however, is that at the meridian of the place and is not necessarily that adopted for everyday purposes To obviate the inconvenience arising through each place using its own local time, the various countries of the world have adopted *Standard Times*, which are referred to the meridian of Greenwich and in most cases differ from Greenwich time by an integral number of hours Thus Greenwich time itself is used in Great Britain, Ireland, France, Belgium, Spain and Portugal Mid-European time is that of the meridian 1^h E Canada is divided into five time belts, the Atlantic, Eastern, Central, Mountain and Pacific, which are respectively 4^h, 5^h, 6^h, 7^h and 8^h W. the last four of these are adopted in the United States A complete list of the standard times adopted in various countries is given in the *Nautical Almanac*.

THE USE OF THE NAUTICAL ALMANAC

The Nautical Almanac.—The *Nautical Almanac*,* in common with similar foreign publications, contains such particulars of the

* Published annually by the Admiralty two years in advance.

varying positions of celestial bodies as are required for the reduction of observations

Commencing with the issue for 1931, the *Nautical Almanac* was re-designed, and furnished with a very complete explanation It will be assumed, in the remarks that follow, that the reader has a copy before him Certain further slight changes were introduced in 1935, so that the examples will be taken from that year

The values of the published elements are continually changing, and in applying any of the quantities to the reduction of an observation its value at the time of the observation is required It is therefore generally necessary to interpolate the value corresponding to a given instant from the tabulated values Further, the values of the elements are given for certain times on the meridian of Greenwich, and it is usually required to obtain their values for a particular instant of local time at a place east or west of Greenwich It is then necessary to express the given time in terms of Greenwich time before the interpolation can be performed

The manner in which the *Nautical Almanac* is used for the working out of common problems is illustrated below.

Notation.—The following notation* is adopted —

Altitude	h	Greenwich apparent time	G A T
Azimuth	A	Greenwich mean time	G M T
Co-altitude or zenith distance	z	Greenwich sidereal time	G S T.
Co-declination or polar ,,	p	Greenwich apparent noon	G A N
Declination ...	δ	Greenwich mean noon	G M N
Equation of time .	E	Greenwich sidereal noon	G S N
Hour angle . ..	t	Local apparent time	L A T
Latitude	ϕ	Local mean time	L M T
Longitude . .	L	Local sidereal time	L S T
Right ascension . ..	R A	*Nautical Almanac*	*N A.*

The astronomical method of writing dates is year, month, day, hour, etc

1. To Interpolate the Value of a Varying Quantity for a Given Instant of Greenwich Time.

The required value may be obtained (a) by simple or linear interpolation between the successive tabulated values on the assumption that the rate of change is uniform and equal to its value at the middle of the interval, (b) by interpolating strictly, taking higher order differences into account

Let f_{-1}, f_0, f_1, etc be successive values of a function that it is desired to interpolate, and let the first differences between these

* The notation here is that commonly used in surveying, and differs in some instances from that used in the *Nautical Almanac*, and in astronomy generally

values be denoted by Δ', with appropriate suffices, *e g* $f_1 - f_0 = \Delta'_{\frac{1}{2}}$. Similarly, the differences between the first differences, known as the second differences, are denoted by Δ'', with suffices, *e.g.* $\Delta'_{\frac{3}{2}} - \Delta'_{\frac{1}{2}} = \Delta''_1$. These may be represented thus

Function	First Difference	Second Difference
f_{-1}		
	$\Delta'_{-\frac{1}{2}}$	
f_0		Δ''_0
	$\Delta'_{\frac{1}{2}}$	
f_1		Δ''_1
	$\Delta'_{\frac{3}{2}}$	
f_2		

Suppose now we wish to find f_n, *i e* the value of the function at a fraction n of the way between f_0 and f_1 The most suitable formula, if third and higher order differences may be neglected (as they may in all the cases with which we shall be concerned), is Bessel's,

$$f_n = f_0 + n\,\Delta'_{\frac{1}{2}} + \frac{n(n-1)}{4}\,(\Delta''_0 + \Delta''_1)$$

Neglect of the last term is equivalent to simple linear interpolation. The effect of such neglect is a maximum when $n = \frac{1}{2}$, and is then equal to one-eighth of the average second difference Hence the criterion for deciding whether second differences must be taken into account is whether one-eighth of the second difference is negligible

The *Nautical Almanac* tabulates the first difference or Δ' in the case of all rapidly-changing quantities It will be observed that $\Delta''_0 + \Delta''_1 = \Delta'_{\frac{3}{2}} - \Delta'_{-\frac{1}{2}}$, so that we require the first difference between two dates, and also the difference between the following and the preceding first differences The Besselian coefficient $\frac{n(n-1)}{4}$, or B'', is given to three decimals (which are sufficient for all surveying purposes) in Table XVIII This coefficient is always negative Hence the working formula reduces to

$$f_n = f_0 + n\Delta'_{\frac{1}{2}} + B''(\Delta'_{\frac{3}{2}} - \Delta'_{-\frac{1}{2}})$$

Example —To find the value of the sun's declination at 1935 July $4^d\ 11^h\ 50^m$ G M T , from the following data on page 14 of the *N A*

Date	Declination			
	°	′	″	″
1935 July 3	+23	03	36·9	
				−273·8
4	+22	59	03·1	
				−297·9
5	+22	54	05·2	
				−321·8
6	+22	48	43·4	

From Table IX of the *N.A* (pp. 724–5) $11^h\ 50^m = 0^d{\cdot}49306$, hence $n = 0{\cdot}49306$ and B'' (p 746) $= -0{\cdot}062$

			°	′	″
$f_0 =$ Dec at July $4^d\ 0^h$	=	+22		59	03·1
$n\Delta'_{\frac{1}{2}} = 0{\cdot}49306 \times -297''{\cdot}9$	=	−		2	26·9
$B''(\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}) = -0{\cdot}062 \times -48''{\cdot}0$	=	+			3·0
Sum $= f_n =$ Dec. at $4^d\ 11^h\ 50^m$	=	+22		56	39·2

In this case the neglect of second differences would have caused an error of 3″, which is about the maximum error that could occur with the sun's declination. The maximum error with the right ascension or equation of time is $0^s{\cdot}1$.

2. To find the Greenwich Time corresponding to a Given Instant of Local Time.

Rule—To the given local time, expressed in astronomical reckoning, apply the longitude in time, adding if west and subtracting if east. The resulting Greenwich time is on the same system, apparent, mean or sidereal, as the local time

Example—Find G A T corresponding to L A T April 24 (*a*) 12 15 p m, (*b*) 5 23 p m at a place in longitude 32° 15′ E

			h	m
(*a*) Given L A T, expressed astronomically	=	Apr 24	12	15
East longitude in time	=	−	2	09
Corresponding G A T	=	Apr 24	10	06
(*b*) Given L A T, expressed astronomically	=	Apr. 24	17	23
East longitude in time	=	−	2	09
Corresponding G A T	=	Apr 24	15	14

3. To find the Local Mean Time corresponding to a Given Instant of Local Apparent Time.

In this case it is required to find the value of E, the equation of time, at the given instant As the time given is apparent, we utilise the G M.T of transit of the sun at Greenwich (pp 22–29), *i.e* the G M T of apparent noon, as the difference between this time and 12^h is the equation of time at apparent noon

Rule.—Find G A.T corresponding to the given L A.T Interpolate the value of G M T. of transit at Greenwich to this G A.T. Then

$$\text{L.M.T.} = \text{L.A.T.} + \text{G.M.T. of transit at Greenwich} - 12^h$$

Example.—A time observation on 1935 Aug. 13, at a place in longitude 64° 45′ W., gave L.A.T. as $17^h\ 34^m$ Find the corresponding L.M T.

			d	h	m
Given L.A.T.	=	Aug.	13	17	34
West longitude	=			+4	19
G.A.T.	=		13	21	53
	=	$13^d\ 12^h + 9^h\ 53^m$			
	=	$13^d\ 12^h + 0^d{\cdot}4118$			

		h	m	s
From *N.A.*, p. 26, G.M.T. at G.A.T. 13^d 12^h	=	12	04	53·20
0·4118 × −10^s·58	=		−	4·36
−0·061 × −1^s·11	=		+	0·07
Sum = G M T at G.A.T. 13^d 21^h 53^m	=	12	04	48·91
L.A.T.	=	17	34	00·00
Sum − 12^h = L M T	=	17	38	48·91

4. To find the Local Apparent Time corresponding to a Given Instant of Local Mean Time.

Rule—Interpolate the value of *E* from those given for mean midnight, and add the value so obtained algebraically to the L.M.T.

Example—Find L A T corresponding to L M T 1935 Feb 25^d 21^h 06^m at a place in longitude 64° 45′ W

			d	h	m
Given L M T	=	Feb	25	21	06
West longitude	=			+ 4	19
G M.T.	=	Feb	26	01	25
	=	Feb	26^d·0590		

		m	s
From *N.A.*, p 8, *E* for Feb. 26^d 0^h	=	−13	13·10
0·0590 × +9^s·77	=	+	0·58
−0·014 × +1^s·11	=	−	0·02
Sum = *E* for Feb 26^d 01^h 25^m	=	−13	12·54

			d	h	m	s
Given L M T	=	Feb	25	21	06	00·00
E (Apparent − Mean)	=			−	13	12·54
Sum = Required L A T	=	Feb.	25	20	52	47·46

5. To find the Sun's Right Ascension and Declination at any Instant of Local Time.

Rule—First obtain the corresponding G T If this is G.M T interpolate the values given for every midnight. If the time obtained is G.A.T , interpolate the values at transit at Greenwich (pp. 22–29), which are, of course, for G A.T. = 12^h.

Example—Find the sun's R A. on 1935 Nov. 1^d 16^h L A T at a place in longitude 87° 15′ E.

			d	h	m
Given L A.T	=	Nov.	1	16	00
East longitude	=			− 5	49
G A T.	=	Nov.	1	10	11
	=	Oct. 31^d 12^h + 22^h 11^m			
	=	Oct. 31^d 12^h + 0^d·92431			

		h	m	s
From *N A*, p 28, R A. at G.A T. 31^d 12^h	=	14	19	06·98
0·92431 × 234^s·15	=		+ 3	36·43
−0·017 × +1^s·55	=		−	0·03
Sum = R.A. at G A.T. Nov. 1^d 10^h 11^m	=	14	22	43·38

6. To find the Local Mean Time of Local Apparent Noon.

Rule.—Interpolate the G M T. of transit at Greenwich to the given longitude

Example —Find the approximate L M T of L A N in longitude 5^h 42^m East on 1935 April 25

		h	m	s
From *N A*, p. 24, G M T of transit at Greenwich	=	11	58	03
$-0{\cdot}24 \times -11^s{\cdot}1$	-		+	3
Sum = L M T of L A N	--	11	58	06

7. To find the Local Mean Time of Local Sidereal 0^h.

Rule —Correct the G M T of transit of first point of Aries for the date at the rate of 9^s 8296 for every hour of longitude, adding if east and subtracting if west This is most conveniently done with the aid of Table IV.

Example —Find the L M T of local sidereal 0^h on 1935 Dec 7 at a place in longitude 4^h 23^m E

		h	m	s
From *N A*, p 21, Transit of Aries, Dec 7		18	57	32·04
Correction for 4^h East (p 719)	--		+	39·32
Correction for 23^m East (p 719)	-		+	3 77
Sum = L M T of local sidereal 0^h	-	18	58	15·13

8. To find the Local Mean Time corresponding to a Given Instant of Local Sidereal Time.

Rule —Find the L M T of the preceding local sidereal 0^h, as in the previous example, and to it add the mean time equivalent (from Table IV) of the given sidereal time

Example —Find the L.M T corresponding to 1935 Dec 8^d 13^h 08^m $54^s{\cdot}2$ L.S T at a place in longitude 4^h 23^m East

		d	h	m	s
From *Example* 8, L M T of preceding L S 0^h		7	18	58	15·13
Mean time equivalent (p 719) of 13^h			12	57	52·22
08^m	–			7	58·69
54^s	=				53·85
$0^s{\cdot}2$					·20
Sum = L M T.	=	8	08	05	00·1

9. To find the Local Sidereal Time Corresponding to a Given Instant of Local Mean Time.

Rule —Correct the G.S T at 0^h at the rate of $9^s{\cdot}8565$ for every hour of longitude, adding if west and subtracting if east, and to this add the sidereal equivalent of the given L.M.T.

Example.—Find the L S T corresponding to 1935 Dec. 8^d 08^h 05^m $00^s{\cdot}1$ L.M.T., at a place in longitude 4^h 23^m East.

			h	m	s
From *N A*, p 20, G S T at 0^h		=	5	03	17·64
Correction for 4^h East (p 718)		-		—	39·43
Correction for 23^m East (p 718)		=		—	3·78
Sidereal equivalent (p 718) of 8^h		=	8	01	18·85
	5^m	=		5	00·82
	$0^s·1$	=			0·10
Sum = L S T		=	13	08	54·2

10. To find the Local Mean Time of Transit of a Star.

Rule —Since the R A of the star equals the L S T at the instant of its upper transit, convert this L S T to L M T, as in *Example* 8. The lower transit is separated by 12 sidereal hours from the upper.

Note —For the purpose of selecting stars for observation, a rough estimate of the time of transit is usually all that is required It is then sufficient to subtract from the R A of the star, increased if necessary by 24^h, the G S T at 0^h for the date

Example —Find the approximate L M T at which *β Leonis* will be on the meridian on 1935 April 11

	h	m
From *N A*, p 301, R A of *β Leonis*	11	46
From *N A*, p 10, G S T at 0^h	13	13
Difference = L M T of transit	22	33

11. To find the Local Sidereal Time of Elongation of a Star.

Rule —To the R A of the star apply its hour angle at elongation (page 12), in time, adding for west and subtracting for east elongation The result, increased or diminished by 24^h if necessary, represents the L S T of elongation

Example —Find the L S T at which *β Ursæ Minoris* will elongate on the evening of 1935 July 27, at a place in latitude 55° 52′ N.

			h	m	s
From *N A*, p 456, R A of *β Ursæ Minoris*	=		14	50	53·7
δ	=	+74° 25′ 21″			
At elongation $\cos t$	=	$\dfrac{\tan \phi}{\tan \delta}$			
log tan 55° 52′	=	0·168835			
log tan 74° 25′ 21″	=	0·554736			
log cos t	=	9·614099			
t	=	65° 43′ 01″	4	22	52·1
L S T of W		elongation	19	13	45·8

As the sidereal time at 0^h is approximately 20^h 15^m (*N A*, p 14), it is evident that the western elongation is the one that occurs in the evening

Modifications when Standard Time is Used.—It is, generally speaking, more convenient to keep the mean time clock on the standard time used in the country of observation, although the

apparent and sidereal times required will always be local. Conversion from one system of time to another is then done by converting the given time to the corresponding Greenwich time, as in paragraph 2, then converting to the required system of time at Greenwich, and finally reducing the time so obtained to the desired local or standard time. The principal conversions dealt with may be summarised, in the new form, as follows

3a. To find the Standard Mean Time corresponding to a Given Instant of Local Apparent Time.

G.A T. = L A T + west longitude
= L A T − east longitude
G M T = G A T − 12^h + G M T of transit at Greenwich, interpolated from apparent noon to the given G A T.
Standard mean time = G M T − west longitude of standard meridian
= G M T + east longitude of standard meridian

4a. To find the Local Apparent Time corresponding to a Given Instant of Standard Mean Time.

G M T = Standard mean time + west longitude of standard meridian
= Standard mean time − east longitude of standard meridian
G A T = G M T + Equation of time (Apparent − Mean), interpolated from 0^h to the given G M T
L.A T = G A T − west longitude
= G A T. + east longitude.

8a. To find the Standard Mean Time corresponding to a Given Instant of Local Sidereal Time.

G S T. = L S T + west longitude
= L S T. − east longitude
G.M T. = Mean time of the preceding transit of the first point of Aries + mean time equivalent (Table IV) of given G S T
Standard mean time = G M T − west longitude of standard meridian
= G.M.T. + east longitude of standard meridian.

9a. To find the Local Sidereal Time Corresponding to a Given Instant of Standard Mean Time.

G M T. = Standard mean time + west longitude of standard meridian
= Standard mean time − east longitude of standard meridian
G.S.T. = Sidereal time at 0^h + sidereal equivalent (Table III) of given G M T.
L S T. = G.S.T − west longitude
= G S T. + east longitude.

EXAMPLES

1 Compute the value of the sun's declination for the instant on the morning of 1935 Oct 28 at which its hour angle is 50° at a place in longitude 50° E The sun's declination at transit at Greenwich is

		°	′	″	″
1935 Oct	26	−12	13	04·3	
					−1232·8
	27	−12	33	37·1	
					−1221·2
	28	−12	53	58·3	
					−1209·1
	29	−13	14	07·4	

2 Find the true altitude of the sun's centre from an observation which gave an apparent altitude of 55° 34′ 23″ to the sun's upper limb Take the sun's horizontal parallax as 9″ and semi-diameter as 15′ 53″

3 Find the standard mean time of L A N on 1935 July 21, at a place in New South Wales in longitude 142° 30′ E Standard time in New South Wales is 10^h fast on Greenwich

Date		G M T of Transit at Greenwich			
		h	m	s	s
1935 July	19	12	06	04·24	
					+4·11
	20	12	06	08·35	
					+3·58
	21	12	06	11·93	
					+3·05
	22	12	06	14·98	

4 The Greenwich sidereal time at Greenwich mean midnight on a particular day is found from the *N A* to be $7^h\ 20^m\ 35^s$ An observation of a star is taken in longitude 2° west at local sidereal time $17^h\ 30^m\ 50^s$ The correction of S T for longitude is $9^s{\cdot}86$ per hour Find the local mean time at instant of observation

366·2422 sidereal days = 365·2422 mean solar days (Univ of Lond , 1918)

5 At what standard time does ♈ make its upper transit on 1935 Nov 4, at a place in longitude 122° 15′ W in the Pacific time belt (8^h W) ?

G M T of G S N , Nov 4 = $21^h\ 07^m\ 17^s{\cdot}1$

6 Calculate to the nearest second the Indian standard mean time (corresponding to the meridian $5^h\ 30^m$ E) of transit of *β Draconis* (R A $=17^h\ 29^m\ 01^s{\cdot}0$) on 1935 July 2, at a place in longitude 84° 30′ E , given that G S T at 0^h on July $2=18^h\ 36^m\ 25^s{\cdot}4$

7 From the following data calculate the Greenwich mean time of transit of the star A at the place B

	h	m
Right ascension of star A	10	00
Sidereal time of mean midnight at Greenwich	22	00
Longitude of place B		8 W

8. Find, to the nearest second, at what Eastern standard time (corresponding to the meridian 5^h W of Greenwich) γ *Cassiopeiæ* ($\delta = +60° \ 22' \ 09''$, R A $= 0^h \ 52^m \ 51^s{\cdot}3$) elongates on the evening of 1935 Aug 31, at a place in latitude 50° N. and longitude 70° W, and state whether the elongation is eastern or western. The transit of the first point of Aries at Greenwich on Aug. 31 is at $1^h \ 26^m \ 47^s$

9 At what L S.T is β *Ceti* ($\delta = -18° \ 24' \ 47''$, R A $= 0^h \ 39^m \ 40^s{\cdot}5$) on the prime vertical at a place in latitude 22° 32′ S, and what is then its altitude ?

10 At what L S T does α *Bootis* ($\delta = +19° \ 35' \ 35''$, R A $= 14^h \ 12^m \ 05^s{\cdot}7$) attain an altitude of 60° on the east side of the meridian at a place in latitude 32° 17′ N ?

CHAPTER II

FIELD ASTRONOMY—OBSERVATIONS

THE quantities to be obtained by the observations of field astronomy are time, azimuth, latitude and longitude Each can be determined in several ways, and the selection of a suitable method is based chiefly upon the instrumental means available and the degree of precision required

The observations to be described do not exhaust those available for the different determinations The principal methods are given, including those employed for the most refined field determinations as required in geodetic survey It is impossible to state definitely the probable accuracy to be expected from the different methods of determination as it depends very largely upon the capability of the instrument used

The data required in the reduction of observations include, in many of the methods, quantities which necessitate astronomical observation for their evaluation Thus observations for time are made in connection with determinations of azimuth, latitude and longitude In some cases the same observation yields more than one quantity For determinations of low grade it is satisfactory to make one observation serve a double purpose, but in deliberate work each unknown should be observed for independently.

Astronomical and Geodetic Positions.—From the definitions of time, azimuth, latitude and longitude it is evident that the values of those quantities are influenced by the direction of the vertical or the plumb line at the place The results of astronomical determinations therefore include the effect of local deviations of the vertical caused by the irregular distribution of mass in the earth's crust The amount of local deflection cannot be directly measured. It is deduced by comparison of the astronomical position obtained by observation and the geodetic position computed with reference to the spheroid which best represents the form of the whole earth or a particular part of it (page 182) The discrepancies are of great importance in geodetic investigations Although their value is usually very small, it sometimes exceeds 30″, and the possibility

of abnormal deflections must be recognised in applying astronomical checks to geographical surveys.

INSTRUMENTS

The theodolite is the most generally useful instrument for the observations of field astronomy, and all sizes from 3 in to 12 in. are employed for the purpose. For determinations in connection with geographical mapping, the 5-in or 6-in micrometer instrument gives sufficiently accurate results, and is that most commonly used. The astronomical or nautical sextant may be employed for the measurement of altitude For primary determinations, as required in geodetic surveying, the principal instruments used are the portable transit instrument for time, the geodetic theodolite for azimuth, and the zenith telescope, or the theodolite with eyepiece micrometer, for latitude observations

The Theodolite.—The modern large theodolite, described on page 138, is usually entirely suitable for astronomical work, but certain types of engineer's transit theodolite are imperfectly adapted for the measurement of altitude The provision of micrometer reading of the circles instead of verniers is very desirable, but of even greater importance is the necessity for the mounting of a sensitive spirit level on the frame carrying the vertical circle micrometers or verniers This altitude level, being independent of the inclination of the telescope, defines the horizon throughout the observation, and serves to show whether the instrument remains stable Its sensitiveness should be about 5″ per 1·5 or 2 mm division for a 5-in or 6-in theodolite, and must be carefully determined so that altitudes observed with the bubble off centre may be corrected. The use of a striding level to ensure horizontality of the horizontal axis is also necessary

Accessory parts required are . (*a*) means for illuminating the field of view , (*b*) a diagonal eyepiece , (*c*) sight vanes fitted on the upper and lower sides of the telescope tube to give a line of sight parallel to that of the telescope to facilitate pointing to a star ; (*d*) a dark glass to fit on the eyepiece when observing the sun.

For star observations it is necessary to illuminate the field sufficiently to enable the cross-hairs to be seen This is usually effected by having the trunnion axis hollow and attaching a small lamp to one of the standards The light is projected along the axis, and is reflected by a very small mirror in the telescope Alternatively, a paper reflector may be attached in front of the objective. It should be bent over the lens and be provided with an opening to enable the light from the star to enter the telescope. A lamp

is held or placed so that just sufficient light is reflected down the tube.

The diagonal eyepiece (Vol I, page 24) is used when the altitude exceeds about 45°. The proportions of the theodolite must be such that the telescope can be transited without having to disturb the focus of the eyepiece In observing the sun with the diagonal eyepiece, it should be remembered that the prism inverts the image, so that with the usual Ramsden type the sun appears right side up but inverted in azimuth

The Portable Transit Instrument.—This instrument (Fig 22) is an adaptation of the transit circle used in observatories, but is of much smaller size It is set in the plane of the meridian for observing star transits for time and longitude determinations, and is occasionally utilised for the observation of azimuth, the referring object (page 69) being placed sufficiently near the plane of the meridian to be within the range of the eyepiece micrometer The instrument may also be fitted for latitude observations by the Talcott method (page 79)

Various sizes are used, that illustrated being suitable for primary determinations In the larger instruments, the telescope objective has a focal length of 36 to 45 in and an aperture of about 3 in The powers of the various eyepieces provided range up to over 100 diameters The telescope is mounted on a rigid axis resting on wye supports as in the theodolite, and provision is made for reversing the axis without lifting the telescope by hand A sensitive striding level is essential The vertical circles are of small diameter, and are used only for setting the telescope approximately at any required altitude The instrument must have a very stable support, the best form being a masonry pier well founded and insulated from vibrations by having a narrow surrounding air space below the ground

In a design of portable transit instrument, known as the broken-telescope transit, which is used on the Continent and in America, the light which passes through the object glass is reflected through a right angle by means of a prism placed in the trunnion axis This hollow axis is continued beyond the supports, and the eyepiece is fitted at one end A small electric lamp for the illumination of the field is carried at the other end of the axis With this optical arrangement the standards can be made much lower than in the ordinary pattern, so that the instrument is very compact A hanging level is used in place of a striding level, but in its other features, including its adaptability for latitude observations, the instrument does not differ essentially from that illustrated

A transit is observed by taking the time at which the star crosses each of several vertical hairs forming the reticule When the times are taken by the eye and ear method (page 58), no more

than five hairs can be used in order that the intervals between successive passages may permit of the times being booked. When the instants are registered by chronograph, eleven or more passages

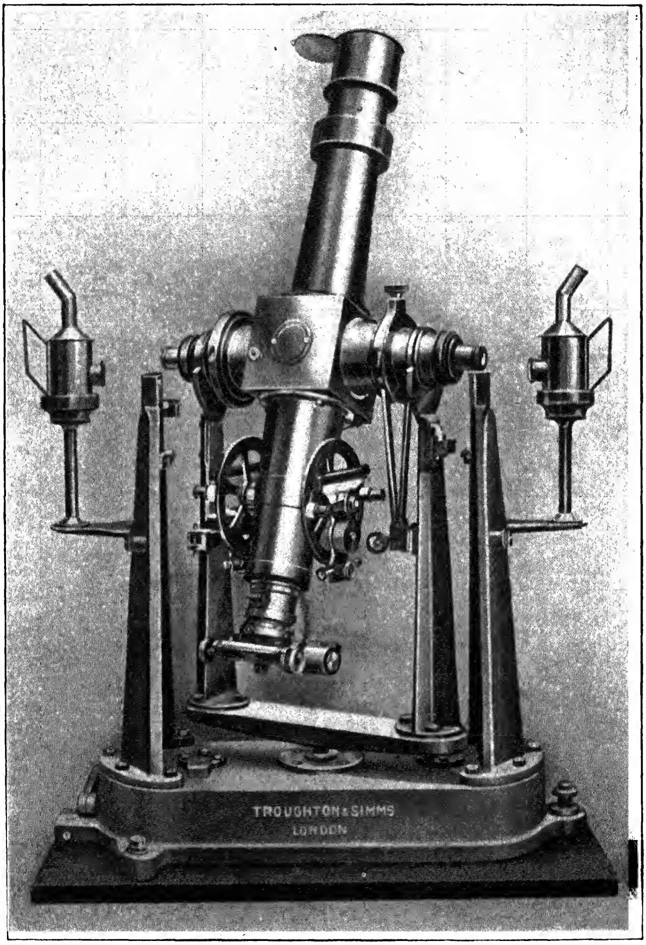


FIG. 22.—PORTABLE TRANSIT INSTRUMENT.

are recorded, and the influence of accidental errors of observation on the mean result is consequently reduced.

Still better results are obtained by the use of the transit micrometer, fitted with its movable vertical hair in the focal plane of the telescope. The micrometer drum carries five contact points, which

make an electric circuit as they pass a fixed contact spring, and the instants of the contacts are registered on the chronograph sheet. In observing a transit it is only necessary, as the star crosses the field of view, to keep it continuously bisected by the movable hair. Two milled heads are provided for actuating the micrometer screw so that, by using both hands, a steady motion may be imparted to the hair. An automatic cut-out is fitted, and no record is transmitted except while the star is traversing the middle part of the field defined by two fixed vertical hairs. Four complete revolutions of the micrometer screw are required to carry the hair across this space, so that twenty contacts are made and registered on the chronograph sheet.

The Chronograph.—The chronograph (Fig. 23) is an instrument which reproduces the time record of a chronometer in graphical form. A sheet of paper to receive the record is wrapped round a cylinder

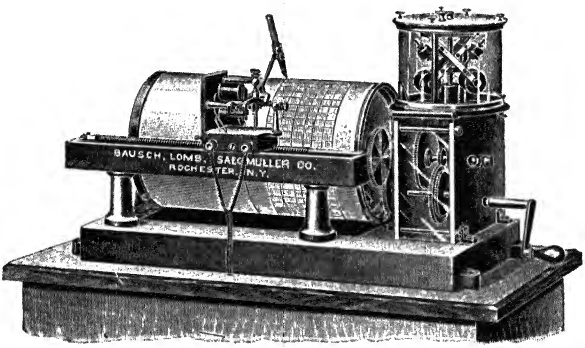


FIG. 23.—CHRONOGRAPH.

which is rotated by the action of a descending weight at the uniform rate of either one or two revolutions a minute. The speed is controlled by a governor. The pen carriage is mounted on a screw, which is also rotated by the descending weight, so that the pen moves uniformly along the cylinder. A helix is therefore traced upon the paper, but by means of an electrical connection with a chronometer a sharp break in the line is made every second by the chronometer automatically breaking the circuit. Whole minutes are usually recorded by the absence of the regular mark at the fifty-ninth second. The chronometer may be arranged to break the circuit at the even seconds only, and the whole minutes are then indicated by an additional mark at the fifty-ninth second.

For taking transits with the aid of the chronograph, there is provided in the chronograph circuit a key or button, which the observer

holds in his hand and depresses at each passage of the star across a hair. The circuit is thereby broken, and the instants are recorded by the pen in the same manner as the breaks made by the chronometer. The positions of these additional marks relatively to the

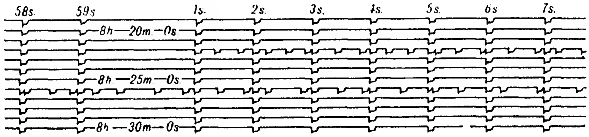


FIG. 24.—CHRONOGRAPH RECORD.

second or two-second marks can be scaled with considerable precision, since the speed of rotation of the cylinder is sufficiently steady that it may be assumed uniform between adjacent chronometer marks. When a transit micrometer is used, the chronograph sheet is automatically marked at the instants at which the movable hair, bisecting the star, reaches the positions corresponding to contacts on the micrometer head (Fig. 24).

In a compact form of the instrument, known as the tape chronograph (Fig. 24A), the record is received upon a continuous strip of

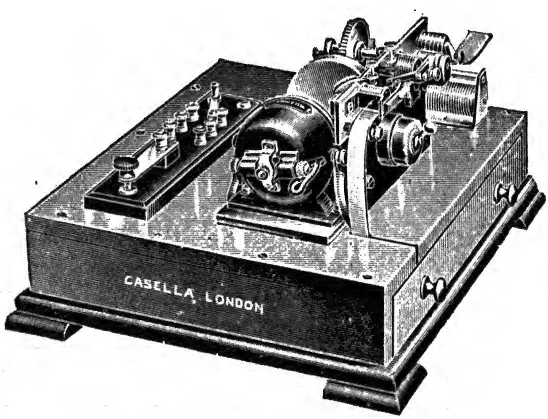


FIG. 24A.—TAPE CHRONOGRAPH.

paper which is fed through at a uniform rate. The instrument illustrated is operated by a small electric motor, and the speed is regulated by means of a rheostat. The record of the micrometer contacts is made alongside that of the chronometer seconds instead of being superimposed upon it. In some instruments the armatures carry needle points instead of pens, and a series of fine punctures forms the record.

The Zenith Telescope.—This instrument (Fig. 25) is used for precise determinations of latitude by the Talcott method of measuring small differences of meridian zenith distance of pairs of stars (page 79). The telescope is similar in size to that of the portable transit instrument, and is mounted either centrally on, or at one end of, a short horizontal axis levelled by a striding level. An

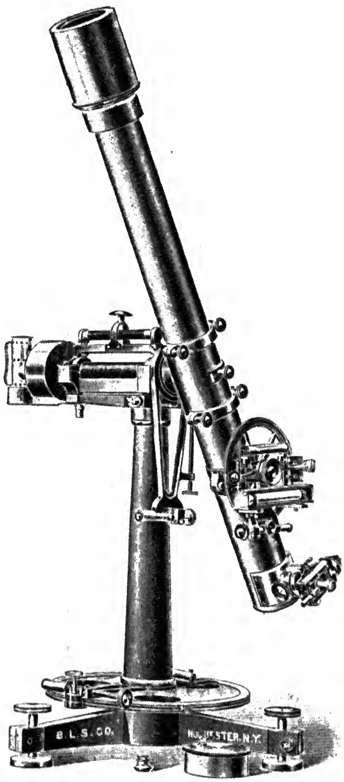


FIG. 25.—ZENITH TELESCOPE.

eyepiece micrometer, with movable horizontal hair, is an essential feature. The value of one division of the micrometer head is about $0''{\cdot}5$, and the range of the movable hair about $20'$. The vertical axis, carrying the supports for the horizontal axis, is made long to ensure a true motion of the telescope in azimuth, its deviation from true verticality being measured by means of one or two chambered latitude levels attached to the telescope and having a sensitiveness of about $0''{\cdot}8$ per mm. The vertical circle or arc is used simply

for setting the telescope approximately to any required altitude. The horizontal circle enables the line of sight to be set approximately in the meridian, and azimuth stops are provided for clamping on to the circle so that the telescope may be quickly swung through 180° from a north to a south star. The instrument support must be very stable, and should preferably be of masonry.

The Astronomical or Nautical Sextant.—The framework of this instrument (Fig. 26) is a gun-metal casting, the curved limb of which carries the graduated arc *1*. At the centre of curvature there is fitted an axis about which the index arm *2* rotates. The latter is provided with a vernier reading against the arc, and is

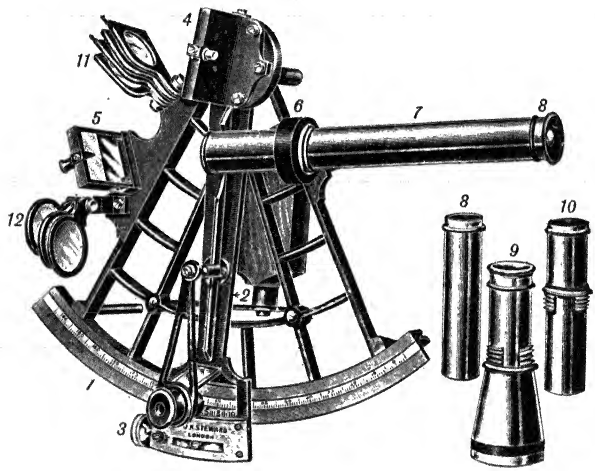


FIG. 26.—ASTRONOMICAL SEXTANT.

controlled by a clamp and tangent screw *3*. It also carries the index glass *4*, the vertical axis of which is collinear with the axis of rotation of the arm. This glass is wholly silvered, and is fixed in a metal tray in a manner permitting its adjustment perpendicular to the plane of the arc. The horizon glass *5*, mounted on the frame, is silvered on the lower half and of plain glass on the upper half.

The arc has a radius of from 6 to 8 in., and is graduated to 20′ or 10′ according to the size of the instrument. In accordance with the principle of the sextant (Vol. I, page 53), the divisions are figured at twice their actual value so that readings may represent the angles measured. Subdivision is carried to 20″ or 10″ by the vernier, which is usually of the extended type (Vol. I, page 39). The graduation of the arc is continued for a few degrees beyond the zero to form an arc of excess (see index error, page 44).

The telescope is screwed into the collar *6*, formed on a short pillar, called the up-and-down piece This can be raised or lowered by a screw to vary the position of the line of sight relatively to the upper edge of the silvered part of the horizon glass and so equalise the brightness of the direct and reflected images Two telescopes accompany the instrument—the long or inverting telescope *7* and the star telescope *9* The former is generally provided with two eyepieces *8*, of different powers The reticule usually consists of four lines forming a square in the centre of the field The star telescope is of low power and wide field, and is convenient in identifying stars and for terrestrial observations In addition a plain tube *10*, with a pin-hole sight, is provided, but its utility is chiefly confined to terrestrial observations and to preliminary sights in astronomical work Two sets of coloured shades, *11* and *12*, are fitted on hinges so that they may be brought into action when observing the sun When it is desired to reduce the brightness of the direct and reflected images equally, as when making a solar observation by artificial horizon, a dark glass is fixed on the telescope eyepiece

Testing and Adjustment of Sextant.—The geometrical relationships of the sextant which can be established by adjustment are

(1) The index glass should be perpendicular to the plane of the arc

(2) The horizon glass should be perpendicular to the plane of the arc

(3) The line of sight should be parallel to the plane of the arc

(4) The mirrors should be parallel to each other when the index reads zero

The following further requirements influencing the accuracy of the instrument are not adjustable, but the errors arising from their non-fulfilment can be ascertained and allowed for

(5) The axis of rotation of the index arm should pass through the centre of graduation of the arc

(6) The glasses forming the mirrors and shades should have their faces accurately parallel.

(7) The graduation of the arc should be uniform.

1. Adjustment of Index Glass.—*Object* —To set the index glass perpendicular to the plane of the arc

Test.—(1) Clamp the index arm near the middle of the arc.

(2) Place the eye just above the plane of the arc and near the index glass. The part of the arc seen by reflection in the index glass should appear continuous with the arc itself.

Adjustment.—If the reflected part appears to rise (fall) from the arc, the glass is leaning forward (backward) Turn the adjusting screw at the back of the glass in the direction thus indicated until the test is passed

2. Adjustment of Horizon Glass.—*Object* —To set the horizon glass perpendicular to the plane of the arc

Test —(1) Point the telescope on a star

(2) Move the index arm to either side of the zero This should cause the reflected image to pass exactly over the direct image

Adjustment.—Turn the adjusting screw at the top of the horizon glass until the test is fulfilled

Note —Although a star forms the best object for sighting, the test may be performed by observing the sun If the instrument is used for terrestrial observations, as in the case of the sounding sextant (Vol I, page 433), it is sufficient to sight any distant straight line, making the direct and reflected images appear continuous On tilting the instrument, the continuity should be maintained

3. Adjustment of Telescope.—*Object* —To make the line of sight parallel to the plane of the arc

Test —(1) Fit the inverting telescope, and turn the eyepiece until two of the wires are approximately parallel to the plane of the arc

(2) Sight one star directly, and bring the reflected image of another, not less than 90° distant, into coincidence on one of the wires.

(3) Move the instrument until the images appear on the second parallel wire The contact should remain perfect

Adjustment —Alter the inclination of the telescope by the opposing screws controlling the collar

Note —A simple alternative test can be performed indoors as follows. Set the sextant on its legs on a table Place on the arc two small objects of exactly equal height to serve as temporary sights Sight along them, and mark where the line of sight meets a vertical surface at least 20 ft distant. Now sight through the telescope, and note how far the telescope line of sight strikes above or below the mark This difference should be the same as the difference between the height of the temporary sights and the vertical distance between the centre of the telescope and the plane of the arc

4. Index Error.—*Object* —To ascertain the reading of the vernier index when the index glass is parallel to the horizon glass

The position of the index when the mirrors are parallel, *i e* when the direct and reflected images of a very distant object are coincident, is the true zero from which angles are measured on the arc If this does not coincide with the zero of the graduations, the

difference is the index error, positive or negative, which must be applied to all observations alike As its value is liable to change, it is preferable to ascertain the amount of the correction, rather than attempt to keep the instrument free from error

Test—First method Point the telescope at a star and bring the direct and reflected images into coincidence The reading of the vernier is the index error, which is negative when the vernier index is to the left of the zero, or on the arc, and positive when off the arc (on the arc of excess)

Second method (1) Set the index at about 30′ on the arc and sight the sun

(2) The direct and reflected images should appear approximately in contact Complete the contact of the right and left limbs by the tangent screw, and note the reading

(3) Set the index at about 30′ off the arc, make the contact as before, and note the reading

(4) The index error is given by half the difference between the two readings, and is subtractive (additive) when the greater reading is on (off) the arc

Notes—(1) Since index error is applied to all observations, it must be determined with as great refinement as is employed in the observations, and several determinations may be made and the mean adopted As the error changes with change of temperature, it is advisable to ascertain its amount on each occasion of observation

(2) As a check on the observation in the second method, the sum of the readings on and off the arc should equal four times the sun's semi-diameter, as given in the *Nautical Almanac* for the date

(3) As the arc of excess is read in the opposite direction from the arc itself, care must be exercised in reading the vernier off the arc.

(4) In the case of a sextant used for terrestrial observations, the first method of testing is employed with the sight taken on a distant object

Adjustment—If it is desired to eliminate the error, clamp the index at zero, and bring the reflected image of a celestial body into coincidence with the direct image by turning the adjusting screw at the base of the horizon glass

Note—After performing this adjustment, the perpendicularity of the horizon glass should again be tested and, if necessary, re-adjusted, in which case a further adjustment of index error may be found necessary

5. Centering Error.—The error produced by non-coincidence of the axis of rotation of the index arm with the centre of graduation of the arc cannot satisfactorily be detached from the effects of refraction due to non-parallelism of the mirrors and shades, or from errors arising from defective graduation or from flexure It is therefore usual to group all such residual errors under the name of centering error, and from the results of testing to prepare a

table of corrections, for various angles, to be applied in the manner of index error. The examination is most conveniently made by the use of fixed collimators, and in this country it is usual to have the test performed by the National Physical Laboratory, Teddington Flexure of the instrument may, however, affect the constancy of the corrections, which should therefore be determined from time to time, and the surveyor may have to undertake field tests.

One method consists in observing the angular distance between two stars and comparing the result, after correction for index error and atmospheric refraction, with their calculated distance apart. To facilitate making the refraction correction, the stars selected should lie as nearly as possible on the same vertical circle Several pairs of stars are thus observed, and the centering error is ascertained for several points on the arc An alternative method consists in observing latitude by circum-meridian altitudes of north and south stars (page 83), which should be of about equal altitude The difference between the latitude deduced from the north star and that from the south star represents the centering error corresponding to the double altitude observed in the artificial horizon In the northern hemisphere the error is positive or negative according as the latitude given by the south star is the greater or smaller, and *vice versa* for the southern hemisphere

The Artificial Horizon.—It has not been found practicable to fit the sextant with levelling apparatus so that altitudes may be observed directly, and such measurements are made either by observing the angle of elevation from the sea horizon and applying a negative correction for the dip of the horizon, or by the use of an artificial horizon

The artificial horizon consists essentially of a horizontal reflecting surface, and the sextant observation of altitude consists in measuring the angle between the celestial body and its image as seen in the reflector. In Fig 27, let AB be the horizontal reflecting surface, D the position of the sextant, and S a celestial body DE being a horizontal line, the required altitude h is EDS=BCS The angle measured is SDS′, and by virtue of the laws of reflection and the parallelism of AB and DE, SDS′ is evidently twice the required altitude

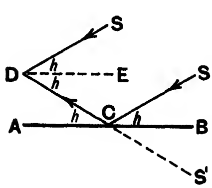


FIG. 27.

There are two general types of artificial horizon according to whether the reflector is a glass plate or the free surface of mercury in a tray The former instrument consists of a small plate of black glass having a truly plane surface and mounted without strain on a brass frame fitted with three levelling screws. A loose level tube is laid on the surface in setting the plate

horizontal. The mercurial instrument has the advantage in the certainty of the reflecting surface being level and in the superior brightness of the reflected image. The commonest form consists of a shallow iron or wooden tray (Fig. 28) about 6 in. by 3 in. To shield the surface of the mercury from wind, the tray is covered by a collapsible roof with sloping faces of glass plate worked to a uniform thickness and with truly plane surfaces. The mercury is contained in an iron flask when not in use.

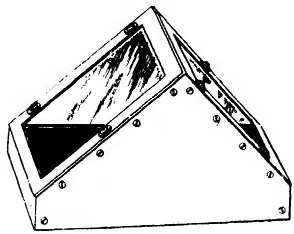


FIG. 28.—MERCURIAL ARTIFICIAL HORIZON.

Observing by Artificial Horizon.—In the measurement of altitude, the artificial horizon is placed on the ground in front of the observer and at a convenient distance for sighting the reflected image. The latter is viewed directly. By moving the index arm, the celestial body is then brought down until the two images are approximately in contact. This preliminary sighting should preferably be done with the blank tube, or simply through the telescope collar, and when the index arm is clamped at the approximate angle, the inverting telescope is screwed on as quickly as possible, and the two images are made coincident by the tangent screw.

In the case of solar observations the required coincidence consists in making one disc just touch the other. To ascertain whether the measurement is being made to the sun's upper or lower limb, the rule is that, with an inverting telescope, the images are continually overlapping in the forenoon and separating in the afternoon for the upper limb, and *vice versa* for the lower.

When taking repeated sights, except in the case of observations of equal altitudes, the glass roof should be reversed for half the observations as a precaution against the effects of possible non-parallelism of the glass plates. Under all circumstances the index correction is applied to the measured angle *before* halving it.

Notes.—(1) Practice is necessary for acquiring speed in observing with the artificial horizon. The use of a sextant stand is helpful, but if this is not available, the observer should sit on the ground and rest the right arm against the knee.

(2) The brilliancy of the reflection depends upon the cleanness and chemical purity of the mercury. Dirty mercury may be cleaned by straining through chamois leather or by pouring it a few times through a paper funnel with a very small opening.

(3) Should the mercury be accidentally spilled, any liquid may be substituted temporarily: viscous liquids such as treacle or heavy oil are preferable to water.

The Prismatic Astrolabe is an instrument for observing stars at equal altitudes. The original pattern (Fig. 29), designed by MM. Claude and Driencourt, consists of a horizontal telescope with a

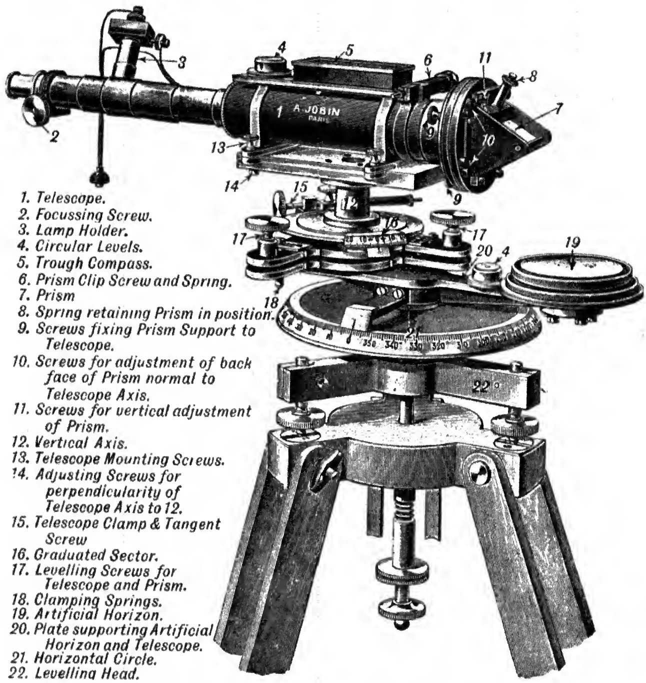


FIG. 29.—PRISMATIC ASTROLABE.

60° prism fitted in front of the object glass. Below and in front of the prism is fixed an artificial horizon. The telescope can be turned about a vertical axis.

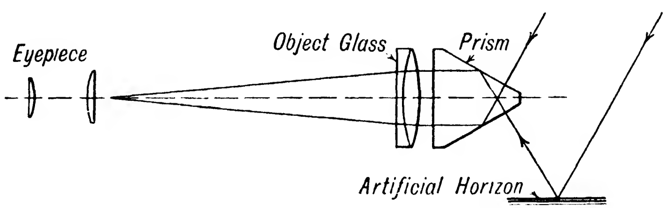


FIG. 30.

Fig. 30 shows the optical arrangement. The face of the prism next the object glass is made normal to the optical axis by means of adjusting screws. If the telescope is pointed at a star which is

approaching 60° in altitude, two images of the star are seen, one by reflection at the lower face of the prism, and the other after successive reflection by the artificial horizon and the upper surface of the prism These images approach each other as the altitude approaches 60°, and touch at the instant when the altitude is equal to the angle of the prism. The apparent velocity of each image is equal to that of the star, so that the relative velocity of the two star images is twice that of the star This facilitates an accurate determination of the time at which the star reaches an altitude of 60° It should be noted that the observation is not dependent upon the levelling of the telescope If the telescope is not quite horizontal, the two star images will coincide at a point above, or below, the centre of the field of view. Prismatic astrolabe attachments, designed by Mr E A Reeves, can be obtained for theodolites By means of these attachments any theodolite may be converted into a prismatic astrolabe

The latest pattern of instrument is known as the 45° prismatic astrolabe, and was designed by Captain T Y Baker, R N As its name implies, it is designed for observing stars at an altitude of 45° Fig 30A shows the optical arrangement The telescope is pointed

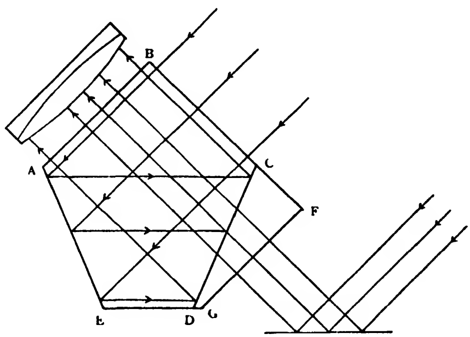


FIG 30A

downwards at an angle of 45°, and a five-sided prism ABCDE is employed to reflect the direct light from the star into the telescope The side AE is completely silvered , the side CD is also silvered, but a circular space equal in area to half the object glass of the telescope is left clear to admit light from the artificial horizon. The prism CFGD has the side GF parallel to the side AB so that light reflected from the artificial horizon passes through the two prisms without deflection The prism reflects the direct beam twice, and the artificial horizon reflects the indirect beam once, so that as the star moves in altitude the two star images move in opposite directions in the field of view, one up and the other down.

A weak prism is fitted over half the aperture of the direct beam, so that half the beam is deflected sideways, and thus forms a second direct image of the star, at the side of the first image and on the same horizontal line As the star moves in altitude, the indirect image passes between the two direct images. When the three images are in line, the star is at an elevation of 45°.

The prismatic astrolabe as originally designed permitted only one observation to be made on each star More accurate results are obtained if several observations are made of the same star at altitudes which differ from each other by known amounts With this object in view, three deflecting prisms are inserted between the artificial horizon and the main prism Each of these deflecting prisms is fixed in a cell, which can be turned through 180° The prisms deflect the indirect beam through angles of 3′, 6′ and 12′ respectively Each prism can be set in two positions, namely edge up or edge down The settings of the three prisms may therefore be combined in eight different ways These combinations produce the following deflections in the indirect beam, +21′, +15′, +9′, +3′, −3′, −9′, −15′, −21′ In the 45° astrolabe, as manufactured by Messrs Cooke Troughton and Simms, the correct sequence in the setting of the deflecting prisms is obtained by turning a handle once after each observation.

When three stars have been observed at equal altitudes, the computation of the observations depends upon the solution of three simultaneous differential equations—one for each star—of the form

$$\Delta\theta + \cot A_0 \sec\phi_0 \Delta\phi - \operatorname{cosec} A_0 \sec\phi_0 \Delta h - \Delta\theta_0 = 0$$

$\Delta\theta$ is the correction to the chronometer, A_0 is the computed azimuth of the star for the assumed latitude ϕ_0 and altitude h_0, and $\Delta\theta_0$ is the difference between the computed time of observation and the observed time—a numerical term The quantities $\Delta\phi$ and Δh are the corrections to the assumed latitude and altitude respectively The most reliable results are obtained when the three stars differ in azimuth by 120°. In practice it is generally the custom to observe from eight to twelve stars—two or three in each of the four quadrants The computation of the observations may then be carried out by graphical methods, or more precisely by forming a differential equation for each star, and using the method of least squares for the solution It should be noted that in using the 45° astrolabe it cannot be assumed that the star was at the mean of the altitudes at the mean of the observed times, and for this reason it is necessary to apply to the mean altitude a correction, which depends upon the latitude of the place and the azimuth of the star.

As a preliminary to observations with the prismatic astrolabe, it is necessary to prepare a programme. The preparation of this programme involves, first, the selection of stars that will be at the

required altitude and azimuth at a suitable time, and next, the computation of the exact moment when each star will be at the required altitude, and its azimuth at that moment. For the 60° astrolabe the preparation of programmes has been facilitated by the publication of a 60° star list by the American Geographical Society. This list gives for each degree of latitude between 60° N. and 60° S. the azimuth, local sidereal time, magnitude and right ascension of every star that is available for observation with the astrolabe.

Solar Attachment.—The solar attachment is a device whereby the astronomical triangle may be solved mechanically. It was invented in America by Burt, in 1836, in the form of the solar compass, as an improvement on the ordinary compass for setting out boundaries along meridians in land survey. Several patterns are now made in a form suitable for attaching to an ordinary theodolite.

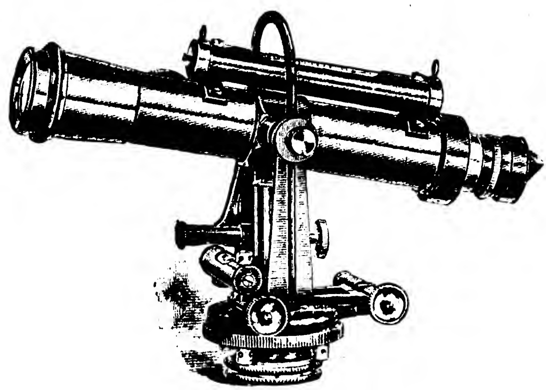


FIG. 31.—SOLAR ATTACHMENT.

They are used principally for the rough determination of azimuth, but, unless a large number of solar azimuths are likely to be required, it is doubtful whether the saving of labour in reducing ex-meridian observations outweighs the disadvantages of extra weight and additional adjustments.

Fig. 31 illustrates a simple telescope form of solar attachment designed by Saegmuller. It is fitted on the top of the theodolite telescope, and differs from other forms in not having an arc on which to set off the sun's declination. The solar telescope, besides having a motion about its horizontal axis, can be rotated about the upright axis of the attachment, called the polar axis, which must be accurately perpendicular to the horizontal axis of the theodolite and to the line of sight of the main telescope. A graduated hour-circle, divided in hours with 5^m subdivisions, is

fitted round the polar axis in some forms, and serves to give time roughly.

Evidently, if the theodolite is oriented in the meridian, and the co-latitude is set off on the vertical circle, the line of sight of the main telescope will lie in the plane of the equator, and the polar axis in the earth's axis (Fig 32), parallax being considered negligible If the solar telescope were parallel to the main telescope, rotation of the former about the polar axis would cause its line of sight to sweep out the plane of the equator. But if the solar telescope is turned about its horizontal axis through an angle equal to the sun's declination for the time of observation, with an allowance for refraction, the solar line of sight may be brought on to the sun by turning the solar telescope about the polar axis This can be done only when the polar axis is pointing to the pole, so that, when the declination and co-latitude are set off, an observation for meridian consists in turning the solar telescope about the polar axis and the main telescope about the vertical axis until the sun appears in the field of the former When, on manipulating the respective tangent screws, the sun is placed in the square of the reticule of the solar telescope, the line of sight of the main telescope has been placed in the meridian, and the hour-circle reads the time In the absence of a declination arc, the declination is set off by first bringing both telescopes into the same vertical plane by sighting any convenient point with each, setting the corrected declination on the vertical circle by depressing the main telescope for a north declination and elevating it for a south declination in the northern hemisphere, and then bringing the solar telescope horizontal by reference to its attached level

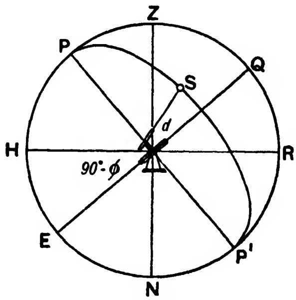


FIG 32

Timekeepers.—The most precise portable timekeeper is the box chronometer as employed in observatories and on board ship In field astronomy it is used for primary longitude determinations and in other circumstances where it can be kept either stationary or on board a vessel The instrument is too delicate to maintain a uniform rate during transport by land The pocket chronometer is better adapted for land journeys, but is liable to stoppage if subjected to shocks in travelling over rough country. The most serviceable timekeeper for field use is that known as the half-chronometer watch, which with due care gives results of a suitable accuracy for mapping A stop-watch is frequently a convenient accessory in observing

Watch Error and Rate.—The result of an observation for time gives, by comparison of the computed time with that indicated by the watch, the watch error at the mean instant of observation. The magnitude of the error is immaterial for purposes of time-keeping If the watch were regulated to keep mean or sidereal time, the L M.T , Standard M T. or L S T for the place of observation could be obtained at any instant by application of the error to the watch reading It is, however, unnecessary that the error should maintain a constant value What is expected of a good timekeeper is that, while subject to a particular set of conditions, it should go at uniform speed, so that its error changes uniformly The daily change of error is known as the rate, which may be either a gaining or a losing one

The length of time a good timekeeper will maintain a sensibly constant rate depends largely upon the degree of uniformity in the conditions to which it is exposed It should be wound carefully and at the same hour each day The rate is liable to variation by change of temperature, no matter how carefully compensated the watch or chronometer may have been, and is also affected by change of atmospheric humidity unless the instrument is enclosed in an air-tight case It should be kept in either the horizontal or the vertical position, as the rate differs for the two The standing rate, or rate when the instrument is stationary, differs from the travelling rate developed during transport by land or sea

Determination of Standing Rate.—The value of the standing rate is obtained by taking at the same place at least two sets of time observations separated by an interval of a few days A test of the constancy of the rate may be made by means of time observations every night or every second night for a period of seven to ten days Each time determination should, as far as possible, be made in the same manner The rate is computed from the observed change of error as follows

Example—A time observation on Oct 20 at about $22^h\ 21^m$ L M T. showed the watch error to be $46^m\ 21^s{\cdot}2$ slow, and at the same place on Oct 26 at about $20^h\ 13^m$ the error was $45^m\ 49^s{\cdot}4$ slow. Compute the rate, and find the L M T at which the watch reads $6^h\ 42^m\ 54^s{\cdot}2$ on the morning of Oct. 23

		d	h	m	
	Oct	26	20	13	
	Oct	20	22	21	
Interval between observations	=	5	21	52	= $5^d{\cdot}911$

Change of error = $46^m\ 21^s{\cdot}2 - 45^m\ 49^s\ 4 = 31^s{\cdot}8$ gained

$$\text{Rate} = \frac{31{\cdot}8}{5{\cdot}911} = 5^s{\cdot}38 \text{ gaining}$$

To find the error on Oct 23 at $6^h\ 42^m\ 54^s{\cdot}2$ watch time

Watch time of determination on Oct 20 was about $22^h\ 21^m$ less $46^m\ 21^s{\cdot}2$, say $21^h\ 35^m$

Watch interval $= 23^d\ 06^h\ 43^m - 20^d\ 21^h\ 35^m = 2^d\ 09^h\ 08^m = 2^d\cdot 38$ = mean time interval with sufficient accuracy

Gain $= 2\cdot 38 \times 5^s\cdot 38 = 12^s\cdot 8$

Required error $= 46^m\ 21^s\cdot 2 - 12^s\cdot 8 = 46^m\ 08^s\cdot 4$ slow

and required L M T $= 6^h\ 42^m\ 54^s\cdot 2 + 46^m\ 08^s\cdot 4 = 7^h\ 29^m\ 02^s\cdot 6$

Determination of Travelling Rate.—Three cases occur, according as the journey—

(1) Lies between two points of which the longitude difference is known

(2) Returns to the starting point

(3) Does not return to the starting point

Case (1) occurs in land journeys when travelling between two points at which longitude difference can be obtained by telegraph or is otherwise known, the travelling rate being required for longitude determinations at intermediate points on the route The watch error is determined before starting and again on arrival, but the change of error is due in part to the difference between the local times of the two places The latter, being known, is eliminated, and the residual change falls to be divided by the time interval between the observations, which is given with sufficient accuracy by the watch In the case of land journeys, the travelling rate conditions include the normal rest at night If, however, the surveyor is compelled to remain for a few days at a point on the route, and the watch is kept stationary, the circumstance must be allowed for in computing the travelling rate (see *Example* 2 below)

In case (2), since the observations for watch error at start and finish are taken at the same place, the calculation is performed as for standing rate Case (3) necessitates the determination of the travelling rate before commencement of the journey The method is the same as for standing rate, except that during the period between the two time observations the watches are carried on a daily march of the same duration as the average daily march is likely to be during the journey, and under as nearly as possible the same conditions The opportunity should be taken, by means of a series of time observations between those from which the rate is computed, of testing the constancy of the travelling rate, which is more liable to irregularity than the standing rate

Example 1.—A survev party travels eastwards from A to B, the difference of longitude between which is $2°\ 12'\ 20''\cdot 8$ Before starting, a time observation showed the standard watch to be $6^m\ 14^s\cdot 2$ fast, and after arrival at B the error was $1^m\ 56^s\cdot 1$ slow The interval between the time observations, as shown by the watch, was 9·07 days Compute the travelling rate

$$\text{Difference of longitude in time} = \frac{2°\ 12'\ 20''\cdot 8}{15} = 8^m\ 49^s\cdot 4$$

and, since B is east of A, the change of longitude makes the watch less fast by this amount

But observed change of error $= 6^m\,14^s{\cdot}2 + 1^m\,56^s{\cdot}1 = 8^m\,10^s{\cdot}3$ lost
change of error due to rate $= 8^m\,49^s{\cdot}4 - 8^m\,10^s{\cdot}3 = 39^s{\cdot}1$ gained

whence travelling rate $= \dfrac{39{\cdot}1}{9{\cdot}07} = 4^s{\cdot}31$ gaining

Example 2 —A time observation at A on Nov 7 at about $20^h\,52^m$ watch time showed the error of a watch on L M T to be $18^m\,20^s{\cdot}5$ fast On the following morning a journey started towards B, and on the evening of arrival on Nov 15 at about $21^h\,43^m$ the error on L M T was found to be $6^m\,49^s{\cdot}6$ fast The watch was kept stationary while the party remained at B, and on Nov. 17 at about $20^h\,16^m$ the error was $6^m\,37^s{\cdot}9$ fast. A return journey to A commenced on the morning of Nov 18, and A was reached on the evening of Nov 28, the watch error then being found to be $16^m\,57^s{\cdot}8$ fast at about $22^h\,12^m$. Compute the travelling rate

Interval between observations at A
$=$ Nov $28^d\,22^h\,12^m -$ Nov. $7^d\,20^h\,52^m = 21^d\,01^h\,20^m$, of which the rest between the observations at B amounted to
Nov $17^d\,20^h\,16^m -$ Nov $15^d\,21^h\,43^m = 1^d\,22^h\,33^m$

Period during which travelling rate developed
$= 21^d\,01^h\,20^m - 1^d\,22^h\,33^m = 19^d\,2^h\,47^m = 19^d{\cdot}116$

Change of error between observations at A
$= 18^m\,20^s{\cdot}5 - 16^m\,57^s{\cdot}8 = 1^m\,22^s{\cdot}7$ lost
but of this there was lost during the wait at B
$6^m\,49^s{\cdot}6 - 6^m\,37^s{\cdot}9 = 11^s{\cdot}7$
change of error due to travelling rate $= 1^m\,11^s{\cdot}0$ lost.

$$\text{Travelling rate} = \frac{1^m\,11^s{\cdot}0}{19{\cdot}116} = 3^s{\cdot}71 \text{ losing.}$$

Note —Instead of having the watches on standing rate while at B, they might have been subjected to travelling conditions by being taken on a march during each of the two days the party remained at B In calculating the travelling rate, no account would then have to be taken of the wait at B

Comparison of Watches.—When the error of one watch is known, that of another can be obtained by comparing their simultaneous readings The comparison is best made by two observers, each having one of the watches At a whole second on his watch, observer A makes a sharp tap, at which B estimates the reading of the other to $0^s{\cdot}1$ or $0^s{\cdot}2$ Each notes his reading This is repeated two or three times, and then the same number of signals is made by B after an exchange of watches. The precision of such a comparison will probably be not less than that of the known error.

If the comparison is made by a single observer, he must measure the short interval between the reading of one watch and that of the other, either by counting the ticks of the first watch or by means of a stop-watch. The number of ticks per second, usually $2\frac{1}{2}$, $4\frac{1}{2}$ or 5, being known, the watches, A and B, to be compared are laid side by side, and the observer, reckoning from a whole number of seconds on A, starts his count of A with the first tick thereafter He immediately turns to B, and reads the position of the seconds hand at the instant corresponding to one of the numbers of the count. The time equivalent of the ticks counted is added to the

reading of the A watch at the start of the count, and gives the time to be compared with the reading of the B watch When a stop-watch is used, it is started from $0^m\ 0^s$ when the seconds hand of the first watch reaches a graduation, and is stopped at the instant of reading the second watch With either method, several repetitions are required

Comparison of Chronometers.—A highly accurate comparison of box chronometers can be made if one keeps mean time and the other sidereal time Box chronometers beat every half-second, and since sidereal time gains on mean time by nearly 10^s an hour, there is a steadily varying interval between the beats of a mean time chronometer and those of a sidereal time chronometer The beats synchronise about every 3^m, and each coincidence marks an instant at which the times on the two chronometers differ exactly by their readings taken to half-seconds

To compare a mean time with a sidereal time chronometer, each is read by a separate observer at the instant the beats are in unison. Since the readings are required only to half-seconds, the precision of the comparison is dependent upon the selection by one of the observers of the coincident beat at which both readings are taken The exact coincidence is difficult to distinguish, since several beats almost synchronise, but with practice it is possible to detect a lack of coincidence as small as $0^s{\cdot}03$, and the probable error is likely to be much less, particularly if two or three coincidences are noted An observer working alone can make the comparison by watching one chronometer and counting the beats of the other from a whole second near the coincidence In applying the method to the comparison of two mean time chronometers, each is compared with a sidereal time chronometer, while a mean time chronometer is similarly used as the medium for the comparison of sidereal time chronometers

GENERAL PROCEDURE IN OBSERVATIONS

The routine in observing depends upon the precision required and the instrument used. The theodolite is generally employed for field determinations, and certain items of routine, common to several observations, are described here.

Observing by Theodolite.—In preparing to observe, attention should be given to the stability of the theodolite support. If the instrument must be supported on an ordinary tripod instead of an observing pillar, the tripod should rest on well driven pegs if the ground is at all yielding. Needless to say, the theodolite should be in adjustment (Vol I, page 85, and Vol. II, page 140), but nevertheless the routine should be designed to eliminate instrumental errors

as far as possible. In general, observations must consist of an equal number of face right and face left sights, and each circle reading must be obtained from both micrometers or verniers In the case, however, of certain observations, such as equal altitudes, the aim is to preserve constant instrumental conditions, and a change of face is not required

Level.—Before observing, the vertical axis must be made truly vertical by manipulating the levelling screws and, if necessary, the clip screws until the bubble of the altitude level remains central during a rotation It is usual to find that this bubble does not preserve a central position throughout a series of observations. The readings of both ends of the bubble should therefore be taken immediately after each observation and before reading the circles. If the bubble is considerably off centre, it should be returned approximately to the central position by the clip screws before taking the next sight The correction to altitude for dislevelment is as follows

For a tube graduated in both directions from the centre

Let ΣE = sum of the successive readings of the eye end of the bubble for the n individual observations, both F R. and F.L

ΣO = sum of the corresponding readings of the object glass end

d = value of one division of the level

c = level correction to average altitude

Then $c = \dfrac{\Sigma O - \Sigma E}{2n} \cdot d$

For a tube graduated continuously from the eye end

Let G = reading of centre graduation.

Then $c = \dfrac{\Sigma(O+E)-2G}{2n} \cdot d$

In azimuth observations the striding level must be used, and the azimuth correction for horizontal axis dislevelment is computed from the readings of the bubble (page 72)

Taking Times.—Whenever possible, the observer should have the assistance of a recorder, who books the level and circle readings and, when necessary, reads the times of observations The observer should, as far as possible, allow the celestial body to make its own passage or contact, and a few seconds in advance warn the recorder to be prepared At the instant of passage or contact he calls out to the recorder, who reads the timekeeper.

Times are most easily taken on a chronometer. Chronometers beat half-seconds, so that the rhythm is not difficult to follow The best method is to observe the hand and count with the beat up to 10, thus—1 *and* 2 *and* 3 *and* . . After a little practice, the

occurrence of a particular instant can be estimated to $0^{s}\cdot 1$ with considerable accuracy Since the half-chronometer watch or an ordinary watch is more generally used in field astronomy, times cannot usually be taken more accurately than to about $0^{s}\cdot 2$ In reading a watch it is well to keep count of the ticks while observing the seconds hand Half-chronometer watches usually make $4\frac{1}{2}$, and ordinary watches 5 ticks a second.

If the observer has not the assistance of an efficient recorder, he must himself take the times In the " eye and ear " method, which is applicable when the chronometer is used, the observer places the chronometer so that it can be heard and seen from the instrument, reads it before the observation, and starts counting from the reading He keeps up a continuous count in unison with the beats, and estimates the instant of the observation from his count, verifying the minutes on the chronometer immediately afterwards With a half-chronometer or ordinary watch, a common method is to hold the watch near the ear while observing and to start counting the ticks at the instant of bisection or contact The watch is then looked at, and the count is stopped when the seconds hand reaches a convenient division. The required time is the watch reading at that instant less the time equivalent of the ticks counted.

A simpler method is for the observer to use a stop-watch, which he starts at the instant of passage or contact He then carries the stop-watch to the chronometer or watch, and stops it when the latter reads an exact second. The chronometer reading less that of the stop-watch is the required time of the observation

Note-keeping.—The observer should maintain a regular order of reading and calling out quantities to the booker, thus level, eye end, object end ; vertical circle, micro I, micro II , horizontal circle, micro A, micro B Only minutes and seconds are read and booked for the second micrometer or vernier reading of a pair For observations with a small micrometer theodolite, it should be decided at the outset whether the quality of the determination and the state of the instrument as regards adjustment of the micrometers warrants correction of the readings for run (page 143) In refined observations both back and forward readings are taken, and columns are provided in the angle book for the entry of run corrections and the means of the corrected readings Quantities entered in addition to those read out by the observer are object observed, face, time, barometer and thermometer readings (for evaluation of the refraction correction), date and place of observations, as well as any further data required in reducing the observations.

Relative Merits of Star and Sun Observations.—Star observations are likely to yield much more accurate results than those derived

from observations of the sun A star appears in the field of the telescope as a point of light, and presents an ideal mark for bisection The great choice of stars of known position makes it possible to multiply, without undue delay, observations on stars well situated for the determination in hand Errors arising from uncertainties in refraction are effectively reduced by pairing and balancing of observations on east and west stars for time and azimuth, and on north and south stars for latitude, more particularly as the observations forming a pair can be taken within such a short time of each other that it is reasonable to assume that the change of refraction between them is negligible Balancing of sun observations for time and azimuth, on the other hand, necessitates a wait from morning till late afternoon, and it is impossible to make any balance in determining latitude by the sun

Nevertheless, solar observations are very useful to the surveyor when a refined result is not required, as it is sometimes awkward to have to remain away from camp after dark In malarial regions, especially, star observations are not generally undertaken if determinations of sufficient precision can be obtained from the sun

DETERMINATION OF TIME

In determining local time, the chronometer times of the individual observations are noted, and the result gives for the average instant of the observation the error of the chronometer on sidereal or solar time, according as a star or the sun is observed Determinations are made from meridian or ex-meridian observations In the former case, the local sidereal time of the instant of transit of a star is given by the star's right ascension, and the chronometer error is obtained by comparison of that quantity with the chronometer reading at transit For the sun, the instant involved is that of local apparent noon, which is reduced to local mean time for the evaluation of the watch error on mean time When a celestial body is observed out of the meridian, the observations are directed to obtaining sufficient data to enable the astronomical triangle to be solved for the hour angle, from which the sidereal or solar time is obtained.

Methods.—The principal methods are by

(1) Transits

(2) Ex-meridian altitudes of stars or the sun

(3) Equal altitudes of stars or the sun

Time by Star Transits.—Observation of the instants of star transits forms the most direct method of obtaining local time, and

is that employed for primary field determinations. The method can be applied to minor determinations with small theodolites, but is not commonly employed, since the observations necessary to enable the telescope to be placed in the plane of the meridian would themselves yield the approximate watch error.

Primary Determinations.—Primary determinations are made by means of a portable transit instrument (page 37) The best results are obtained by automatic registration on a chronograph by means of the transit micrometer Otherwise the instants of the passages of the star across the successive hairs are transmitted to the chronograph record by key In the absence of a chronograph, the times are taken by the eye and ear method For a precise determination, such as would be required in connection with primary telegraphic longitudes, twenty-four to thirty-two star transits are observed These are arranged in groups of four or six, the telescope being reversed between each group.

The approximate direction of the meridian having been set out in advance, the telescope is first placed in that plane preparatory to closer adjustment An error in azimuth has only a small effect upon the observed time of transit of a star near the zenith, and the chronometer time of transit of such a star is first observed and an approximate value for the chronometer error obtained The telescope is then directed towards a slow moving or circumpolar star which is about to transit The chronometer time of transit is computed, allowing for the approximate error, and the middle vertical hair is kept upon the star, the telescope being adjusted in azimuth by the slow motion screws until the chronometer indicates that the star is on the meridian A second approximation is made by again observing a star transit near the zenith, followed by an adjustment on a circumpolar The line of sight is now very nearly in the plane of the meridian, and the amount of the residual error is discovered and allowed for in the reduction of the time observations

The chronometer error on sidereal time cannot be obtained by direct comparison of the right ascensions of the stars with their observed times of transit, because it is impracticable to secure that the instrumental line of sight lies exactly in the plane of the meridian The observed times are subject to three principal corrections (1) the azimuth correction, due to the line of sight not being oriented precisely in the plane of the meridian; (2) the level correction, due to dislevelment of the horizontal axis, (3) the collimation correction, due to the line of sight not being perpendicular to the horizontal axis.

The amounts of these corrections to the observed time of transit of a star of zenith distance z and declination δ are as follows.

$$\textit{Azimuth correction} = a \sin z \sec \delta = aA$$

where a is the error of azimuth in seconds of time. a is considered positive (negative) if the line of sight is too far east (west) when the telescope is pointed south When the telescope is pointed north, the line of sight is changed to the other side of the meridian.

$$\textit{Level correction} = b \cos z \sec \delta = bB$$

in which b is the inclination of the horizontal axis in seconds of time. The value of b is given not only by the readings of the striding level in the direct and reversed positions, but also includes the inclination caused by inequality of the trunnion pivots, if appreciable. b is positive (negative) when the west (east) end of the axis is the higher.

$$\textit{Collimation correction} = c \sec \delta = cC$$

where c is the error of collimation in seconds of time, regarded as positive (negative) when the line of sight is to the east (west) of the meridian

In these corrections the quantities a, b and c represent instrumental errors, while the factors A, B and C depend upon the position of the star, and their values are given in tables of star factors The signs of A, B and C for the northern hemisphere are

Position of star	A	B	C
South of zenith	+	+	+
Between zenith and pole	−	+	+
Below pole	+	−	−

The correction bB can be computed for each observation from the readings of the striding level and the pivot inequality constant, and is applied at the outset to the observed time of transit In the most precise work small corrections are also made for diurnal aberration and chronometer rate The former arises from the effect of the diurnal rotation of the earth on the relative velocity of light, and the correction to the observed time of transit is given by $0^s{\cdot}02 \cos \phi \sec \delta$, where ϕ is the latitude of the observer, and δ the declination of the star This correction is negative for all stars observed at upper transit and positive for stars observed at lower transit The rate correction is required when the set of observations before reversal of the telescope extends over a considerably longer or shorter period than that after reversal The chronometer time of transit so corrected is now in error only by the effects of azimuth and collimation error, and these are determined from the time observations themselves.

Let T = the chronometer reading, corrected for level, aberration and rate

e = the required chronometer error, positive if slow, negative if fast

The finally corrected chronometer time of transit $= T+aA+cC$, $=$ the chronometer reading which would have been obtained if the line of sight were actually in the meridian $=$ R A. $-e$.

$$e = \text{R A.} - \text{T} - \text{a A} - \text{c C}$$

For each transit we have an observation equation with three unknowns, e, a and c. The number of observations involved in a determination exceeds the number of unknowns, the most probable values of which are therefore most accurately derived by the method of least squares. An approximate solution is obtained with less labour by solving simultaneous equations equal in number to the unknowns, the equations being formed by addition of groups of the observation equations

The stars forming a set are selected with a view to a good solution of the equations There may be included in each set one or two slow moving or azimuth stars, from the observation of which a is principally derived Otherwise all stars are time stars, and contribute equally to the result The time stars of a set should be situated north and south of the zenith, and their declinations should have such values that the algebraic sum of their A factors is as nearly zero as possible, while their right ascensions should be such that the transits occur at convenient intervals

Minor Determinations by Star Transits.—Refinement in the reductions is not warranted when the observations are made with a small theodolite. The setting of the instrument in the plane of the meridian may be performed by zenith and azimuth stars as described above, unless the direction of the meridian is known with sufficient precision Residual errors of adjustment are approximately eliminated by making the time observations on a pair of stars of about the same altitude, one north and the other south of the zenith.

Time by Ex-meridian Altitudes of Stars.—The method of determining time by ex-meridian altitudes of a star or stars or of the sun is that most commonly used by surveyors In its simplest form, the star observation consists in measuring the altitude of a known star at some distance from the meridian and noting the chronometer time of the measurement If the observer's latitude is known, the astronomical triangle PZS (Fig 9) can be solved for the hour angle, from which is derived the local sidereal time corresponding to the chronometer reading. A close approximation to the value of the latitude is required.

Instead of relying upon a single altitude of the star, several altitudes are observed on alternate faces in quick succession, the chronometer time of each being noted. The mean of the altitudes will correspond with the mean of the chronometer times if the

observations are completed within a few minutes The star selected should be one which is changing rapidly in altitude, i e it should be near the prime vertical. The influence of errors in observed altitude, as well as in the value of the latitude, is a minimum when the star is actually on the prime vertical For the avoidance of excessive refraction, the altitude of the star should be at least 20°, and practically it is generally necessary to observe stars at some distance from the prime vertical For the more effectual reduction of error in the assumed value of the refraction coefficient, as well as of instrumental errors, the observation of an east star should be balanced by an observation of a west star of similar altitude

Observation of Ex-meridian Altitudes of Stars.—Stars suitable for observation are selected in advance When a star is on the prime vertical its altitude is given by

$$\sin h = \frac{\sin \delta}{\sin \phi}$$

This relationship shows that the declination of the star must be of the same sign as the latitude, and must be less than the latitude In order, however, that h may not be less than 20°, δ should be greater than $\sin^{-1} \sin \phi \sin 20°$ As the star may be observed out of the prime vertical, the foregoing serves to define only roughly the requirements as to declination The selection must be further based upon the availability of stars for observation at convenient times

The observation of each star of a pair consists of two or three successive altitude measurements, each consisting of a face right and a face left observation The details of each measurement are performed according to the routine described on page 56 The chronometer times are taken to the nearest $0^s{\cdot}2$, or to $0^s{\cdot}1$ if possible

Computation.—The astronomical triangle is conveniently solved for the hour angle t by means of

$$\tan \frac{t}{2} = \sqrt{\cos s \sin (s-h) \operatorname{cosec} (s-\phi) \sec (s-p)}$$

$$\text{where } s = \frac{h+\phi+p}{2}$$

In applying the formula, ϕ is always treated as positive, and p is measured from the elevated pole. The computed value of t is measured the shorter way from the meridian It is added to or subtracted from the star's right ascension, according as the star is west or east of the meridian, to give the sidereal time corresponding to the mean of the chronometer readings. If the chronometer is a mean time one, the computed sidereal time must be converted to mean time (page 32). The same calculation is performed

for the other star of the pair, and the mean of the computed chronometer errors is accepted as the error at the average time of the observations. The observation of two or three pairs of stars with a 5-in. micrometer theodolite should ensure a result well within one second of the truth. The precision of the method is greater in low than in high latitudes.

Example.—On 1935 March 3, at a place in latitude 15° 12′ 36″ N and longitude 25° 03′ 20″ E., an observation of γ Orionis (R.A. $5^h\ 21^m\ 40^s{\cdot}3$, $\delta = +6°\ 17'\ 33''{\cdot}5$) west of the meridian gave a mean corrected altitude of 35° 22′ 10″·5. Find the L.S.T. of the observation.

	°	′	″		
h =	35	22	10·5		
ϕ =	15	12	36		
p =	83	42	26·5		
Sum = $2s$ =	134	17	13·0		
s =	67	08	36·5	log cos =	9·5893071
$s-h$ =	31	46	26·0	log sin =	9·7214548
$s-\phi$ =	51	56	00·5	log cosec =	0·1038623
$s-p$ =	−16	33	50·0	log sec =	0·0184068
				Sum =	9·4330310
				log tan $\frac{t}{2}$ =	9·7165155

$\frac{t}{2}$ = 27° 30′ 07″·6

		h	m	s
t =	55° 00′ 15″·2 =	3	40	01·0
	R.A. of star =	5	21	40·3
	L.S.T.	9	01	41·3

Time by Ex-meridian Altitudes of the Sun.—In applying the method of ex-meridian altitudes to solar observations, the required balancing is effected by measuring a succession of altitudes both in the morning and afternoon. The requirements as to the position of the sun during observation are the same as for a star, and the most suitable times are between 8 and 9 a.m. and again between 3 and 4 p.m.

The minimum number of observations in each set is four, consisting of a face right and a face left measurement to both the upper and lower limbs, but it is preferable to double this number of observations. The hour angle is computed, as in the case of star altitudes, from

$$\tan\frac{t}{2} = \sqrt{\cos s \sin (s-h) \operatorname{cosec} (s-\phi) \sec (s-p)}$$

but it will be necessary to obtain p by interpolating the value of the sun's declination for the mean instants of the morning and afternoon observations respectively. The interpolation requires a knowledge not only of the longitude of the place of observation, but also of the local time, and, since the latter is being determined, the evaluation

of t should really be performed by successive approximations If, however, local time is known within about two minutes, the value of the sun's declination can be interpolated with sufficient precision for the purpose On reducing the observations, if a greater discrepancy is discovered between the computed and assumed local times, the former is used for a better interpolation of δ, and the computation of t is repeated with the new value

The local apparent time corresponding to the mean of the watch times of the forenoon observations is obtained by subtracting the hour angle from 12^h For the afternoon set, 12^h is added to the hour angle The conversion to mean time is made by the methods of Chapter I The mean of the two watch errors thus determined is the error at the average instant of the two sets

Example —On 1935 May 10, at a place in latitude 36° 55′ 08″ N and longitude 57° 53′ 30″ E , the mean observed altitude of the sun, corrected for refraction, parallax and level, was 39° 54′ 13″ 8 The mean watch time of the observations was $15^h\ 31^m\ 04^s$ 2, the watch being known to be about 4^m fast on L M T Find the watch error on L M T

Value of sun's δ at mean instant of observation

		d	h	m	s
Approx L M T	=	May 10	15	27	04
Longitude	=		− 3	51	34
Approx G M T	=	May 10	11	35	30

		°	′	″
From *N A* , p 10, sun's δ at May $10^d\ 0^h$	=	+17	19	00·2
Change in $11^h\ 35^m\ 30^s$	=	+	7	42·8
Required δ	=	+17	26	43·0
p	=	72	33	17·0

	°	′	″		
h =	39	54	13·8		
ϕ =	36	55	08·0		
p =	72	33	17·0		
Sum = $2s$ =	149	22	38·8		
s =	74	41	19·4	log cos =	9·4217074
$s-h$ =	34	47	05·6	log sin =	9·7562534
$s-\phi$ =	37	46	11·4	log cosec =	0·2129003
$s-p$ =	2	08	02·4	log sec =	0·0003013
				Sum =	9·3911624
				$\log\tan\frac{t}{2}$ =	9·6955812

$$\frac{t}{2} = 26°\ 23'\ 11''{\cdot}9$$

$$t = 52°\ 46'\ 23''{\cdot}8 = 3^h\ 31^m\ 05^s{\cdot}6$$

Conversion to L M T.

		h	m	s
L A T	=	15	31	05·6
Longitude	=	−3	51	34
G A T	=	11	39	31·6
From *N A* , p 24, G M T of G A T 12^h	=	11	56	20·4
Correction = $-0^d{\cdot}014 \times -2^s{\cdot}8$	=			+0 0
L A T	=	15	31	05·6
Sum -12^h = L M T	=	15	27	26·0
Watch time	=	15	31	04·2
Watch error on L.M T	=		3	38·2 fast.

Time by Equal Altitudes of a Star.—The mean of the two chronometer times at which a star attains equal altitudes east and west of the meridian represents the chronometer time of its transit A simple method is thus afforded of obtaining the chronometer reading corresponding to the sidereal time of transit, or right ascension of the star. The determination is independent of the actual value of the altitude, and no correction is required for refraction, but the precision of the result depends upon the refraction having the same value for both observations The observations should be made when the altitude is changing rapidly, *i e* when the star is near the prime vertical, and the method is open to the objection that several hours may elapse between the first and last observations unless the declination of the selected star is nearly equal to the latitude

The observations are made by means of a theodolite or a sextant and artificial horizon In place of observing the star only once on either side of the meridian, a set of four or five altitudes should be taken to reduce errors of observation Since the essential feature of the observation is the equality of the altitudes on either side of the meridian, the theodolite telescope is not reversed, and the bubble of the altitude level on the micrometer arm must be exactly centered either by the level or clip screws at each observation, the angle between the telescope and the level remaining unaltered. Sextant observations give satisfactory results since the use of the artificial horizon avoids possible errors of level, while the settings are in round figures and no correction for index error is required

Instead of taking the observations on the same night, they may be made in the morning and evening If the western altitudes are observed first, the average of the chronometer times represents the chronometer reading at lower transit, the sidereal time of which is 12^h greater than the star's right ascension

Time by Equal Altitudes of Two Stars.—The effect of a possible change in refraction between the two sets of observations at equal altitudes of the same star is greatly reduced, and the inconvenience of waiting is avoided, by making the equal altitude observations on two stars, one east and the other west of the meridian If it were possible to select two stars having the same declination, the mean of their right ascensions would represent an instant of sidereal time with which the mean of the chronometer readings could be compared for the determination of the chronometer error. Actually the two bodies have different declinations, and, in consequence, a correction must be applied to the mean of their right ascensions

The observation of a pair of stars may be completed in a few minutes, and several pairs should be used for a good determination.

Stars selected to form a pair should have a difference in right ascension of at least 6^h, and, on account of the approximations made in computing the correction for difference of declination, the latter should be kept within from 2° to 5° according to the required refinement of the determination. The stars should also preferably be such as are near the prime vertical at the time they reach the same altitude Since the sidereal time at which the two stars are simultaneously at the same altitude is approximately the mean of the two right ascensions, the observer can select pairs of stars which will afford convenient intervals between the observation of one pair and that of the next

If the east star is first observed, the line of sight is set just above it at a few minutes before the time at which the stars are at the same altitude The chronometer reading when the star crosses the horizontal hair is noted, and the telescope is then turned to the west star The latter should appear a little above the horizontal hair, and the chronometer time of its passage is likewise observed If the stars are not selected in advance, a pair may be found by trial of their altitudes

The correction for difference in declination of the two stars is computed as follows

Let c = correction in time to be applied to the average of the two right ascensions to give the sidereal time corresponding to the average of the chronometer readings T_E and T_W

$$t = \tfrac{1}{2}(\text{R.A.}_E - \text{R.A.}_W) - \tfrac{1}{2}(T_E - T_W)$$

ϕ = observer's latitude, positive or negative according as it is north or south

δ_E and δ_W = declinations of the east and west stars

$$\delta = \tfrac{1}{2}(\delta_E + \delta_W)$$

$$\text{Then } c = \frac{\delta_E - \delta_W}{2 \times 15}(\tan\delta\cot t - \tan\phi\operatorname{cosec} t)$$

Time by Equal Altitudes of the Sun.—In applying the method of equal altitudes to solar observations, a series of altitudes is taken about 9 a m , and the same series is repeated in reverse order about 3 p.m Each altitude setting on the instrument yields two observations if the instants are noted at which both the upper and lower limbs touch the horizontal hair

The mean of the times of the forenoon and afternoon equal altitudes does not in this case exactly represent the instant of transit owing to the change in the sun's declination between the observations The average of the watch times of the observed equal

altitudes must therefore be corrected to give the watch time of apparent noon, as follows.

Let c = correction in time to be applied to the average of the watch times of equal altitude to give the watch time of apparent noon

t = half the interval between the times of equal altitude

ϕ = observer's latitude, positive or negative according as it is north or south

δ_E = sun's declination at the average of the morning observations

δ_W = sun's declination at the average of the afternoon observations

$\delta = \frac{1}{2}(\delta_E + \delta_W)$

$$\text{Then } c = \frac{\delta_W - \delta_E}{2 \times 15}(\tan \delta \cot t - \tan \phi \operatorname{cosec} t)$$

The value of ϕ used in computing the correction need be only approximate. The method is available for afternoon observations followed by a series at the same altitudes on the following morning. The result then gives the chronometer error at apparent midnight The value of c is then given by

$$c = \frac{\delta_E - \delta_W}{2 \times 15}(\tan \delta \cot t + \tan \phi \operatorname{cosec} t)$$

Example —On 1935 June 5 in latitude 56° 06′ 11″ N and longitude 4° 59′ 05″ W., equal altitude observations of the sun's lower limb for time gave the results tabulated Find the error of the watch on standard time (British summer time), the approximate error being 2^m slow.

Altitude Micro I F L			Watch time			Watch time		
°	′	″	h	m	s	h	m	s
49	04	26	11	11	07·0	15	21	31·0
49	23	10	11	14	10·5	15	18	26·5
49	35	32	11	16	13·5	15	16	20·0
49	46	01	11	18	00·0	15	14	35·0
Average watch times			11	14	52·8	15	17	43·1

From *N A.*, p. 12, δ at $10^h\ 17^m = \delta_E = +22°\ 28'\ 07''$

δ at $14^h\ 20^m = \delta_W = +22°\ 29'\ 16''$

$\delta_W - \delta_E = +69''$

$\delta = +22°\ 29'$

$t = 2^h\ 01^m\ 25^s$

	h	m	s
Mean of watch times =	13	16	18·0
c =	−		5·1
Watch time of L.A.N. =	13	16	12·9

	h	m	s
From *N.A*, p 25, G M T of G A N =	11	58	10·7
Correction for longitude = 0·0138 × +10^s·3 =			+ 0·1
Sum = L M T of L.A N. =	11	58	10·8
Longitude west =		19	56·3
Correction for summer time =	+ 1		
Sum = B S T of L A N =	13	18	07·1
Watch time =	13	16	12·9
Watch error =		1	54·2 slow

Time by Transit of the Sun.—If the direction of the meridian is known, a rough determination of time may be made by noting the watch times of transit of the west and east limbs of the sun, the mean of the observed times being the watch time of local apparent noon. If only one limb is observed, the time of transit of the sun's centre is obtained by application of the semi-diameter in sidereal time, which is given for every day of the year in the *Nautical Almanac*

DETERMINATION OF AZIMUTH

The determination of azimuth, or the direction of the meridian at a survey station, consists in obtaining the azimuth or true bearing of any line from the station, so that the azimuths of all the survey lines meeting there may be derived An essential feature of the observation is the measurement of the horizontal angle between a celestial body and a terrestrial signal In primary determinations the observed azimuth should be that of a side of the triangulation. Otherwise it is frequently convenient to establish a reference mark, from the observed azimuth of which those of the survey lines are measured

The quality of the angular work of the survey indicates the necessary precision of the observations. The requirements as to probable error range from about 0″·3, in the case of primary triangulation, to over 1′ 0 for determinations made for the frequent control of compass traversing

Reference Mark.—When the azimuth of one of the survey lines is directly observed, the signal is of the form adopted throughout the survey. Otherwise a special signal, called the reference or azimuth mark or referring object (R.O.), is erected. If practicable, it should be placed at a distance of not less than a mile, not only to render negligible small errors of centering of the instrument, but principally to avoid the necessity for changing the focus of the telescope between the sights to the celestial body and to the mark. For the best results the mark should be nearly in the horizontal plane through the instrument, and should be raised at least four feet above the ground. The line of sight to it should be nowhere

less than this distance from the ground, to minimise the possibility of lateral refraction

For solar observations any form of opaque signal is used In the case of stellar observations, the mark must consist of a lamp placed behind a screen or in a box with an open top and provided with air holes in the bottom The light shines through a vertical slit or a circular aperture in the face, the effective width of the signal being such as to subtend at the instrument an angle of about $0''{\cdot}5$ to $1''$ according to the grade of the determination For the measurement in daylight of the azimuths of the survey lines from that of the reference mark, an opaque signal must be provided in the vertical of the aperture This may be arranged by painting the lamp screen with a narrow vertical line centrally with the aperture

Methods.—The principal methods of determining azimuth involve observations of

(1) Close circumpolar stars
(2) Circumpolar stars at elongation
(3) Ex-meridian altitudes of stars or the sun
(4) Hour angles of stars or the sun
(5) Equal altitudes of stars or the sun
(6) Circumpolar stars at culmination

Azimuth by Observations on Close Circumpolar Stars.—The most refined determinations are possible by this method, which embraces several systems of observation, each suitable for primary measurements The observation consists essentially of the application of a method of precise angle measurement (page 148), by direction or repetition, to the observation of the horizontal angle between the star and the terrestrial signal Since the position of the star is continually changing, the chronometer times of the individual observations are required, and the chronometer error and rate must be known From the corrected chronometer times the hour angle of the circumpolar star is obtained, and if the latitude of the place of observation is known, the astronomical triangle PZS can be solved for the azimuth angle A

When the circumpolar star is at elongation, its motion in azimuth is zero A refined determination cannot, however, be obtained by observing at the instant of elongation, since repeated measurements are required for the reduction of instrumental and observational errors. But a number of observations may be taken near elongation, preferably before and after The nearer the star is to elongation, the less is the observation affected by small errors of time, but the influence of an error in the latitude is then a maximum The

latter effect is eliminated by observing the same star near both eastern and western elongation, or, for greater convenience, two stars differing in right ascension by about 12^h are selected, and one is observed near its eastern and the other near its western elongation.

In the measurement of the horizontal angle between the star and the signal the routine as to changing face, swinging right and left, and number of zeros must be that followed throughout the survey (page 149) One complete measurement involves so many observations that a star may be at some distance from elongation during part of them. The routine of angle measurement should not be curtailed, and, when the time is accurately known, entirely satisfactory results may be obtained by observing a close circumpolar star at any hour angle

Observation.—Suitable stars may be selected from the list of close circumpolars of which the apparent positions are given in the *Nautical Almanac* for every day of the year Of the northern stars, *a Ursæ Minoris* (*Polaris*) is used whenever possible, and can be observed by daylight with the theodolites used in primary triangulation, if the sun is not too high The other stars are much fainter, of these 51H *Cephei*, δ, λ and 6B *Ursæ Minoris* are most frequently used The southern close circumpolars are all of smaller magnitude than the fifth The stars σ, υ and ρ *Octantis* are useful, but for determinations by small theodolites it may be necessary to observe β *Hydri* (mag 2·9) or β *Chamæleontis* (mag 4·4), which are much farther from the pole

For observations in the northern hemisphere the times of elongation of *Polaris* are first computed (page 31) If daylight observations are impracticable, it will be ascertained whether it is possible to use *Polaris* during the hours of darkness. If not, it will be necessary to select stars which elongate at a convenient time and such that the right ascensions of paired stars differ by about 12^h

When the direction method of angle measurement is used, the azimuth observations may be made in conjunction with the measurement of the horizontal angles of the survey In each round of angles is included a pointing upon the star The chronometer time of each bisection of the star is noted to $0^s.1$, and the striding level is carefully read If the instrument is fitted with an eyepiece micrometer, the bisections are made by means of the movable hair and not by the tangent screw. The small angle measured by the eyepiece micrometer lies in the plane containing the line of sight and the horizontal axis of the telescope It must be reduced to the horizontal by multiplying it by the secant of the altitude, the approximate value of which should therefore be obtained.

If the repetition system is employed, the angle between the star and one of the survey signals is multiplied in the usual manner,

notwithstanding that the angle is changing. Division by the number of repetitions gives the mean angle corresponding to the mean of the chronometer times of the observations.

A third method, capable of the highest degree of accuracy, consists in measuring by means of the eyepiece micrometer the small angle between the star and a signal placed nearly in the vertical of the star at elongation. The measurement is independent of readings of the horizontal circle, and a large number of face right and face left pointings may be secured in a short time The altitude of the star should be observed at intervals for reduction to the horizontal of the angle given by the micrometer readings

Reduction.—The azimuth angle A between the elevated pole and the star at any hour angle t is given by

$$\tan A = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

$$\text{or } \tan A = \sec \phi \cot \delta \sin t \left(\frac{1}{1-a}\right)$$

$$\text{where } a = \tan \phi \cot \delta \cos t$$

The latter form is due to Albrecht, and in conjunction with his tables of log $\left(\frac{1}{1-a}\right)$ is commonly used

The value to be taken for the hour angle is that corresponding to the mean of the corrected chronometer times of the n observations forming a set The resulting value of A is then corrected for the curvature of the apparent path of the star by the amount

$$\text{Curvature correction for set} = \frac{\tan A}{n} \Sigma \frac{2 \sin^2 \frac{1}{2}\Delta t}{\sin 1''}$$

where Δt represents the angular equivalent of the difference in sidereal seconds between the time of an individual observation and the mean of the set The correction always reduces the numerical value of A whether reckoned eastwards or westwards from the meridian.

If the horizontal axis is inclined during a pointing on the star or the signal, the horizontal circle reading falls to be corrected by

$$\text{Level correction} = \frac{d}{2n}(\Sigma W - \Sigma E) \tan h$$

where d = value of one division of striding level

ΣW and ΣE = sum of west and east end readings, reckoned from centre, of bubble in direct and reversed positions

h = altitude

For the northern hemisphere, if the east (west) end of the axis is the higher, the clockwise reading of the circle is too great (small).

In refined determinations, correction is required for the influence of the aberration of light caused by the rotation of the earth. The effect is to place the apparent position of the star east of its true position by the amount $0''{\cdot}32 \dfrac{\cos\phi \cos A}{\cos h}$, the value of which for close circumpolars is always practically $0''{\cdot}32$.

Azimuth by Circumpolar Stars at Elongation.—The routine followed in a primary determination by observations of circumpolar stars as just described may be greatly simplified for determinations of secondary accuracy If *Polaris*, or other circumpolar, is observed within a few minutes on either side of elongation, the motion in azimuth is scarcely perceptible in the telescope of a moderate size theodolite, and the azimuth of the star may be computed as its azimuth at elongation.

The time of elongation is computed in advance (page 31), and the instrument is set up and carefully levelled at least fifteen minutes before the time of elongation About five minutes before elongation a pointing is made upon the reference mark, and the micrometer readings are taken The star is then bisected, and after the readings are booked the telescope is reversed, and a second observation is made on the star Finally, the reference mark is again bisected A second set may be obtained without having to observe the star at more than five minutes from elongation When local time is not known, the star must be followed with the vertical hair until it appears stationary in azimuth, and the set is then completed If the watch error has been carefully determined, several sets may be secured by extending the range of the observations to, say, 20^m on either side of elongation, and reducing the position of the star to that at elongation

When the star is at elongation,

$$\sin A = \frac{\cos\delta}{\cos\phi}$$

where A is the angle measured the shorter way between the elevated pole and the star An approximate value for the latitude is sufficient.

The correction applicable to A when the star is observed at more than 5^m and less than 30^m from elongation is given in seconds of arc by the approximate formula,

$$c'' = 1{\cdot}96 \tan A \, (t_E - t)^2$$

where $t_E - t$ is the sidereal interval in minutes between the time of observation and that of elongation.

With a 5-in. or 6-in micrometer instrument, the error of the mean result from a single star should not exceed $5''$.

Example —On 1935 July 6 at a place in latitude 55° 50′ 17″ N., an azimuth observation was made on *Polaris* at eastern elongation, with the results tabulated. The referring object was east of the star. Find its azimuth

Object	Face	Horizontal Circle Readings					Mean			Angles between R O and Star		
		Micro A			Micro B							
		°	′	″	′	″	°	′	″	°	′	″
R O	L	246	15	20	15	35	246	15	28	110	41	13
Star	L	135	34	05	34	25	135	34	15			
Star	R	315	33	35	33	40	315	33	38	110	41	27
R O	R	66	14	55	15	15	66	15	05			
									Mean	110	41	20

$$\sin A = \frac{\cos \delta}{\cos \phi}$$

From *N A*, p 309, $\delta = +88° 57' 10''{\cdot}4$

$\log \cos \delta = 8\ 2618460$
$\log \cos \phi = 9\ 7493760$
$\log \sin A = 8{\cdot}5124700$
$A = 1° 51' 54''$

The star is at eastern elongation,
R O is 112° 33′ 14″ east of north,
or azimuth clockwise from south = 292° 33′ 14″

Azimuth by Ex-meridian Altitudes of Stars.—This method is very generally used for determinations of other than primary standard The observation has much in common with that for determining time by ex-meridian altitudes, and the two determinations may be combined if the watch times of the altitudes are noted A good knowledge of the observer's latitude should be available, and the astronomical triangle is solved for the azimuth angle

The star should be observed when changing rapidly in altitude and slowly in azimuth The most favourable situation therefore occurs when the star is on the prime vertical, the influence of errors of observed altitude being then a minimum As in the corresponding observation for time, an east star should be paired with a west star of similar altitude, and suitable stars should be selected in advance

The routine of observation for either star of a pair consists in first bisecting the reference mark, the readings of the horizontal circle being taken on both micrometers The telescope is then directed towards the star Since the star has a slow motion in azimuth and a rapid one in altitude, the telescope should be so pointed that the horizontal hair is in advance of the star. By means of the upper tangent screw the vertical hair is kept upon the star, and the motion in azimuth is stopped when the star reaches

the intersection of the hairs The readings of both micrometers on each circle are noted as well as that of the altitude level The telescope is transited, the star again bisected, and the new horizontal angle to the reference mark obtained as before A second set should be observed in the same manner from a new zero.

The azimuth angle A is conveniently obtained from

$$\tan\frac{A}{2} = \sqrt{\sec s \sin(s-h)\sin(s-\phi)\sec(s-p)}$$

$$\text{where } s = \frac{h+\phi+p}{2}.$$

ϕ is always treated as positive, and p is measured from the elevated pole The resulting value of A is the horizontal angle between the elevated pole and the average position of the star during the observation, and is measured the shorter way, east or west This quantity is applied to the mean horizontal angle between the star and the reference mark to give the angle between the latter and the elevated pole The possibility of confusion as to signs is avoided if the notes include a diagram showing the relative position of the pole, the star, and the reference mark The calculation of the azimuth of the reference mark is repeated from the other star of the pair, and the mean is accepted Determinations made in connection with deliberate mapping should be based upon two or three pairs of stars, when the error of the mean is not likely to exceed 5″

Example —On 1935 June 8 in latitude 56° 06′ 11″ N , ex-meridian altitudes were observed on a pair of stars with the results tabulated A compass bearing showed the referring object to be about 9° west of north Find its azimuth

Star	Mean horizontal angle from R O			Corrected mean altitude		
	°	′	″	°	′	″
η *Ursæ Majoris* (west)	80	52	01	66	54	33
δ *Cygni* (east)	102	06	45	60	02	54

η *Ursæ Majoris* $\delta = +49°\,38'\,10''$

		°	′	″			
h	=	66	54	33			
ϕ	=	56	06	11			
p	=	40	21	50			
Sum = $2s$	=	163	22	34			
s	=	81	41	17	log sec	=	0·8399444
$s-h$	=	14	46	44	log sin	=	9·4066926
$s-\phi$	=	25	35	06	log sin	=	9·6353326
$s-p$	=	41	19	27	log sec	=	0·1243683
					Sum	=	0·0063379
					log tan $\frac{A}{2}$	=	0·0031689

		°	′	″	
A	=	90	25	05	west of north
Angle between R.O and star	=	80	52	01	
R.O. west of north		9	33	04	

δ *Cygni*. $\delta = +44° \; 58' \; 10''$

		°	′	″			
h	=	60	02	54			
ϕ	=	56	06	11			
p	=	45	01	50			
Sum = $2s$	=	161	10	55			
s	=	80	35	28	log sec	=	0·7865380
$s-h$	=	20	32	34	log sin	=	9 5451916
$s-\phi$	=	24	29	17	log sin	=	9·6175282
$s-p$	=	35	33	38	log sec	=	0·0896417
					Sum	.	0·0388995
					log tan $\frac{A}{2}$	=	0·0194497

		°	′	″	
A	=	92	33	54	east of north
Angle between R O and star	=	102	06	45	
R O west of north		9	32	51	
Mean result					
R O west of north		9	32	58	

Azimuth by Ex-meridian Altitudes of the Sun.—In finding azimuth by ex-meridian altitudes of the sun near the prime vertical, observations in the morning should be balanced by a similar set in the afternoon The general features of the measurement are the same as for a star

Since the required altitudes and horizontal angles are those to the sun's centre, the hairs should be set tangential to two limbs simultaneously. After changing face, the opposite limbs are observed The appearance at contact presented by the ordinary Ramsden diagonal eyepiece for the two observations of a set is as in Fig 33 The motion in azimuth is slow, and the vertical hair is kept in contact by the upper slow motion screw, the sun being allowed to make contact with the horizontal hair. If the telescope has no vertical hair, the sun must be placed in the opposite angles as in Fig. 34.

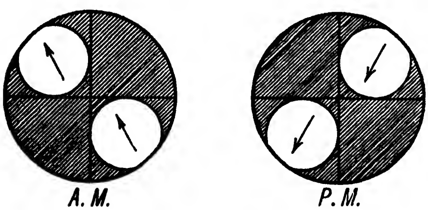


Fig 33.

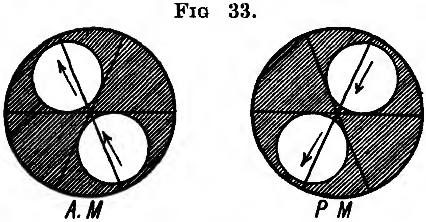


Fig 34.

A set of observations consists in sighting the reference mark and then the sun with telescope direct The telescope is then reversed, and the opposite limbs of the sun and finally the reference mark are sighted. This set may be followed by a second one in which the limbs should be taken in the reverse order The time of each altitude measurement is noted, so that the appropriate value of the sun's declination may be interpolated for use in the computation

The reduction is performed in the same manner as for the corresponding star observation.

Azimuth by Hour Angles of Stars or the Sun.—This method resembles that of ex-meridian altitudes, except that, in place of measuring altitudes, the chronometer times of the observations are recorded Horizontal angles are measured between the reference mark and east and west stars near the prime vertical From the corrected mean of the chronometer readings for a series of observations on a star the hour angle corresponding to the mean horizontal angle is obtained

Various formulæ for A are available, that generally used being

$$\tan A = \tan t \cos M \operatorname{cosec} (\phi - M)$$
$$\text{in which } M = \tan^{-1} \tan \delta \sec t.$$

In applying the formula, t is measured in arc the shorter way from upper transit, and is always positive, ϕ is positive for north latitudes and negative for south, A is the azimuth angle from the south part of the meridian measured the shorter way, east or west. In reducing forenoon and afternoon observations on the sun, δ is interpolated for the times of observation.

The accuracy of the result is largely dependent upon a good knowledge of the chronometer error and rate, and, in consequence, the method is less commonly used than those of ex-meridian altitudes or circumpolar stars at elongation.

Azimuth by Equal Altitudes of a Star.—By measuring the angle subtended between the reference mark and a star in two positions of equal altitude, the angle between the mark and the meridian is given by half the algebraic sum of the two observed angles. The method is independent of a knowledge of the star's co-ordinates, but is open to the objection that the determination involves a wait of several hours

Several observations are made with the star east of the meridian. The reference mark is first sighted, and the star is bisected by both cross-hairs. An equal number of observations are made with the telescope direct and reversed, and the altitude as well as the horizontal angle is noted for each. For the west series the same altitudes

are set on the circle, and for each individual altitude the same face as before is used. At each observation the star is kept bisected by the vertical hair, and the motion in azimuth is stopped when the star reaches the horizontal hair The algebraic mean of all the horizontal angles represents, without correction, that between the reference mark and the north or south part of the meridian, according to the position of the star.

Azimuth by Equal Altitudes of the Sun.—In this case a series of horizontal angles is measured between the reference mark and the sun in the forenoon In the afternoon a similar series is observed with the sun at the same altitudes Since the value of the altitude is not required, it is sufficient to observe the upper or the lower limb throughout, but an equal number of sights should be taken on the right and left limbs and with the telescope direct and reversed.

On account of the change in declination in the interval between the morning and afternoon equal altitudes, the mean of the horizontal angles does not represent that between the reference mark and the meridian The watch time of each observation should be carefully noted, and the value of the correction is computed as follows

Let c = angular correction to be applied to the algebraic mean of the observed angles to give the angle between the reference mark and the meridian

t = half the interval between the times of equal altitudes

ϕ = observer's latitude

δ_E = sun's declination at the average of the morning observations

δ_W = sun's declination at the average of the afternoon observations

Then $c = \frac{1}{2}(\delta_W - \delta_E) \sec \phi \operatorname{cosec} t$

When the sun's declination is changing towards the north (south), the mean of the observed azimuth angles lies west (east) of the meridian for places in north latitudes, and *vice versa* for south latitudes

Azimuth by Circumpolar Stars at Culmination.—A rough determination may be obtained by observing the horizontal angle between the reference mark and a circumpolar star on the meridian The watch error must be known, and the watch times of upper or lower transit of two or more circumpolar stars are computed in advance.

The reference mark is observed shortly before transit of the first star, and the telescope is then pointed to the star, which is

kept bisected by the vertical hair up to the computed instant of culmination The horizontal circle readings are taken, and the reference mark is again bisected as a check on the stability of the instrument No change of face is possible, and the observation of the second star should be made with the telescope reversed

The smaller the polar distance of the stars the better. Since their azimuths are changing rapidly, good results cannot be expected unless a large number of stars are observed The accuracy is largely dependent upon that with which the time is known

DETERMINATION OF LATITUDE

Observations for latitude nearly always consist in measuring the altitude of a celestial body when it is either on or near the meridian The determination of latitude is equivalent to measuring the altitude of the pole, and this may be obtained from the meridian or circum-meridian altitude of a body of known declination

The latitude ϕ of the place of observation is related to the meridian zenith distance z of a celestial body of declination δ as follows

Let ϕ and δ be marked + or − according as they are north or south, and z + or − according as the zenith is north or south of the body for the northern hemisphere, and the reverse for the southern Then $\phi = z + \delta$ when the body is at upper transit, as at S_1, S_2 and S_3 in Fig 35

If the star is at lower transit, as at S_4, the declination should be taken as the angle from the equator through the pole to the star, i e QOS_4 or $180° -$ the tabulated declination Hence $\phi = z + 180° - \delta$

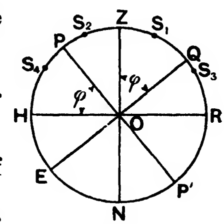


Fig 35

Methods.—The principal observations for determining latitude may be summarised as

(1) Talcott's method of meridian altitudes of stars.

(2) Circum-meridian altitudes of stars or the sun

(3) Meridian altitudes of stars or the sun

(4) Altitudes of *Polaris*

(5) Ex-meridian altitudes of stars

(6) Equal altitudes of stars

Latitude by Talcott's Method.—Precise latitude determinations are made by means of the zenith telescope (page 41) by the method devised by Captain Talcott of the U S Corps of Engineers about the year 1834. Less refined determinations may be obtained on the

same system by the use of a theodolite fitted with an eyepiece micrometer and having a sensitive altitude level

The observation consists in measuring the small difference in meridian zenith distance of two stars culminating on opposite sides of the zenith at very nearly the same altitude and within a short time of each other In Fig. 35 let S_1 and S_2 represent two stars of declination δ_1 and δ_2 respectively, and let their meridian zenith distances in latitude ϕ be respectively z_1 and $-z_2$.

$$\begin{aligned} \text{Then } \phi &= z_1+\delta_1 = -z_2+\delta_2 \\ 2\phi &= \delta_1+\delta_2+z_1-z_2 \\ \text{or } \phi &= \tfrac{1}{2}(\delta_1+\delta_2)+\tfrac{1}{2}(z_1-z_2) \end{aligned}$$

The declinations being known, ϕ is therefore derived from the measurement of the difference of the meridian zenith distances.

The method lends itself admirably to precise determinations The north and south stars selected to form a pair are such as have nearly the same zenith distance, so that z_1-z_2 can be accurately measured by means of the eyepiece micrometer The refraction correction required is only the difference between those applicable to the individual zenith distances, and the effect of uncertainty in the value of the refraction coefficient is therefore negligible. In addition small errors of pointing, arising from error of collimation, dislevelment of horizontal axis, or inaccuracies of setting in the meridian, exercise very little influence on the measurement of the small quantity z_1-z_2.

Observations in Talcott's Method.—A primary determination having a probable error of measurement not exceeding $\pm 0''\cdot 1$ involves the observation with the zenith telescope of from ten to forty pairs of stars, and requires from one to three or four nights' work. An observing list of pairs of stars is first prepared, and for this purpose the latitude should be known to the nearest 1′. Suitable stars are selected from a modern star catalogue, and must fulfil the following conditions —

(1) The zenith distance should not exceed 45° in order to guard against too great uncertainties in refraction

(2) The difference in zenith distance of stars forming a pair should be within range of the micrometer.

(3) The interval between the times of transit of the stars of a pair, or their difference in right ascension, should for accuracy and convenience of observation be not less than 1^m or greater than 20^m.

(4) The interval between the transits of the second star of one pair and the first of the next should be not less than 2^m.

(5) The sum of the north zenith distances in a series should balance as nearly as possible that of the south zenith distances to minimise errors arising from inaccurate knowledge of the value of a micrometer division and irregularities in the screw.

The setting of the telescope in zenith distance for the observation of a particular pair of stars is the mean of their zenith distances or half their difference in declination The setting angle for each pair is tabulated to the nearest 1′ in the observing list, and from it and the known value of a turn of the micrometer screw the number of turns required to bisect each star of a pair is estimated in advance and entered in the list. The right ascensions are also tabulated to the nearest second to show the sidereal interval between transits and to aid in the identification of faint stars

The telescope is placed in the plane of the meridian, and the azimuth stops are set by first sighting a previously established meridian mark The adjustment is finally effected when the times of transit of two or more stars, referred to a sidereal time chronometer of known error, agree with their right ascensions. In observing the first of a pair of stars, the movable hair is placed at the position at which the transit is expected When the star reaches the centre of the field, it is bisected, and the micrometer and latitude level readings are taken The telescope is then swung through 180° in azimuth, the clamp block being brought gently up to the second azimuth stop The hair is approximately set for the second star, which is similarly observed at transit In the interval between the two observations of a pair no change must be made in the inclination of the telescope relatively to the latitude level. Occasionally a star has to be observed a little out of the meridian although within the field of the telescope as set The sidereal time of the observation is then noted so that the observed zenith distance may be reduced to the required meridian zenith distance

Reduction.—The fundamental formula

$$\phi = \tfrac{1}{2}(\delta_1+\delta_2)+\tfrac{1}{2}(z_1-z_2)$$

in which the subscripts 1 and 2 refer respectively to the south and north stars of a pair, may be rewritten to express z_1-z_2 in terms of the corresponding micrometer readings It therefore becomes

$$\phi = \tfrac{1}{2}(\delta_1+\delta_2)+\tfrac{1}{2}(m_1-m_2)T$$

in which m_1 and m_2 are the micrometer readings for the south and north stars respectively, and T is the angular value of one turn of the micrometer screw. This expression applies if the micrometer readings increase with increasing zenith distance; otherwise the signs of m_1 and m_2 are reversed. The values of the declinations must be computed for the nights of the observations

from the elements given in the star catalogue unless the star happens to be in the *Nautical Almanac* list In the case of a star below the pole $180° - \delta$ is inserted in the formula in place of δ

In the reduction of each pair of observations it is necessary to apply corrections for the inclination of the latitude level during the observations and for refraction.

Level Correction —The correction to altitude for a single observation being $\frac{1}{2}d(O-E)$ (page 57), that to be added to zenith distance is $\frac{1}{2}d(E-O)$ The respective corrections for the south and north stars are therefore $\frac{1}{2}d(E_1-O_1)$ and $\frac{1}{2}d(E_2-O_2)$, and the correction to $\frac{1}{2}(z_1-z_2)$ or to ϕ is

$$\frac{d}{4}\Big((E_1-O_1)-(E_2-O_2)\Big)$$

The level readings are commonly booked as north and south end readings of the bubble, in which case, writing N_1 for E_1, S_1 for O_1, S_2 for E_2, and N_2 for O_2, the level correction may be expressed as

$$\frac{d}{4}\Big((N_1+N_2)-(S_1+S_2)\Big)$$

Refraction Correction This correction is the difference between the refraction corrections applicable to the individual zenith distances, and, as its value is small, it is sufficient to assume mean atmospheric conditions and to use the formula $r = 58'' \tan z$ If r_1 and r_2 are the respective refraction corrections for the south and north star observations, the additive correction to $\frac{1}{2}(z_1-z_2)$ or to ϕ is $\frac{1}{2}(r_1-r_2)$ which, since z_1-z_2 is small, may be evaluated from

$$58''.\tfrac{1}{2}(z_1-z_2)\sin 1' \sec^2 z = 0''{\cdot}017\ \tfrac{1}{2}(z_1-z_2)\sec^2 z$$

where z_1-z_2 is expressed in minutes.

With the level and refraction corrections applied, the latitude formula therefore becomes, with T expressed in seconds

$$\phi = \tfrac{1}{2}(\delta_1+\delta_2)+\tfrac{1}{2}T(m_1-m_2)\,(1+0{\cdot}00028\sec^2 z) + \frac{d}{4}\Big((N_1+N_2)-(S_1+S_2)\Big)$$

Reduction to the Meridian When a star is observed out of the meridian, the line of sight of the telescope being maintained in the meridian, the correction to be applied to the zenith distance of the star, as observed, to give the meridian zenith distance is

$$\sin^2 \tfrac{1}{2}t \sin 2\delta \operatorname{cosec} 1''$$

in which t is the hour angle of the star at the instant of observation, and δ is its declination. Half of this amount represents the

correction to the latitude, the sign always being positive in the case of stars of north declination and negative for stars of south declination and stars below the pole. In the event of both stars of a pair being observed out of the meridian, the value of the correction is computed for each

Latitude by Circum-meridian Altitudes of Stars.—This is the most accurate of the methods commonly used in field determinations for mapping purposes It consists in observing a series of altitudes at noted times of each of several stars for a few minutes before and after transit From the hour angle of the star at the instant of each observation the measured circum-meridian altitudes are reduced to the meridian, and the mean result furnishes a better measurement of meridian altitude than can be obtained directly

The effects of an erroneous value for the refraction correction, personal errors of bisection, and instrumental flexure are reduced to a minimum by observing an equal number of north and south stars in pairs of similar altitude The method requires a good knowledge of the chronometer error

The stars selected are such as have a meridian altitude preferably not less than 40° to avoid irregular refraction The reductions are simplified if an upper limit is assigned to the altitudes, and this may be put at 80°. From an approximate knowledge of the latitude of the place the limits of declination of suitable north and south stars are known, and from an approximate value of the longitude the required right ascensions are obtained so that the transits will occur at a convenient time and at suitable intervals An observing period of about 20^m should be allowed for each star.

The observation of each star is commenced about 10^m before the computed time of transit The altitudes are observed in rapid succession, the chronometer time of each being recorded to the nearest $0^s{\cdot}1$ or $0^s{\cdot}2$ Observations are continued for about 10^m after transit. The measurements for each star may consist of an equal number of face right and face left altitudes, but change of face is unnecessary and inadvisable if the observations are adequately paired on north and south stars There should be the same number of observations on either side of the meridian in order to reduce the effect of an erroneous value of the chronometer error.

The reduction to the meridian is computed as follows.

Let z_0 = observed circum-meridian zenith distance, corrected for refraction, and positive or negative according as the zenith is north or south of the star

z = required meridian zenith distance

z_1 = approximate meridian zenith distance, deduced from the greatest observed altitudes

ϕ_1 = approximate latitude derived from z_1, positive or negative according as it is north or south
t = hour angle of star when observed
δ = declination of star
$A = \cos\phi\cos\delta\operatorname{cosec} z = \cos\phi_1\cos\delta\operatorname{cosec} z_1$ very nearly
$$m = \frac{2\sin^2\frac{1}{2}t}{\sin 1''}$$

Then $z = z_0 - Am$

With greater accuracy, $z = z_0 - Am + Bn$

$$\text{where } B = A^2\cot z, \text{ and } n = \frac{2\sin^4\frac{1}{2}t}{\sin 1''}.$$

The approximate formula $z = z_0 - Am$ is usually sufficient provided the observations are confined within a period of 10^m on either side of transit and the star is not too near the zenith The value of A used is that derived from the approximate values of the zenith distance and latitude, and, strictly, a second approximation should be made with the closer values of z and ϕ obtained from the first Since m is a function of $\sin^2\frac{1}{2}t$, and t is small, the correction to z_0 varies as the square of the hour angle, and, in consequence, the mean of a number of observed altitudes or zenith distances does not correspond with the average of their chronometer times. The observations cannot therefore be averaged in the usual way

The values of the variable m should be taken out for the individual observations, either by calculation or from tables for the reduction of circum-meridian observations to the meridian.

If z_0' = the arithmetical mean of the observed zenith distances, each corrected for refraction
m' = the arithmetical mean of the individual values of m
then $z = z_0' - Am'$
and $\phi = z + \delta$

The value of z may be worked out for each observation in order that the degree of consistency in the values may be ascertained.

With a 5-in. or 6-in. micrometer theodolite, the error of latitude derived from a single pair of stars should not be more than about 3″. Observation of ten pairs of stars should give the latitude correct to about 1″.

Latitude by Circum-meridian Altitudes of the Sun.—In applying the method to solar observations, the precision is much decreased since the observations cannot be paired. The altitudes are measured to the upper and lower limbs alternately. In reducing to the

meridian, the value of the declination should, strictly, be computed for the instant of each observation, but practically it is sufficient to use the declination corresponding to the mean instant of the set.

Example—On 1935 June 15, in longitude 4° 59′ 05″ W, circum-meridian observations of the sun gave the results tabulated. The altitudes given represent the means of the two micrometer readings corrected for level. The error of the watch was known to be $17^m\ 04^s{\cdot}4$ fast on L M T. Find the latitude.

Object	Face	Watch Time			Observed Altitude			Pair of Max Observed Altitudes			Remarks
		h	m	s	°	′	″	°	′	″	
$\underline{\odot}$	L	12	09	53·0	56	53	06·5				Bar 30·1 in.
$\overline{\odot}$	R	12	12	59·6	57	25	43·0				Ther 72°F
$\underline{\odot}$	R.	12	15	39·2	56	54	40·0	56	54	40·0	
$\overline{\odot}$	L	12	18	31·8	57	26	13·0	57	26	13·0	
$\underline{\odot}$	L	12	21	32·0	56	53	58·5				
$\overline{\odot}$	R	12	23	55·4	57	24	54·0				
	Means	12	17	05·2	57	09	45·8	57	10	27	

(1) *True mean zenith distance, z'_0*

		°	′	″
Mean observed altitude	=	57	09	45·8

Mean refraction = 37·6
Barometer correction = +0·1
Temperature correction = −1·7

Refraction	=	−		36·0
Parallax 8″·67 × cos 57°	=	+		4·7
True altitude	=	57	09	14·5
z'_0		32	50	45·5

(2) *Watch time of L A N*

	h	m	s
From *N A*, p 25, G M T of G A N	12	00	06·9
Correction for longitude			+0·2
L M T of L A N	12	00	07·1
But watch is fast on L M T by		17	04·4
Watch reading at L A N	12	17	11·5

(3) *Sun's δ at mean instant of observation.*

		h	m	s
Mean of observed watch times	=	12	17	05·2
Watch time of L.A.N.	=	12	17	11·5
Mean instant of observation is before L A N by				6·3
G.A.T. of observation = $12^h\ 19^m\ 56^s{\cdot}3 - 6^s{\cdot}3$	=	12	19	50·0

		°	′	″
From *N.A.*, p 25, δ at G A N.	=	+23	17	11·7
Correction	+			2·2
δ	=	+23	17	13·9

(4) *Approximate meridian zenith distance z_1 and latitude ϕ_1.*

		°	′	″
Mean of two greatest observed altitudes	=	57	10	27
Refraction and parallax	=	—		31
Approximate meridian altitude		57	09	56
z_1	=	32	50	04
δ		23	17	14
ϕ_1		56	07	18

(5) *Am′*.

Sun's hour angle at each observation = difference between watch time of observation and watch time of L A N

t		m
m	s	″
7	18·5	104·8
4	11 9	34 6
1	32·3	4·6
1	20·3	3·5
4	20·5	37·0
6	43·9	89·0

Mean = m' = 45·6

$\log m'$	=	1·6590
$\log \cos \phi_1$	=	9·7462
$\log \cos \delta$	=	9 9631
$\log \operatorname{cosec} z_1$	=	0·2658
$\log Am'$	=	1·6341
Am'	=	43″·1

(6) *Latitude*

		°	′	″
$z = z'_0 - Am'$	=	+32	50	02
δ	=	+23	17	14
ϕ	=	+56	07	16

Latitude by Meridian Altitudes of Stars.—The measurement of a meridian altitude does not enable accidental errors of observation to be reduced so effectively as in the case of the multiple observations of circum-meridian altitudes. The reduction of meridian observations, however, is so simple that the number of stars observed can be suitably increased without undue labour.

The direction of the meridian may not be accurately known, and the watch time of transit should be computed in advance. Strictly, one observation of each star is all that can be obtained, but it is usually possible to secure two readings, since the altitude of the star is changing slowly near transit If the watch error is

unknown, the star may be followed with the horizontal hair until it appears stationary in altitude After the level and circle readings are taken, a second altitude is measured as soon as possible, on the reverse face in the case of an unpaired star.

Errors of observation, refraction and instrument are effectively reduced by observing pairs of north and south stars of similar altitude culminating within a short time of one another. The observation is then similar in principle to Talcott's method, and the pairs of stars are selected on similar lines The altitudes of the north and south stars of a pair being nearly equal, it is unnecessary to change face, since only the difference of the corrected zenith distances is required in computing the latitude With a 5-in or 6-in micrometer theodolite, the latitude deduced from meridian observations of a pair of stars should be within 4″ or 5″ of the truth

Latitude by the Meridian Altitude of the Sun.—Measurement of the sun's altitude at apparent noon affords only a rough value of the latitude owing to the impossibility of balancing the observation. The method is, however, very useful for rough determinations, and is that employed in navigation

Both faces must be used in a theodolite measurement. The watch time of apparent noon should be computed in advance, so that the altitude may be taken as nearly as possible on the meridian Otherwise, the horizontal hair is kept upon the upper or the lower limb until the greatest altitude is attained, or a series of altitudes is measured in quick succession on alternate faces at about the time of transit, the greatest altitude derived from a F.R. and F.L. pair being accepted as the meridian altitude. The difference between the greatest altitude and the meridian altitude, due to changing declination, is neglected.

Example —Find the approximate latitude given by the following meridian observations of the sun made on 1935 June 13 in longitude 4° 59′ 05″ W The observations given are the pair showing the greatest altitude

Object	Face	Altitude								Level		Remarks
		Micro I			Micro II		Mean			E	O	
		°	′	″	′	″	°	′	″			
$\underline{\odot}$	L.	56	47	20	47	25	56	47	22·5	13	14	Bar 30·1 in Ther 70°F
$\overline{\odot}$	R	57	19	45	19	35	57	19	40	15	12	Level div = 8″
							57	03	31	28	26	

$$\text{Level correction} = \frac{26-28}{2\times 2}\times 8'' = -4''$$

Mean observed altitude	=	57°	03′	27″
Mean refraction = 37″				
Barometer correction = +1				
Temperature correction = −2				
Refraction	=			−36
Parallax $8''{\cdot}7 \times \cos 57°$	=			+ 5
True altitude	=	57	02	56
z	=	32	57	04

Sun's Dec. at L A N.

West longitude in time = $19^m\ 56^s$

From *N A*, p 25, δ at G A.N.	= +23°	10′	42″·9	
Correction	+		2·8	
δ at L A N	= +23	10	46	
z	= +32	57	04	
ϕ	= +56	07	50	

Latitude by Meridian Altitudes of a Circumpolar Star at Upper and Lower Transits.—If the altitude of a circumpolar star can be measured at both upper and lower transits, the latitude is given by half the sum of the altitudes separately corrected for refraction. A knowledge of the declination of the star is not required.

The method is of limited value in the field. The two observations are separated by twelve sidereal hours, so that they can be made with small instruments only when the hours of darkness exceed twelve.

Latitude by Altitudes of Polaris.—If the error and rate of the chronometer are known, latitude may be determined by observing the altitude of *Polaris* at any instant

The observation consists in taking a set of alternate face right and face left altitudes in succession and noting the chronometer time of each The average of, say, two face right and two face left altitudes is assumed to correspond with the average of the chronometer times, and from these the latitude is computed. Several such sets should be observed, and if the resulting latitudes are consistent, their mean is accepted

The latitude ϕ is computed from

$$\phi = h - p\cos t + \tfrac{1}{2}p^2 \sin 1'' \sin^2 t \tan h$$

where h = observed altitude, corrected for refraction

p = polar distance, in seconds

t = hour angle = L S T. of observation − R A of *Polaris*

The second and third terms of the formula constitute two corrections, in seconds of arc, to the measured altitude The sign

of the first is controlled by that of $\cos t$; the second is always positive. Further corrections are omitted since their sum is always less than 1″

Since the observation of low altitudes is to be avoided on account of uncertainties in refraction, the method cannot be applied in low latitudes. Errors of refraction affect the result by their whole amount unless the observation is paired with an observation of circum-meridian altitudes of a south star of similar altitude, when the accuracy of the determination becomes little inferior to that of circum-meridian altitudes of paired stars.

Example—On 1935 June 8 in longitude 4° 59′ 05″ W, the mean of four observed altitudes of *Polaris* was 55° 21′ 44″, the average of the local mean times being $23^h\ 38^m\ 04^s{\cdot}5$ The barometer and thermometer readings were 30·1 in and 57° F respectively Find the latitude of the station

$$\phi = h - p\cos t + \tfrac{1}{2}p^2 \sin 1'' \sin^2 t \tan h$$

h

			°	′	″
Observed altitude	″	=	55	21	44
Mean refraction =	39				
Barometer correction =	+1				
Temperature correction =	−1				
Refraction		=			−39
h		=	55	21	05

p

		°	′	″	
Dec of *Polaris*	=	88	57	12·2	
p	=	1	02	47·8	= 3768″

t

		h	m	s
L M T	=	23	38	04·5
L S T (as in *Example* 9, p 30)		16	43	49
R A of *Polaris*	=	1	39	10
Hour angle	=	15	04	39

First correction

log p	=	3·5761
log cos t	=	9·8405n
Sum	=	3·4166n
First correction	=	+2610″ = +43′ 30″

Second correction

log ½	=	9·699
2 log p	=	7·152
log sin 1″	=	4·686
2 log sin t		9·716
log tan h	=	0·160
Sum	=	1·413
Second correction	=	+ 26″

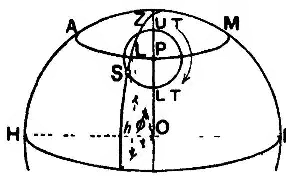


FIG 36

ϕ = 55° 21′ 05″ + 43′ 30″ + 26″
= 56° 05′ 01″ N

Note—Fig 36 illustrates the position of the star for the above observation APM represents the almucantar, or small circle of equal altitudes, through P. The sum of the corrections is represented by the angle SOL.

Latitude by Ex-meridian Altitudes.—Evidently an ex-meridian altitude of a star of known declination would enable the astronomical triangle to be solved for ϕ if the time of the observation were known An accurate knowledge of t and therefore of the watch error at the instant of observation is required. The observation should consist of altitude measurements of pairs of north and south stars disposed on both sides of the meridian. The method, however, is little used, and is much inferior to that of circum-meridian altitudes, since the effect of errors of time and altitude on the derived latitude increases from a minimum when the body is on the meridian to a maximum when it is on the prime vertical

An accurate knowledge of the watch error is not required if observation is made of two altitudes, either of the same star or of different stars. In this case, latitude and time can be obtained if the interval between the observations is accurately known from the watch rate For, if S and S′ be the two stars, triangle PSS′ can be solved for SS′ and angle PSS′ Triangle ZSS′ can be solved for angle ZSS′, which enables triangle PZS to be solved for PZ, the co-latitude, and ZPS, the hour angle of S. The accuracy of the derived latitude is largely dependent upon having one of the stars near the meridian

Latitude by Equal Altitudes.—Several methods of latitude determination are based upon observation of the times of equal altitudes of stars Until recently these methods had not been applied to any extent in field astronomy, but they have now been successfully employed on the Peru-Bolivia and Brazil-Bolivia boundary surveys, and have been extensively used in Egypt * Since the actual altitude is not required, the results are independent of circle readings. The observation is that for which the prismatic astrolabe (page 48) has been primarily designed

Equal Altitudes of Two Stars.—The altitude, declination and hour angle of a star, and the latitude of the place have the relationship

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t$$

If, therefore, two stars of declinations δ_1 and δ_2 respectively are observed to have the same altitude h with hour angles t_1 and t_2, we have

$$\sin \phi \sin \delta_1 + \cos \phi \cos \delta_1 \cos t_1 = \sin \phi \sin \delta_2 + \cos \phi \cos \delta_2 \cos t_2$$

whence ϕ is derived independently of h.

The stars forming a pair must be situated north and south of the

* "Modern Methods of Finding the Latitude with a Theodolite," by Dr. Ball *The Geographical Journal*, June 1917.

zenith, and should not differ greatly in right ascension. They should be observed near the meridian, and the watch error and rate must be accurately known.

Equal Altitudes of Three Stars.—To avoid the necessity for an exact knowledge of the watch error, observation may be made of three or more stars at equal altitudes Both latitude and time may then be obtained, and this method is better adapted for field determinations than the last The stars should be near the meridian, and one of them on the remote side of the zenith from the others In the northern hemisphere it is best to have one north star very near the meridian and two south stars not more than an hour from it. The watch rate must be known so that the intervals between the observations may be accurately obtained If these intervals are denoted by i_1 and i_2 for a three-star observation, then

$$\sin h = \sin \phi \sin \delta_1 + \cos \phi \cos \delta_1 \cos t_1$$
$$\sin h = \sin \phi \sin \delta_2 + \cos \phi \cos \delta_2 \cos (t_1 + i_1)$$
$$\sin h = \sin \phi \sin \delta_3 + \cos \phi \cos \delta_3 \cos (t_1 + i_2)$$

from which, by subtraction, are obtained two simultaneous equations for ϕ and t_1 The solution is considerably simplified by assuming values for ϕ and the watch error and computing the value of h for each observation The results, h_1, h_2 and h_3, will in general differ from the true altitude h because of errors in the assumed values, but if

$$d\phi = \text{error of assumed latitude}$$
$$dt = \text{error of watch}$$

$d\phi$, dt and h are obtainable from the simultaneous equations

$$h_1 - h + \cos A_1 d\phi + \cos \phi \sin A_1 dt = 0$$
$$h_2 - h + \cos A_2 d\phi + \cos \phi \sin A_2 dt = 0$$
$$h_3 - h + \cos A_3 d\phi + \cos \phi \sin A_3 dt = 0$$

where A is the azimuth of each observation, either observed or computed. If more than three stars are observed, the reduction may be performed by treating the observations in groups of three rather than by the rigorous method of least squares

Instead of observing three different stars, the method may be modified by using two stars and observing one of them twice. The double observation on the same star on opposite sides of the meridian gives the watch error, and the reduction is simplified.

Dr. Ball states that, if carefully carried out, the three-star

method applied to a single set of stars not more than an hour or so from the meridian and with a level sensitive to 5″ per division, usually gives the latitude to within 3″ of the truth

DETERMINATION OF LONGITUDE

Since the longitude of a place is the difference between its local time and that at the reference meridian at the same instant, observation of the difference of time between two places determines their difference of longitude.

Determinations may be classed as relative or absolute By a relative measurement is meant one in which the longitude of one place is found from the known longitude of another by observing the difference of time between them. In an absolute determination astronomical observations are made for Greenwich time, and the required longitude is obtained directly from the meridian of Greenwich. Absolute methods are all of inferior precision, and need not be further considered The only relative method that is now employed is the comparison of local time, as found by astronomical observations, with Greenwich mean time, as broadcast from one or more wireless stations

Wireless Signals.—Greenwich mean time signals are sent out, at stated hours, from a number of powerful long-wave transmitting stations, situated in various parts of the world The *Admiralty List of Wireless Signals,* published annually, gives full particulars of the time of emission, wave length, and type of signal sent out by each station Any changes are notified in the weekly *Notices to Mariners.* Although various types of signal are sent out, the "Standard Rhythmic" signal is the only type which is generally made use of by surveyors, because it is the only type which permits of an easy and accurate comparison between the wireless signal and a chronometer. The following Standard Rhythmic signals are clearly audible in all parts of the British Isles

Station	Times of emission						
	h m		h m		h m		h m
Bordeaux	8 01	to	8 06	and	20 01	to	20 06
Paris (Eiffel)	9 31	to	9 36	and	22 31	to	22 36
Rugby	9 55	to	10 00	and	17 55	to	18 00
Nauen	12 01	to	12 06	and	0 01	to	0 06

The Standard Rhythmic signal consists of a series of Morse dots, which are sent out at the rate of 61 dots per minute of mean time, for

five consecutive minutes. The zero of the series, and the end of each minute, is marked by a dash instead of a dot.

Reception of Wireless Signals.—The method of receiving the time signals is as follows The surveyor must be equipped with either a mean time or a sidereal time chronometer, beating half seconds. We will assume in the first instance that he has a mean time chronometer Since the wireless signal consists of 61 beats to the minute, he will hear the wireless beats overtaking the chronometer beats until the two coincide Coincidence appears to be maintained for a few seconds, and then the wireless beats go ahead of the chronometer. The surveyor must then determine which particular beat of the wireless coincided with which beat of the chronometer To do this he counts the wireless beats , the first beat after the dash is " one," the second beat " two," and so on As the wireless and chronometer beats approach coincidence, he notes the correspondence For instance, he finds the 28th wireless beat corresponds with the 4th second of the chronometer The beats appear to coincide exactly from the 30th to the 34th wireless beat He therefore records coincidence as occurring on the 32nd wireless beat This beat corresponds with the 8th second on the chronometer

The following is a record of a complete observation .—

	Rugby Time Signal			Mean Time Chronometer		
	h	m	beat	h	m	s
	9	55	32	9	54	08
	9	56	33	9	55	09
	9	57	33	9	56	09
	9	58	33	9	57	09
	9	59	32	9	58	08
Mean	9	57	32·60	9	56	08·60

The wireless beats must be reduced to seconds , since there are 61 beats to a minute 32·60 beats equal $32{\cdot}60 \times 60/61$ seconds, i e 32·07 seconds Hence the Greenwich time of the mean wireless signal was $9^h\ 57^m\ 32^s{\cdot}07$ and the chronometer time $9^h\ 56^m\ 08^s{\cdot}60$. The chronometer was therefore $1^m\ 23^s{\cdot}47$ slow on Greenwich mean time.

If a sidereal time chronometer is used the coincidences will occur at intervals of approximately 72 sidereal seconds.

Example—The following coincidences were observed on 1933 June 1, with a sidereal time chronometer.

Rugby Time Signal					Sidereal Time Chronometer		
	h	m	beat		h	m	s
	9	55	16		2	36	42
	9	56	29		2	37	54
	9	57	43		2	39	07
	9	58	56		2	40	19
			Mean		2	38	30·50
The mean of the Rugby signals				=	9	56	30+36 beats
36 beats = 36×60/61 seconds				=			35·41
Mean of Rugby signals				=	9	57	05·41
$9^h\ 57^m\ 05^s{\cdot}41$ mean time				=	9	58	43·50 S T
G S T of 0^h on June 1				=	16	36	06·17
G S T of mean Rugby signal				=	2	34	49·67
Mean of chronometer times				=	2	38	30·50
Chronometer is fast						3	40·83 on G S T

It is possible to observe coincidences on the half-second beat of a chronometer as well as on the full second But in most instruments the half-second beats do not fall exactly at the half second It is advisable therefore to use only the full-second beats

Equipment.—Special long-wave wireless receiving sets fitted with frame aerials are made for the reception of time signals A surveyor in any part of the world equipped with one of these sets should be able to hear the time signals from at least one station

Precise Reception of Wireless Signals.—Accuracy in the reception of time signals is best attained by reducing, or eliminating, the personal errors made by the observer in estimating the moment of coincidence Personal errors can be eliminated by recording both the chronometer and wireless beats on a chronograph But the recording apparatus introduces a time lag which may be a source of serious errors The best and simplest method is to arrange for the chronometer to short circuit the wireless set on each full-second beat. As coincidence approaches, the wireless dots are progressively shortened until at coincidence one or two are completely suppressed. In this way the exact moment of coincidence can be determined with great accuracy

Although every precaution is taken to send out time signals at the scheduled time, small errors, amounting to a few hundredths of a second, are unavoidable The daily corrections to be applied to the signals sent out from the principal stations are published in subsequent issues of the Admiralty *Notices to Mariners* and the *Bulletin Horaire.*

COMBINED DETERMINATIONS

The principal observations that serve for the determination of more than one quantity are those of ex-meridian altitudes and

equal altitudes for time and azimuth, of equal altitudes for time and latitude, and of transits for time, azimuth and latitude. Such observations are commonly made on the sun, and are frequently useful for approximate determinations

In observing for time and azimuth, the routine is the same as for azimuth alone, except that the watch times of the observations are noted. The method of obtaining time, azimuth and latitude consists in observing a series of circum-meridian altitudes with the corresponding watch times and horizontal angles from a referring object. Rough results are obtained by taking the greatest observed altitude as the meridian altitude, the corresponding watch time as that of transit, and the horizontal angle as the azimuth of the referring object. The accuracy is improved by plotting the circum-meridian altitudes on a watch time base, drawing a smooth curve through them, and interpolating the meridian altitude, the watch time of transit, and the required azimuth *

* See "Meridian Diagrams," by C A A Barnes, *Min Proc Inst C E*, Vol CLXI

EXAMPLES

1 In a rough determination of the watch error on 1935 October 15, at a place in longitude 170° 12′ 15″ E, the sun was observed to transit at $11^h\ 50^m\ 17^s$. Find the error of the watch on L M T and on New Zealand standard time, which is $11^h\ 30^m$ E of Greenwich E at October $15^d\ 0^h$ G M T is $+13^m\ 54^s{\cdot}0$, change in 24^h is $+13^s{\cdot}7$

2 Describe and explain the method of determining time by an ex-meridian observation of the sun, stating what data are required in the reduction

On 1935 July 2, at a place in longitude 6° 45′ W, an ex-meridian measurement of the sun gave an hour angle of 68° 15′, the watch time of the observation being $17^h\ 05^m$ The value of the equation of time for 0^h G M T on July 3 is $-3^m\ 47^s{\cdot}9$, the change being $-11^s{\cdot}5$ per 24^h Find the error of the watch on G M T

3. On 1935 April 21, at a place in latitude 36° 55′ 08″ N and longitude 57° 53′ 30″ E, the mean observed altitude of the sun corrected for refraction and parallax was 42° 32′ 14″·3 The mean watch time of the observation was $14^h\ 58^m\ 56^s{\cdot}8$ What was the watch error on L M T ?

Date	G M T of G A N.		Dec at G A N	
	h m s	s	° ′ ″	″
April 19	11 59 16·95		+10 56 45·6	
		−13·36		+1246·2
20	11 59 03·59		+11 17 31·8	
		−12·93		+1235·2
21	11 58 50·66		+11 38 07·0	
		−12·50		+1223·7
22	11 58 38·16		+11 58 30 7	

4 The observed watch times of equal altitude of *α Aurigæ* (R A = $5^h\ 10^m\ 57^s{\cdot}8$) at a place in longitude 28° 04′ E were $21^h\ 04^m\ 26^s{\cdot}2$ and $24^h\ 58^m\ 03^s{\cdot}4$ Find the error of the watch on L M T, if G S T of 24^h G M T for the date was $6^h\ 03^m\ 57^s{\cdot}7$.

5 In a comparison between a S T and a M T chronometer at a place in longitude 85° W, the readings at a coincidence of beat were respectively $12^h\ 45^m\ 29^s{\cdot}5$ and $22^h\ 06^m\ 07^s{\cdot}5$ The error of the S T chronometer was $2^m\ 14^s{\cdot}6$ slow on L S T What was the error of the M T chronometer on L M T if G S.T of 24^h G M T for the date was $14^h\ 45^m\ 20^s{\cdot}5$?

6 On the afternoon of May 12 (5 p m by Greenwich mean time) at a place A, whose latitude was 51° 30′ 20″ N, the altitude of the sun's centre was found to be 23° 05′ 20″, whilst the horizontal angle between the fixed line AB and the direction to the sun's centre was 18° 20′, the sun having crossed the line over an hour before

Determine the azimuth of the sun from the south at the time of the observation, and the azimuth of the line AB, having given —

Correction for refraction = 58″ × tangent zenith distance.

Date	Sun's Dec at 0^h G M T	
	° ′ ″	″
May 11	+17 34 53·9	
		+936·0
12	+17 50 29·9	
		+918·1
13	+18 05 48·0	
		+899·8
14	+18 20 47·8	

Sun's horizontal parallax = 8″·7

7. The following observations of the sun were taken for azimuth of a line in connection with a survey.—

Mean time $16^h\ 30^m$. Mean horizontal angle between sun and referring object 18° 20′ 30″ The sun is west of R O.

Mean corrected altitude 33° 35′ 10″.

Declination of sun from *N A* +22° 05′ 36″ Latitude of place 52° 30′ 20″.

Determine the azimuth of the line (Univ of Lond, 1918)

8 At a place in latitude 30° 08′ 17″ N the horizontal angle between *Orionis* ($\delta = +7°\ 23'\ 32''$, R A $= 5^h\ 50^m\ 57^s{\cdot}7$) and a referring object was observed to be 84° 35′ 52″, the L S T of the observation being $1^h\ 31^m\ 14^s{\cdot}2$ The star was east of the meridian, and the R O was nearly due south of the observer Find the azimuth to the R O

9 A circumpolar star, declination +80° 17′, right ascension $9^h\ 49^m\ 11^s$, is observed at western elongation in the evening in latitude 60° 04′ N, longitude 127° 30′ W, when its whole circle bearing from a reference line OA is found to be 207° 47′ Find the bearing of OA from the meridian, also the local mean time at which elongation is to be expected, if the G S T of 24^h G M T is $16^h\ 54^m\ 13^s$ The difference between sidereal and mean time intervals may be taken as 10 seconds per hour

10 On the evening of 1920 December 23 the meridian altitude of *Polaris* was observed to be 57° 03′ 25″, and at lower transit the following morning the observed altitude was 54° 50′ 00″ Find the latitude of the place, taking refraction as 58″ cot h

11 Describe the procedure for obtaining latitude by observing the meridian altitude of the sun

An observation gave the meridian altitude of the lower limb of the sun as 28° 57′ 52″, looking towards the south point of the horizon Apply the following corrections and compute the latitude of the station —

Sun's semi-diameter	16′ 05″
Sun's parallax	8″
Refraction	1′ 47″
Sun's declination .	−9° 17′ 30″

(Univ of Lond, 1912)

12 Find the latitude from the following data —

Observed meridian altitude of *α Gruis*, 40° 16′ 15″

Declination of *α Gruis*, − 47° 22′ 16″ 5

Vertical arc level readings—Eye end, 16½ divisions, object end, 15½ divisions Value of 1 division of vertical arc level, 10″

Refraction may be taken as 1′ 07″·3

Star is south of the observer (Inst C E, 1915)

13 In an observation on *Polaris* (δ +88° 53′ 06″, R A $= 1^h\ 31^m\ 44^s$ 8) for latitude, the mean corrected altitude was found to be 48° 12′ 16″ The average reading of the sidereal chronometer was $10^h\ 41^m\ 04^s{\cdot}2$, the chronometer being $1^m\ 00^s{\cdot}4$ fast on L S T Find the latitude of the place

14 On a certain night the first beat of the Eiffel Tower wireless time signals was transmitted at $11^h\ 30^m\ 04^s{\cdot}15$ G M T, and was received by a survey party at $12^h\ 59^m\ 40^s{\cdot}28$ on a M T chronometer which was $3^m\ 07^s{\cdot}8$ slow on L M T What was the longitude of the receiving station ?

15 A time observation at A on Jan 18 at $22^h\ 42^m$ showed a watch to be 2^m slow on L M T After travelling to B, the longitude of which is 30′ 15″·0 east of A, the watch error on Jan 25 at $22^h\ 12^m$ was found to be unchanged. Find the travelling rate of the watch (R T C, 1919)

16 The record of the time determinations on a route traverse as given by the reference watch are as follows —

Place	Date	Approx Watch Time		Error of Watch Fast	
		h	m	m	s
A	Aug 14	10	18	24	16·3
B	16	11	26	25	56 7
C	17	10	43	27	03·8
D	19	10	15	28	20·1
E	20	11	08	26	47·3
A	23	10	32	22	44·2

Compute the longitudes of B, C, D and E relative to A as given by this watch

17 At A, the longitude of which is 20° 10′ 32″·3 E, an observation on July 1 showed a mean time chronometer to be 3^m $02^s·4$ fast at 22^h chronometer time On July 5, at the same place of observation, it was found to be 2^m $36^s·2$ fast at 22^h chronometer time, it having been carried on a daily march in the interval The chronometer was then transported to B, where on July 8, at 23^h chronometer time, it was found to be 1^m 16^s slow Calculate the travelling rate of this chronometer and the value of the longitude of B given by the observations (R T C, 1914)

18 On a certain day in June the altitude of the sun was found to be 45° 27′, the time being noted as 14^h 56^m 10^s G M T The *Nautical Almanac* gave for that time the declination of the sun +22° 35′ 50″, and the equation of time 1^m 42^s 5, to be subtracted from apparent solar time Refraction 1′, and parallax of sun 6″ Latitude at place of observation 51° 30′ N Find the longitude of the place of observation (Univ of Lond, 1919)

REFERENCES

Admiralty List of Wireless Signals, The London Annually
Annuaire du Bureau des Longitudes Paris Annually
Auxiliary Tables of the Survey of India, Part 2 Dehra Dun, 1924
BAKER, T Y "The 45° Prismatic Astrolabe" *The Geographical Journal*, Vol 77, No 5, 1931
BALL, J "Modern Methods of Finding the Latitude with a Theodolite" *The Geographical Journal*, Vol 49, No 6, 1917
— *Astrolabe Diagram* Cairo, 1919
BALL, J, and KNOX-SHAW, H *A Handbook of the Prismatic Astrolabe* Cairo, 1919
— "The Prismatic Astrolabe" *The Geographical Journal*, Vol 54, No 1, 1919
BARNES, C A A "Meridian Diagrams" *Min Proc Inst C E*, Vol CLXI
BECKER, L "On a New Artificial Horizon for Sextants" *Monthly Notices of the Royal Astronomical Society*, Vol 79, No 8, 1919
BOSS, L *Preliminary General Catalogue of* 6188 *Stars for the Epoch* 1900 Washington, 1915
BOULNOIS, P K "Field Longitudes by Wireless" *The Geographical Journal*, Vol 63, No 4, 1924
BOWIE, W "Determination of Time, Longitude, Latitude and Azimuth" U S Coast and Geodetic Survey, Special Publication No 14 Washington, 1913
— "Study of Time Errors in Precise Longitude Determinations by the U S Coast and Geodetic Survey" U S Coast and Geodetic Survey, Special Publication No 92 Washington, 1923
CHAUVENET, W *A Manual of Spherical and Practical Astronomy* Philadelphia, 1891
CLAUDE, A, and DRIENCOURT, L *Description et Usage de l'Astrolabe à Prisme* Paris, 1910
COWIE, G D, and ECKHARDT, E A "Wireless Longitude" U S Coast and Geodetic Survey, Special Publication No 109 Washington, 1924
DAVIDSON, G "The Run of the Micrometer" U S Coast and Geodetic Survey, Report for 1884, Appendix No 8 Washington
DOOLITTLE, C L *A Treatise on Practical Astronomy as applied to Geodesy and Navigation*. New York, 1900
DOWNES, R H B "Practical Astronomy as applied to Land Surveying" *Min Proc Inst C E*, Vol CVIII
— "The Determination of Longitude in Land Surveying" *Min Proc Inst C E*, Vol CXXXIII
EDWARDS, H A "Further Frontier Work on the Bolivia-Brazil Northern Boundary" *The Geographical Journal*, Vol. 45, No 5, 1915
FRENCH, O B "Longitude Determinations of Great Accuracy" *Eng News*, Vol 74, No 13, 1915
Hand-book of Professional Instructions for the Trigonometrical Branch Survey of India Department Calcutta, 1902
HAYFORD, J F *A Textbook of Geodetic Astronomy* New York, 1898
— "A Test of a Transit Micrometer" U S Coast and Geodetic Survey, Report for 1904, Appendix No 8 Washington
HERBERT, W H "Description, Adjustments and Methods of Use of the Six-inch Micrometer Block Survey Reiterating Transit Theodolite, 1912 Pattern." Department of the Interior, Canada, Topographical Surveys Branch, Bulletin No. 34. Ottawa, 1916.

HINKS, A. R. "Wireless Time Signals for the Use of Surveyors" Royal Geographical Society. Technical Series, No 3, 1925

MORSE, F, and FRENCH, O B "Determination of the Difference in Longitude between each Two of the Stations, Washington, Cambridge and Far Rockaway" U S Coast and Geodetic Survey, Special Publication No. 35 Washington, 1916

Nautical Almanac, The. H M Stationery Office, London Annually.

NEWCOMB, S *A Compendium of Spherical Astronomy* New York, 1906

NORTON, A P. *A Star Atlas and Telescopic Handbook.* London, 1933

REEVES, E A "A Combined Theodolite and Prismatic Astrolabe" *The Geographical Journal,* Vol 61, No 1, 1923

— "A Device for Observing Stars with the Theodolite without Cross-wires or Field Illumination" *The Geographical Journal,* Vol 64, No 6, 1924

RINES, D "The Prismatic Astrolabe" *Engineering News,* Vol 72, No 15, 1914

ROSS, L "Azimuth by Direct Solar Observation" *Engineering News,* Vol, 69, No 10, 1913

SAMPSON, R A "Determination of Longitude by Wireless Telegraphy" *Monthly Notices of the Royal Astronomical Society,* Vol 80, No 7, 1920

SCHOTT, C A "On the Determination of an Azimuth from Micrometric Observations of a Close Circumpolar Star near Elongation by means of a Meridian Transit, or by means of a Theodolite with Eye-piece Micrometer" U S Coast and Geodetic Survey, Report for 1891, Appendix No 2 Washington

60° Star List American Geographical Society School of Surveying Publication No 4 New York, 1930

STEWART, L B "Certain Useful Forms for the Azimuth Formula" University of Toronto, Faculty of Applied Science and Engineering School of Engineering Research, Bulletin No 4 Toronto, 1924

WADE, E B H "A Method of Determining Small Differences of Latitude" Ministry of Public Works, Egypt, Helwan Observatory Bulletin No 29 Cairo

WARD, H R "Meridian and Latitude Determination" *Engineering News-Record,* Vol 95, No 1, 1925

WOODROFFE, A J "The Astrolabe and Wireless" *The Geographical Journal,* Vol 47, No 5, 1916

WRIGHT, T W, and HAYFORD, J F *The Adjustment of Observations* New York, 1906

CHAPTER III

GEODETIC SURVEYING

Geodesy.—Geodesy in its modern aspect covers a wide field, the chief divisions of the science and the subjects of investigation being: (1) Map making, (2) The dimensions and figure of the earth; (3) The mean density of the earth, (4) Variations in the force of gravity, (5) Deflections of the plumb line caused by the irregular distribution of mass on the earth's surface and in the crust, (6) The structure of the crust, (7) The intensity of magnetic force and magnitude of declination and dip over the earth; (8) Variations in latitude due to changing direction of the rotation axis of the earth; (9) Vertical movements of the crust; (10) Variations in mean sea level; (11) Tidal observations.

The great practical importance of the science of geodesy has led in all civilised countries to the maintenance of a State survey department. The primary object of such an organisation is the production of maps, but the information acquired adds to the data available for the closer determination of the figure of the earth. Every large national survey department also devotes part of its organisation to the furtherance of pure geodesy by conducting investigations on the subjects enumerated above, which, although not so immediately useful to the general user of maps, are of the greatest service in promoting accuracy in practical geodesy and in the sciences of astronomy, physics, geology, and meteorology. The data acquired by national surveys are co-ordinated by the International Geodetic Association, to which all civilised nations belong, and at whose conferences they are represented.

The civil engineer is not directly concerned with the more purely scientific branches of geodesy, but in conducting extended surveys he may have occasion to employ the methods of geodetic surveying, whether the results approach geodetic accuracy or fall far short of it.

Geodetic Surveying.—Geodetic surveying has for its object the precise measurement of the positions on the earth's surface of a system of widely separated points. The positions are determined both relatively, in terms of the lengths and azimuths of the lines joining them, and absolutely, in terms of the co-ordinates, latitude, longitude, and elevation above mean sea level. These geodetic points form control stations to which topographical and other

surveys may be referred, so that their error is limited to that propagated between the geodetic stations.

The area embraced by a geodetic survey is such as to form an appreciable portion of the surface of the earth, and, in consequence, the sphericity of the earth cannot be disregarded The reductions therefore involve a knowledge of the results of previous investigations of the dimensions of the earth.

GENERAL PRINCIPLES

Triangulation.—To avoid an embarrassing accumulation of error in parts of a geodetic survey due to the great distances involved, the observations and reductions must be performed with a much greater degree of accuracy than might appear necessary for practical map making. Because of the impracticability of conducting linear measurements with the requisite degree of precision over all kinds of country, the survey must be executed by triangulation

Triangulation is the method of location of a point from two others of known distance apart, given the angles of the triangle formed by the three points By repeated application of the principle, if a series of points form the apices of a chain or network of connected triangles of which the angles are measured, the lengths of all the unknown sides and the relative positions of the points may be computed when the length of one of the sides is known.

The field work of any triangulation survey therefore possesses the following essential features :

(1) The selection of the stations to form a system of connected triangles ,

(2) The measurement of one of the sides, known as the base line ;

(3) The observation of the horizontal angles

In geodetic and other large surveys, there must be added the astronomical observations necessary to determine the absolute positions of the stations Their geodetic positions and the azimuths of all the lines can be computed if the latitude and longitude of one station and the azimuth of one side of the triangulation are observed. In practice, however, astronomical observations should be made at intervals throughout the survey

Grades of Triangulation.—Triangulation is classified as primary, secondary, and tertiary, according to the degree of accuracy required.

Primary Triangulation constitutes the highest order of triangulation, and is that employed both for surveys executed primarily for figural determinations and to furnish the most precise control for mapping. As it is independent of external checks, no precaution can be neglected in making the linear and angular measurements or in performing the reductions. The length of base line is from 3 to 12 miles, and that of the sides of the triangles ranges from 10 to over 100 miles. The triangular error should average less than

1 second and should seldom reach 2 seconds. The probable error of computed distance will lie between about 1 in 60,000 and 1 in 250,000.

Secondary Triangulation is designed to furnish points closer together than those of the primary triangulation. It may cover extensive areas, but, as it is tied to the primary net at intervals, the operations may be conducted with rather less refinement. The lengths of the sides range between about 5 and 25 miles. The triangular error may reach 5 seconds, and the probable error of distance will vary from 1 in 20,000 to 1 in 50,000.

Tertiary Triangulation is run between the stations of the secondary system, and forms the immediate control for the detail surveys. The lengths of the sides range from less than a mile to about 6 miles. The triangular error may amount to 15 seconds, and the probable error in the computed sides usually lies between 1 in 5,000 and 1 in 20,000.

Note —The above classification is based only upon the degree of accuracy to be attained Every survey which can be classed as geodetic involves work of primary standard, but, on the other hand, the triangulation forming the main control of even an extensive topographical survey may be of secondary, tertiary, or lower grade

Triangulation Schemes.—For a geodetic triangulation of moderate extent the whole area of the survey may be covered by primary triangles, which are extended outwards in all directions from the

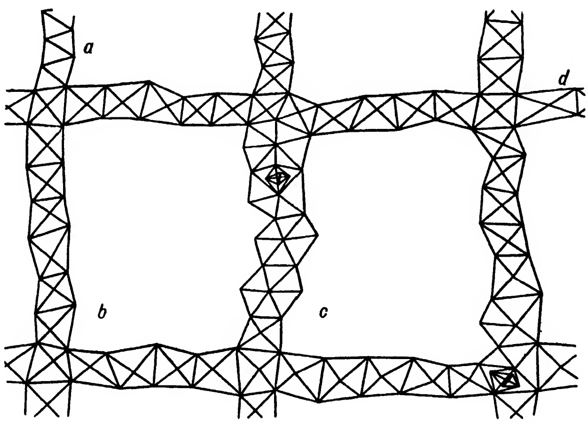


FIG. 37.

initial base. In very extensive surveys, however, it is preferable to lay out the primary triangulation in two series of chains of triangles, which are usually placed roughly north and south, and east and west respectively (Fig. 37). The areas enclosed remain to be filled by secondary and tertiary triangulation.

The first, or central, system, of which a notable example is the Ordnance Survey of the British Isles, has the merit of affording close control over the secondary work. The second, or gridiron, system has, however, the important advantages that the most favourable country can be selected for the primary triangulation and the work can proceed comparatively rapidly, while the subsequent reduction is less complicated.

Chain Figures.—The simplest form of chain is composed of a single system of triangles as at *a* (Fig 37), but this arrangement, although economical, does not conduce to the requisite precision of primary work, since the number of conditions to be fulfilled in the figure adjustment is relatively small. Much better results are obtained by linking the triangles to form more complex figures. In point of accuracy, the quadrilateral with four corner stations and observed diagonals (*b*, Fig 37) forms the best figure When the topography is unsuited for the development of these figures, there may be substituted quadrilaterals, pentagons, or hexagons with a central station (*c*, Fig 37), which form satisfactory figures.

In any component figure one of the unknown sides is that from which the chain is extended, and the degree of accuracy with which distance is transmitted along the chain depends upon the precision with which that side is computed from the known side. For a given standard of angle measurement, the accuracy of transmission in each figure depends upon the number of geometrical relationships which must be fulfilled by its parts and upon the size of the distance angles, or angles used in the sine ratio calculation of the one side from the other. In order that minimum effect may be produced by uncertainties in the distance angles, they should be such as have small logarithmic differences of sine, *i e.* they should approach 90°, and should be within the limits, 30° and 150° In the case of a chain of quadrilaterals, figures very long in the direction of the chain in proportion to their breadth, as at *d* (Fig 37), should therefore be avoided. For a chain of single triangles the equilateral form is the most advantageous, and a lower limit of 30° should be set upon the angles.

When alternative systems of triangulation are possible, the best scheme may be arrived at by evaluating and comparing the strengths of the proposed chains * and estimating their relative costs.

Base Line Sites.—The importance of having bases favourably situated necessitates careful investigation of the relative merits of possible sites. Greater latitude as to the character of the ground surface is permissible when the measurement is to be performed by tape as against rigid bars, but the chief desiderata are:

(1) The site should permit of a line, of length suitable to the requirements of the survey, being laid out over firm and smooth

* See Crandall, *Text-book on Geodesy and Least Squares.*

ground with longitudinal slopes not exceeding about 1 in 12 and with the end stations either intervisible at ground level or such that intervisibility can be secured at small expense.

(2) The surrounding country should be suitable for the development of a well-conditioned connection between the base and the main triangulation.

In very flat country a considerable choice of sites may be available, and the location may be made to suit a proposed scheme of triangulation. In rugged country, on the other hand, the choice may be very limited, and the triangulation must be adapted to suit the location of the base line

Base lines can be measured with a greater degree of precision than can be maintained throughout the triangulation The uncertainties introduced by errors of angle measurement increase as the work progresses from the initial base line, and check bases should therefore be introduced at intervals The distance between them should be such that the discrepancy between the measured length and that computed from the preceding base is within the allowable error set for the survey. In extensive primary surveys the base interval ranges from 100 to over 200 miles In the greater national surveys the number of primary base lines per 100,000 square miles varies from about 1 to 8.

Length of Base Lines.—The length of base should depend primarily upon that of the sides of the triangulation, and should bear as large a ratio to the latter as practicable in order to minimise the loss of precision inherent in the connection of a short base to the triangulation system. The length of the great majority of existing bases lies between a tenth and a fifth of that of the average side of the triangulation The modern development in the use of tapes and wires has tended to increase the length of bases, but in most cases the length is strictly limited by difficulties of site.

Connection of Base Line to Triangulation.—In connecting the comparatively short base line to the main triangulation, badly conditioned figures must be avoided by expanding the base in a series of stages Fig. 38 shows a strong base net connecting the base AB to the main stations G and H. The first expansion yields the line CD, the second EF, and the third the side GH of the main system.

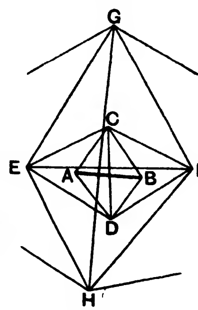


FIG 38

Selection of Stations.—To lay out a system of triangulation with strong figures so that the survey may be conducted economically demands a careful study of the topography of the country. Stations should be placed upon the most elevated ground, so that long sights through undisturbed atmosphere may be secured at minimum ex-

pense for observing towers and signals. Commanding situations are also of advantage for the control of subsidiary triangulation and for possible future extension of the principal system. In very flat country the topography, because of its uniformity, has less influence on the exact location of the stations, and the length of the lines is limited by the cost of erecting high structures to overcome the effect of the curvature of the earth. If such country is heavily wooded, the difficulties are greatly increased, and a considerable amount of clearing may have to be undertaken.

To prevent difficulty of sighting and to avoid the effects of irregular atmospheric refraction, stations must be situated so that lines of sight will not pass over towns, factories, furnaces, or the like, nor graze any obstruction They must be placed on firm ground where, as far as can be judged, they will remain undisturbed and permanently accessible

PRELIMINARY OPERATIONS

Reconnaissance.—This includes all the operations required in examining the country to be triangulated, fixing sites for base lines, selecting and temporarily marking stations, determining intervisibility, ascertaining the required height of observing towers and signals and the amount and direction of clearing The economical execution of a triangulation is very largely dependent upon an exhaustive reconnaissance, and the work should be directed by a person whose judgment can be relied upon.

Advantage should be taken of existing maps of the region. If none are available, it will usually be necessary to undertake a rapid preliminary reconnaissance to ascertain the general location of possible schemes of triangulation suited to the topography These schemes are examined in detail in the reconnaissance proper.

The latter is conducted as a rough triangulation and plotted as the work advances The equipment carried should be as light as possible, and the instrumental outfit will not exceed the following : small theodolite with magnetic needle, sextant for observing from tree tops, good telescope, two heliotropes for testing intervisibility, prismatic compass, one or two aneroids for levelling, steel tape, and drawing instruments and materials.

The highest points are occupied, and horizontal and vertical angles are observed to all salient features likely either to serve as triangulation stations or to influence their selection. To facilitate progress, considerable use is made of magnetic bearings as an aid to identifying points previously occupied, and, where convenient, fixes are made by resection. Great care must be exercised to ensure that reliable data are obtained with reference to the necessary height of stations and that stations judged to be intervisible are really so, as a mistake might seriously retard the progress of the observing party.

Plotting may be performed by protracting the angles or by computing the sides, a scale or base being obtained with sufficient approximation from astronomical observations or by tangential tacheometry from measurement of the vertical angles to a signal at an adjacent station. The essential features of the topography are sketched in The possible triangulation schemes are then laid down, their relative strength and cost are examined, and a final decision is reached. Points selected for stations are flagged, and particulars of their exact location are recorded.

During reconnaissance, ample notes should be made of all circumstances likely to influence the economy and rapidity of the operations to follow. These will include information regarding access to stations, means of transport, supplies of food and water, camping ground or nearest suitable accommodation, material for building towers, etc Descriptive sketches or photographs showing the appearance presented at each station by the surrounding country are useful as an aid to the identification of adjacent signals, and serve to show the extent of territory which can be controlled from the station.

Direction of Clearing.—When one station cannot be seen from another because of the obstruction presented by trees, etc which can be cleared, it becomes necessary to ascertain from one or both stations the direction in which to clear a path for the observation. This direction can be set out by making use of two auxiliary stations.

In Fig 39, let A and B represent the stations between which the obstacle intervenes C and D are selected intervisible points from each of which both A and B are visible, and at which the angles marked 1, 2, 3, and 4 are measured. Calling the distance CD unity, triangle ACD is solved for AC and triangle BCD for BC The angles CAB and CBA can now be evaluated by solution of triangle ABC, and these are set out at A and B respectively for the direction of clearing

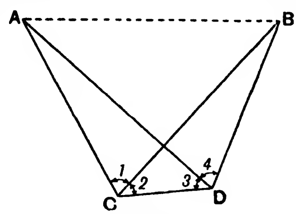


FIG 39

The above procedure is not required if sufficient data are available from the reconnaissance observations made at neighbouring triangulation stations. Thus, if C be such a station, there may be known with sufficient accuracy angle 1 and the distances CA and CB, the latter having been obtained from other triangles of which they form sides.

Intervisibility and Height of Stations.—It is commonly possible to ascertain whether proposed stations are actually intervisible by direct observation either at ground level or from tree tops or guyed ladders. If, however, intervisibility necessitates elevating the station considerably above the ground, the reconnaissance party may not be able to determine by observation the practicability of a proposed

site, and the question must be decided by calculation. The height to which both instrument and signal must be raised above the ground depends principally upon the distance between the stations, their relative elevations, and the profile of the intervening country.

Distance between Stations —The deviation of the level line from the horizontal due to the curvature of the earth in a distance D amounts to $\frac{D^2}{2R}$, where R is the mean radius of the earth. Owing to terrestrial refraction, however, a line of sight is not straight, but, except in abnormal cases, is concave to the earth's surface, and approximates to a circular arc of radius about seven times that of the earth. The combined effect of curvature and refraction is given by

$$h = (1-2k)\frac{D^2}{2R},$$

where k is the coefficient of refraction The value of k may vary considerably (page 209), but for the present purpose should be taken as not exceeding 0 07 for sights over land and 0·08 over water, unless it is known more exactly for the locality. Taking $k=0{\cdot}07$, then $h={\cdot}574D^2$, where h is in feet, and D in miles The accompanying table gives the value of h in feet for distances from 1 to 80 miles for $k=0{\cdot}07$.

TABLE OF CURVATURE AND REFRACTION

Distance in Miles	Curvature and Refraction in Feet	Distance in Miles	Curvature and Refraction in Feet	Distance in Miles	Curvature and Refraction in Feet
1	0 6	28	449 9	55	1735 8
2	2 3	29	482 6	56	1799 5
3	5·2	30	516 4	57	1864 4
4	9 2	31	551 4	58	1930 4
5	14 3	32	587 6	59	1997 5
6	20 7	33	624 9	60	2065 8
7	28·1	34	663 3	61	2135 2
8	36 7	35	702 9	62	2205 8
9	46 5	36	743·7	63	2277 5
10	57 4	37	785 6	64	2350 4
11	69 4	38	828 6	65	2424 4
12	82 6	39	872 8	66	2499 6
13	97·0	40	918 1	67	2575 9
14	112 5	41	964·6	68	2653 4
15	129 1	42	1012 2	69	2732 0
16	146·9	43	1061·0	70	2811 7
17	165 8	44	1110·9	71	2892 6
18	185 9	45	1162·0	72	2974·7
19	207·2	46	1214·2	73	3057 9
20	229 5	47	1267 6	74	3142 2
21	253 1	48	1322·1	75	3227 8
22	277·7	49	1377 8	76	3314 4
23	303·6	50	1434 6	77	3402·2
24	330·5	51	1492·5	78	3491·2
25	358 6	52	1551 [illegible]	79	3581·3
26	387·9	53	1611·9	80	3672·5
27	418·3	54	1673·3		

The tabular figures show the elevation above datum of a signal which at various distances can just be seen from datum level, if the intervening ground presents no obstruction, and, conversely, the distance of the visible horizon from a station of known elevation above it.

Relative Elevation of Stations—When the ground below the line of sight is level, the table may be used to give the necessary elevation of a station at known distance, so that it may be visible from another of known elevation. Thus, if h_1 (Fig 40) is the known elevation of station A above the level ground, the distance D_1, at which the line of sight becomes tangent to the ground, is obtained by interpolation from the table, and the remaining distance, $D_2 = (D - D_1)$,

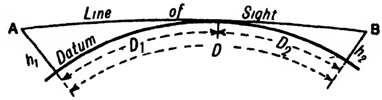


FIG. 40.

is ascertained. The required elevation, h_2, corresponding to D_2, is then extracted, and, the ground level at B being known, it will be found whether it is necessary to elevate the station above the surface, and, if so, the required height of tower is obtained. If the height of neither station is fixed, then for level intervening ground the minimum total height of scaffolding, when the ground level is the same at both stations, occurs when the towers are of equal height. When the difference between the ground levels does not exceed this height, the stations should be brought to the same level: for greater differences in ground level, the minimum is secured by erecting scaffolding only at the lower station.

The line of sight should not graze the surface at the tangent point but should pass above the strata of disturbed air. In fixing station heights, allowance should be made for keeping it nowhere less than 6 ft. above the ground, and preferably 10 ft in primary work.

Example—The elevation of station A is 812 ft., and that of B, 30 5 miles distant, is 857 ft. The intervening ground may be assumed a level plain of elevation 709 ft Find the minimum height of signal required at B, so that the line of sight may not pass nearer the ground than 6 ft.

Minimum elevation of line of sight $= 709 + 6 = 715$ ft.

Taking this elevation as a datum, the elevation of A $= 812 - 715 = 97$ ft., which, from the table, corresponds to a tangent distance $D_1 = 13$ miles.

The remainder $D_2 = 30\ 5 - 13 = 17\ 5$ miles, in which distance the ordinate $h_2 = 175\ 7$ ft. (by calculation or interpolation from table)

$\therefore$ Minimum height of signal above ground at B

$$= 175{\cdot}7 + 715 - 857 = 33{\cdot}7, \text{ say } 34 \text{ ft.}$$

Profile of Intervening Ground.—The elevation and position of peaks which might offer obstruction must be ascertained, and a comparison of their elevations with that of the proposed line of sight at the same distances will exhibit whether that line of sight is practicable. An approximate method of solution, which, in view of the variability o refraction, yields sufficiently good results, is best demonstrated by an example.

Example.—The proposed elevations of two stations A and B, 70 miles apart, are respectively 516 and 1,428 ft above the datum of mean sea level The only likely obstruction is situated at C, 20 miles from B, and has an elevation of 598 ft Ascertain by how much, if any, B should be raised so that the line of sight may clear C by 10 ft

A horizontal sight Ac_1b_1 (Fig 41), through A, strikes the datum surface at d, 30 miles from A, and the ordinates cc_1 and bb_1 corresponding to dc and db respectively are, from the table,

$$cc_1 = 229\ 5 \text{ ft. and } bb_1 = 918{\cdot}1 \text{ ft.}$$

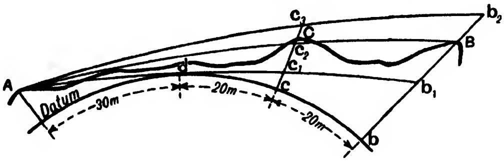


Fig. 41.

A line of sight between A and B would be represented by Ac_2B, and, to test whether it clears the peak C, we have approximately

$$\frac{c_1c_2}{b_1B} = \frac{Ac_1}{Ab_1} = \frac{50}{70},$$

but $b_1B = 1428 - 918\ 1 = 509\ 9$ ft, whence $c_1c_2 = 364\ 2$ ft, and the elevat on cc_2 of the line of sight at the obstruction is

$$229\ 5 + 364\ 2 = 593\ 7 \text{ ft.}$$

The line of sight therefore fails to clear C by $598 - 593\ 7 = 4\ 3$ ft.

To clear by 10 ft, it should be raised at C by the amount $c_2c_3 = 14\ 3$ ft

If Ac_3b_2 represents the required line of sight, then, approximately.

$$\frac{c_2c_3}{Bb_2} = \frac{50}{70}.$$

Whence $Bb_2 = 20$ ft. = the minimum station height above the ground at B.

The relative heights of the two towers for minimum total height is best obtained by trial calculation. In the case of a single obstruction the minimum is secured by erecting a tower only at the station nearer the obstruction.

Observation Towers.—When the station must be elevated above the ground, a rigid support is required for the instrument and the signal. It is sometimes possible to utilise church spires or other lofty structures, with or without the addition of scaffolding, but usually the erection of a masonry pier or a timber or steel scaffold is necessary.

Masonry is very suitable for small heights, but otherwise the cost is prohibitive. Timber scaffolds are most commonly adopted, and have been constructed to heights of over 150 ft Scaffolds (Fig 42) must be designed in the form of two towers, securely founded and efficiently braced and guyed : the inner tower forms the support for the instrument, and the outer carries a railed platform for the observing party, and supports the instrument awning. The two structures must be entirely independent of each other, so that there is no possibility of vibration being transmitted from the outer to

the inner. The signal is sometimes erected on the instrument tower, a position necessitating its removal when the station is occupied, and a convenient alternative is to mount it on the observer's scaffold above the instrument. On extensive surveys standard scaffold designs are worked to.*

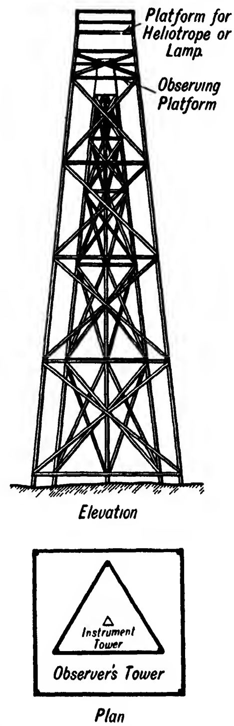


FIG 42 —OBSERVATION SCAFFOLD.

Signals.—The term, signal, includes any object or device used to define for the observer the exact position of a station. The various kinds may be classified as opaque or luminous. Opaque signals comprise several forms of mast or target signals, and are used for comparatively short sights. Luminous signals, including the heliotrope for day observations and different varieties of lights for night work, are greatly used in primary and secondary triangulation. They are indispensable for long lines, but are also economical for relatively short sights through hazy atmosphere

A signal of any class should fulfil the following essential requirements

(1) It should be conspicuous.

(2) It should present a well-defined outline of suitable width for accurate bisection.

(3) It should be capable of being accurately centered over the station mark

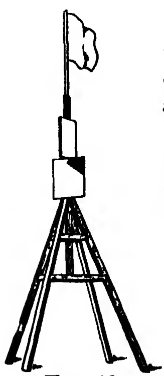


FIG. 43. QUADRIPOD SIGNAL.

Opaque Signals.—This is the usual form of signal for sights of less than about 20 miles, although under favourable conditions it may be employed for much greater distances When the station is not elevated above the ground, the most common arrangement is to have the signal mast supported by a tripod or quadripod trestle (Fig 43), so that the instrument may be centered over the station mark without disturbing the mast. The legs of the trestle are spiked to stakes driven well into the ground, and may be close-boarded or have canvas stretched between them to protect the instrument against the effects of sun and wind. Tall masts must be secured with wire guys.

* For examples see Reports of United States Coast and Geodetic Survey, 1903, Appendix No. 4; 1893, Appendix No 9, pp 406–14; 1882, Appendix No. 10; and Special Publication No. 4, 1900.

The signal should subtend about 1·0 to 1·5 sec. at the observer, corresponding to a width of 0·3 to 0·46 in per mile, but for short sights considerations of rigidity govern the diameter of the pole and necessitate a greater angular width. On the other hand, a mast to subtend those angles at long distances would be inconveniently heavy for erection, and beyond about 15 or 20 miles the width must be increased by nailing on thin strips of wood or targets of rectangular or diamond shape, formed of boarding or of canvas stretched on a frame. In order that targets may be visible from different directions, they should be fixed in pairs at right angles to each other.

To render a signal conspicuous, its height above the station should be roughly proportional to the length of the longest sight upon it, and usually lies between 10 and 30 ft. It should be of dark colour for visibility against the sky, and should be painted white, or in white and black stripes, against a dark background. To afford greater prominence, the top of the mast should carry a flag, a whitewashed bundle of brushwood, a tin cylinder or cone, or other device.

Except when viewed against the sky, many forms of signal permit of accurate bisection only when the sun is in the plane of the line of sight. Under lateral illumination the signal may be partly illuminated and partly in shadow, and, since the observer sees only the bright portion and makes his pointing upon it, an error, termed *phase*, is introduced. The effect cannot be avoided in cylindrical signals nor with square masts unless one side of the latter directly faces the observer. In target signals it is caused by the shadow of the upper target falling on the lower. If the direction of the sun is known, the phase correction, in the simple case of a cylindrical signal, may be formulated as on page 154, and the error eliminated.

A phaseless signal is obtained by using a single target normal to the line of sight, an attendant being told off to turn it to face the observer. In the ordinary target, signal phase is reduced by making the depth of the targets great in proportion to their width and by using lozenge forms rather than rectangular. On the other hand, well-defined phase is obtained with a signal consisting of a tin cylinder to reflect the sun's rays, the pointings being made on the bright line.

The Heliotrope.—The essential features of the heliotrope are a plane mirror to reflect the sun's rays and a line of sight to enable the attendant to transmit the reflected beam in the direction of the observing station. A very simple heliotrope may be improvised with an ordinary mirror mounted on a horizontal axis, the reflected sunlight being projected over the top of a stake or pole marking the required direction. In the usual forms of the apparatus the mirror is of worked parallel glass with rack motions about both horizontal and vertical axes, and the line of sight may be either

telescopic or defined by a sight vane with an aperture carrying cross wires.

In the telescopic form (Fig. 44) the mirror is mounted on the telescope, and two rings are fixed so that the axis through them is parallel to the line of sight. The telescope being directed towards the distant station, the mirror is adjusted until the reflected beam passes centrally through the rings, as evidenced by a concentric annulus of light on the second ring, which is of smaller aperture than the first. In non-telescopic forms, the sight vane may be mounted on the base-board or tripod head supporting the mirror and at about 2 ft. from the latter, or it may be erected on a pole several yards from the station and in the correct line. In aligning the instrument, the cross wires are viewed through a small eyehole in the centre of the mirror, formed by removal of the silvering. The mirror is adjusted to project the beam through the vane so that the wires are illuminated except at their intersection, which is in line with the eyehole. This type of heliotrope does not require the attendance of a skilled assistant, and is largely used.

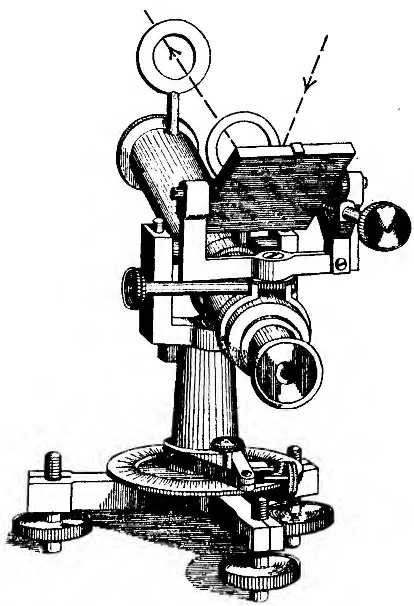


Fig. 44.—Telescopic Heliotrope.

Accurate bisections cannot be made when the signal is too bright, and the size of mirror or aperture of vane should not be greater than will show a clear star of light. The size required is proportional to the distance between the stations, but also depends upon the clearness of the atmosphere and the quality of the observing telescope. If D is the length of sight in miles, the effective diameter or width of mirror has varied in different surveys from about 0·04 D to over 0·1 D inches for average atmospheric conditions, corresponding to an angular width of signal of about $\frac{1}{7}$th to $\frac{1}{3}$rd second. In regions of exceptionally clear atmosphere smaller signals have been successfully used. A mirror of 6 or 8 in. diameter is commonly employed, and the effective area is reduced by a diaphragm fitted on the vane or, in the telescopic form, on the outer ring.

Use of Heliotrope.—The reflected rays form a divergent beam having an angle equal to that subtended by the sun at the mirror,

viz. about 32 min. The base of the cone of reflected rays has therefore a diameter of about 50 ft. per mile of distance, and the signal is visible from any point within that base. In consequence, great refinement in pointing the heliotrope is unnecessary, as the signal will be seen provided the error of alignment is less than 16 min.

The heliotrope must be centered over the station mark, and the line of sight directed upon the distant station Centering is facilitated by means of the "self-centering" beacon introduced by the Survey of Egypt. The device consists of a brass casting with three radial V-shaped grooves, 120° apart, which receive the feet of the heliotrope or the levelling screws of the theodolite. When the distant station cannot be seen or its direction located within about 16 min. of arc, a reference pole is erected with the top just below the line and at least 100 ft. from the heliotrope. Flashes are sent from the observing station to enable the direction to be established, or the pole may be lined in by theodolite, as the bearing will usually be known with sufficient accuracy for the purpose.

The duty of the heliotroper consists in projecting and maintaining the beam in the proper direction. Because of the motion of the sun, the mirror must be adjusted on its axes about every minute When the sun's rays cannot be received directly on the mirror, they must be reflected upon it from a second mirror, which is provided with all forms of heliotrope This mirror is placed in a convenient position facing the sun, and must be adjusted to follow the sun's motion. To avoid misunderstandings and delays, the observing party carries one or two heliotropes or heliographs, and a simple code of signals is adopted for conveying orders to the heliotropers

The heliotrope can, of course, be used only in sunshine, whereas the most favourable atmospheric conditions for angle measurement occur on cloudy days The best results with the heliotrope are obtained from observations taken towards sunset.

Night Signals.—The advantages of night observations (page 147) have been realised since the early days of geodetic triangulation. On the Ordnance Survey of the United Kingdom the longer lines were observed by means of the Drummond or lime light. In the Great Trigonometrical Survey of India blue lights and vase lights* were employed in the earlier work, and latterly Drummond lights and Argand reverberatory lamps with 12-in parabolic reflectors were adopted.

Various forms of oil lamps with reflectors or optical collimators have been used for lines of less than about 40 miles, but in some surveys acetylene lamps are now preferred.† These were employed in the measurement of the arc of meridian in Uganda in 1908–09, in which the longest sight was 46·6 miles The McCaw lamps used

* See Thuillier and Smyth, *Manual of Surveying for India*

† See U.S. Coast and Geodetic Survey Report, 1903, Appendix No. 4.

had a candle power of about 600, and the diameter of aperture in inches was one-sixth the length of sight in miles. The same lamps were employed by the Ordnance Survey on the test triangulation in N.E. Scotland in 1910–11, the longest side being 46·9 miles. Fig. 45 shows a recent pattern of the McCaw lamp.

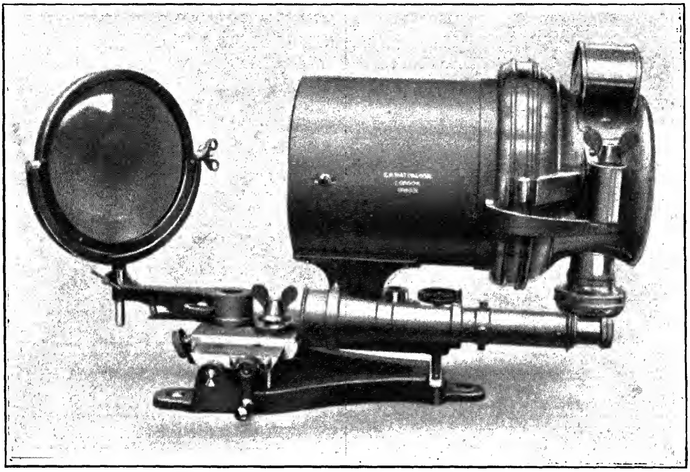


FIG. 45.—McCAW LAMP.

For lines of over 50 miles there are available the Drummond light, the electric arc, and the magnesium lamp. The first two are not generally suitable on account of the weight of the apparatus and the necessity for the attendance of a skilled person. The magnesium lamp with parabolic reflector is very portable and easily manipulated, and gives excellent results.* The lamp burns magnesium ribbon, which is delivered by clockwork at any desired rate. The magnesium is somewhat costly for a continuous light, but the expense of this item may be reduced by showing signals intermittently according to a prearranged time-table.

Alignment of night signals is effected by means of an attached telescope, as illustrated, or more usually by the aid of the line mark set for the heliotrope. The best angular width of signal is rather greater than for heliotropes, and should be determined by trial.

Marking of Stations.—Stations should be marked in a permanent manner to facilitate their identification for future occupation. The general methods are the same for all grades of triangulation, but various details are adopted in different surveys according to local conditions.

In solid rock it is usual to drill a hole about a foot deep, into which a copper or brass bolt is fixed with cement or lead. A number of

* See U.S. Coast and Geodetic Survey Report, 1880, Appendix No. 8.

arrows should be chiselled in the surrounding rock to point to the station. In ground which can be excavated the mark is made on a stone, preferably foreign to the locality, which, to prevent its being disturbed, is buried to a depth of about three feet, and bedded in cement mortar. The centre mark in the stone is a small hole filled with lead, or may consist of a brass screw, an empty cartridge case, or a copper or iron pin cemented in. The sub-surface mark is covered with earth, and a similarly marked stone is then laid with the upper surface at ground level. The whole is covered with a cairn of stones, or, as in the Survey of India, by an observing pillar of masonry or concrete.* The sub-surface mark is referred to only when there is reason to suspect that the surface mark has been disturbed. To enable the station to be recovered when the surface mark is lost, two or three reference or witness marks should be established at some little distance, and note is made of the bearing and distance of the station from each

Station Description.—Stations, besides being numbered, should be given a concise local name for convenience of reference The record of the results of the survey should include a description of the site of each station, its marking, and the positions of the witness marks, the description being amplified by sketches as required. In addition, full particulars should be given regarding access routes, camping grounds, subsistence, etc likely to prove of service to parties occupying the stations.

BASE LINE MEASUREMENT

Standards of Length.—The fundamental standard of length for geodetic operations is the International Metre established by the International Geodetic Association. It is marked on a platinum-iridium bar deposited at the Bureau International des Poids et Mesures at Sèvres, and copies have been allotted to various national surveys. This standard differs slightly from the French legal metre, which was intended to represent 1/10,000,000 of the length of a meridian quadrant of the earth.

In Great Britain the legal unit is the Imperial Yard as defined by Act of Parliament in 1855. There are several official copies of the standard, consisting of a bronze bar with gold plugs, the distance between engraved lines on which represents the standard length at a defined temperature. The relationship between the yard and the metre has been established as

1 Imperial Yard = ·91439180 Legal metre†
= ·9143992 International metre.‡

* See *Handbook of Professional Instructions for the Trigonometrical Branch*, Survey of India Department, or Ordnance Survey Professional Paper, No. 2.

† Clarke, "Comparison of the Standards of Length," 1866.

‡ "Détermination du Rapport du Yard au Mètre." *Travaux et Mémoires du Bureau International des Poids et Mesures*, Vol. XII, 1902.

Forms of Base Measuring Apparatus.—The two main types of base measuring instruments are (*a*) Rigid bars, (*b*) Flexible apparatus. The former may be divided into.

(1) Contact apparatus, in which the ends of the bars are brought into successive contacts;

(2) Optical apparatus, in which the effective lengths of the bars are engraved on them and are observed by microscopes which serve to mark their successive positions.

According to the means adopted for reducing the uncertainties of temperature correction, rigid bars may also be classified as·

(1) Compensating, in which by a combination of two or more metals the bar is designed to maintain a constant length under changing temperature,

(2) Bimetallic, non-compensating, in which the two measuring bars act as a bimetallic thermometer,

(3) Monometallic, in which the temperature is either kept constant at the melting point of ice or is otherwise ascertained.

Flexible apparatus consists of (1) Steel tapes or wires, (2) Invar tapes or wires, (3) Steel and brass wires

Comparative Merits of Rigid and Flexible Apparatus.—In the earlier days of modern geodesy bars were almost exclusively favoured for base measurement, but of recent years the merits of tapes and wires have become more fully appreciated The most far-reaching difference between the two classes of apparatus lies in the much greater length of tapes and wires. The smaller number of contacts required has the advantages that:

(1) A wider choice of base sites is available since rougher ground can be utilised, gorges, etc of smaller width than the tape length offering no obstacle.

(2) The measurement proceeds much more rapidly, and expense is reduced

(3) By reason of the above considerations, longer bases can be used, and bases of verification can be introduced at closer intervals.

The difficulty of ascertaining the actual temperature of the apparatus is common to all forms, except the iced bar, and is akin to the difficulty of ensuring exact compensation in compensating bars. The temperature of a steel tape cannot be measured with sufficient accuracy by mercurial thermometer except in densely cloudy weather or at night. M Jaderin, of Stockholm, one of the pioneers in the application of steel tapes to precise measurement, found that better results could be obtained by the employment of two wires, of steel and brass respectively, to form a bimetallic thermometer. The value of flexible apparatus for geodetic measurement has, however, been greatly enhanced by the discovery of the alloy, invar, which, because of its extremely low coefficient of expansion, does away with the necessity for precise temperature determination. With the assistance derived from the increased

number and length of base lines rendered possible by tape or wire measurement, the ultimate accuracy is little, if any, short of that attained by the use of rigid apparatus, other than the iced bar, which, however, on account of the time and expense it involves, is better adapted for the calibration of apparatus than for direct use in the field.

Standardisation of Base Apparatus.—Standardisation of base measuring apparatus consists in (1) ascertaining its length, in terms of the allotted prototype bar, under definite conditions of temperature, etc., (2) determining the laws governing its change of length under change of those conditions Standardisation is performed by the International Bureau and also by survey and standards departments in various countries. In Great Britain it is undertaken by the Ordnance Survey Office and by the National Physical Laboratory.

The utmost refinement is required in standardising apparatus for geodetic measurements The unknown length is compared with the known standard by means of a comparator, usually in an underground chamber free from vibrations and designed to afford a steady temperature and perfect illumination for microscope reading. The comparator for bar standardisation is a heavy and elaborate apparatus carrying micrometer microscopes For flexible apparatus it consists of a base, usually not more than 100 ft. long, having the terminals marked on invar plates. It is divided into sections, each of length equal to that of the standard bar, and micrometer microscopes are mounted at the intermediate points and the terminals. The operation of standardising a tape or wire consists in repeatedly measuring the base with it and with the standard bar alternately. Expansion coefficients are determined by immersion of the apparatus in water or glycerine heated by hot water piping and kept in circulation, microscope readings being taken through the liquid.

Standardisation of any form of apparatus may also be performed by measuring with it a base line, the length of which has been determined by standardised apparatus, while the coefficient of expansion may be obtained by making repeated measurements at widely different temperatures.

Accuracy of Base Measurement.—Sources of error in base measurement may be summarised as (1) constant error of standardisation of apparatus, (2) accidental field errors due to uncertainties in reading, levelling, temperature, tension, etc.

The degree of accuracy attained in standardisation limits the final precision of a base measurement, *e g* if the probable error of apparatus is 1/1,000,000, no base measured by it can have a probable error as small as 1/1,000,000. The quality of the field work may be gauged by making repeated measurements of a base with the same apparatus, but the degree of consistency in the results does not

indicate the real probable error, which should include all known errors, with allowances for those which cannot be rigidly evaluated.

It may be taken that 1/5,000,000 is the smallest error which can be attained in the standardisation of bar apparatus. Tapes or wires, because of their greater length, cannot be standardised in terms of the short prototype bar with greater precision than about 1/3,000,000. The final uncertainty of field measurement must therefore exceed those limits, and for geodetic work usually lies between 1/500,000 and 1/2,000,000. This standard of accuracy cannot be maintained in the computed sides of the survey, but is gradually lowered, by errors of angle measurement, as the triangulation proceeds from the base.

Base Bars.—An account of the principal forms of rigid apparatus which have been used in various countries will be found in the references on page 191. These instruments are now mainly of historical interest, but, to illustrate their general features, one example each of compensating, bimetallic non-compensating, and monometallic apparatus is here described.

The Colby Apparatus.—This apparatus (Figs 46 and 47) was designed by Maj -Gen. Colby, formerly director of the Ordnance

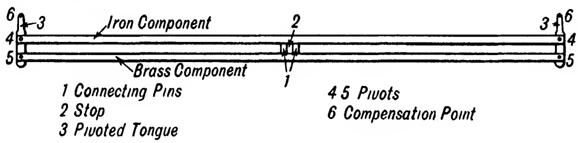


Fig 46 —Colby Apparatus

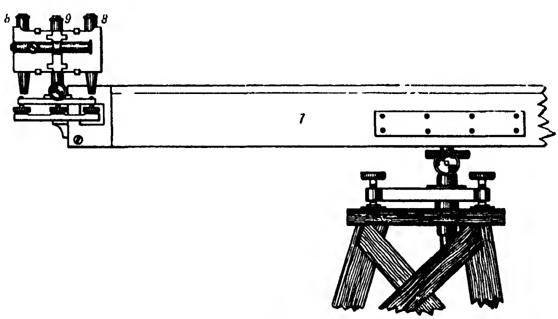


Fig. 47.—Colby Apparatus.

Survey, and was employed in the Ordnance and the Indian Surveys. It is compensating and optical, and consists of an iron and a brass bar, each 10 ft. $1\frac{1}{2}$ in. long, $\frac{1}{2}$ in. broad, and $1\frac{1}{2}$ in. deep. These are firmly fixed together at the middle by two steel pins *1*, and the compound bar rests on rollers in a wooden box *7*, mounted on

trestles in a manner permitting horizontal and vertical adjustment of the box. The bar is held in the box at the middle of its length by the stop *2*, and is free to expand or contract under change of temperature A spirit level is placed on the bar, and is observed through a window in the top of the box.

Near each end of the compound bar is a flat steel tongue *3*, about 6 in. long, which is supported by double conical pivots held in the forked ends of the bars. Each tongue carries at *6* a small platinum plug having a microscopic dot marking the effective end of the bar. To secure compensation, the ratio $\frac{6-4}{6-5}$ is made equal to $\frac{\text{coeff. of lin. expansion of iron}}{\text{coeff. of lin. expansion of brass}}$, so that, since the tongue is free to pivot, the position of the dot remains constant under change of temperature (Fig 48), the distance between these reference points being 10 ft. Evidently this cannot be strictly true for all temperature changes, but the error is very small.

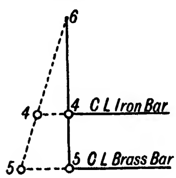


FIG. 48.

Six compound bars are simultaneously employed in the field. The gap between the forward mark of one bar and the rear mark of the next is made a constant length of 6 in. by using a compensated bar 7 in long, the tongues of which carry two vertical microscopes *8*, 6 in. apart A small telescope *9*, parallel to the microscopes, is fixed at the middle of this bar for use in sighting reference marks on the ground. Each compensated microscope bar is supported on the case containing the main bar, and has provision for horizontal and vertical alignment.

The Eimbeck Duplex Apparatus.—The contact apparatus designed by Mr. Eimbeck of the U.S. Coast and Geodetic Survey, and shown diagrammatically in Fig. 49, forms a good example of a bimetallic, non-compensating instrument. The measuring components are

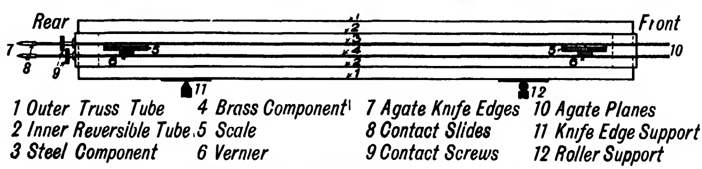


FIG. 49.—EIMBECK APPARATUS

two 5 m. long and ⅝ in. diameter nickel-plated tubes, of steel and brass respectively, the metal of the latter being the thicker so that both may undergo temperature change at the same rate. These are supported within a brass tube of about 2⅝ in. diameter, which is in turn enclosed within another of about 4 in. diameter mounted on trestles and carrying an aligning telescope and an inclination sector. The inner, or reversing, tube is arranged to rotate about

its axis through 180° for reversing the positions of the components to minimise the influence of unequal lateral heating. The components project beyond the tubes at both ends, and each is fitted with contact surfaces of agate in the form of a vertical plane at one end and a horizontal knife-edge at the other. The contacts, steel to steel and brass to brass, are made by turning the screws D, which move the components against the action of springs

Two bars are employed in the field, and, when the contacts are made, the forward bar remains stationary while the other is carried forward and brought into line and contact The inner tube with the components is reversed at regular intervals Unless the temperature happens to be that at which the components are of equal length, one will continually gain on the other, and, when the difference becomes inconvenient in making the contacts, the brass component is set forward or backward relatively to the steel The shift is measured on scales attached near each end of the steel component, and is read through windows in the tubes against verniers on the brass component The relative positions of the components at the beginning and end of a measurement are read on these scales.

Two measures of a base are thus obtained by means of the steel and the brass components respectively, and from the instrumental constants and the difference between the results the true measure, corrected for temperature, is deduced. The constants obtained by calibration are the expansion coefficients and the length and temperature of the components when they are of equal length To furnish a check, each bar is provided with three mercurial thermometers, from the indications of which the base length may be obtained from the result given by either component independently of the other

The Woodward Iced Bar Apparatus.—This apparatus, designed by Prof. Woodward, formerly of the U S. Coast and Geodetic Survey, to obviate uncertainties of temperature correction and compensation, is monometallic and optical. The bar is of steel, 5·02 m long, 32 mm. deep, and 8 mm broad, with the upper half cut away for a length of about 2 cm. at each end to enable the terminal lines, engraved on platinum-iridium plugs, to be placed on its neutral surface or surface of no strain For the alignment of the bar, similar plugs, carrying a centre line, are inserted in its top surface.

The bar is supported at intervals of 0·5 m. in a Y-shaped trough, and is controlled at each support by one vertical and two lateral screws (Fig 50) The trough is mounted on two bogies, which run on a portable narrow gauge track made in 5-m. sections. It is completely filled with crushed ice, which is kept in close contact with the bar by reason of the form of the trough and the vibration caused in rolling the bogies forward. The mounting of the trough

on the bogies permits of vertical, longitudinal, and lateral movements, with fine adjustment, the gradient of the bar being given on a sector. The successive positions of the ends of the bar are marked by micrometer microscopes mounted on trestles or stout posts. The microscopes are set vertical by levelling screws, and can be given a small motion along and transverse to the line of measurement. Micrometer readings are estimated to ·0001 mm.

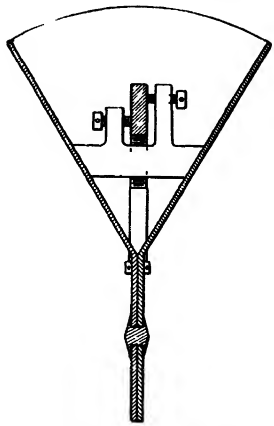


FIG 50 —WOODWARD APPARATUS.

In measuring a base, the microscopes having been erected approximately 5 m. apart, the small distance between the base terminal and the first microscope is read through the latter on a special scale, and the bar is then adjusted until its rear mark is centered on the micrometer wires. The observer at the other end of the bar brings his microscope into position for sighting the forward end mark, bisects it with the micrometer wires, and notes the reading. The forward microscope remains stationary while the bar is rolled forward until its rear mark is focussed under that microscope.

Measurement by Flexible Apparatus.—Wires and tapes applied in precise measurement are usually from 24 m. to 100 m. long. They are stretched under a constant tension, and, to avoid uncertainties arising from friction and from unevenness of the ground surface, they must be suspended in the air. The most accurate method, and that adopted with the shorter lengths, is to suspend the wire or tape in one catenary between end supports. Under constant conditions of temperature, tension, etc., the straight line distance between its terminal marks is invariable, and forms the standardised measuring unit, alterations in the conditions being the subject of corrections. Long tapes are sometimes stretched over one or several intermediate supports.

The question as to whether wires or tapes are to be preferred is a matter of opinion. Wires expose less surface to the action of wind, while tapes have the advantages that twist is easily detected and that the terminal graduations can be engraved directly on the tape, whereas a wire must carry attached scales.

The increasing importance of the catenary method of precise measurement, and the now all but universal application of invar thereto, warrants a description of the apparatus and methods usually employed.

Invar.—The researches of Dr. Guillaume, of the Bureau International des Poids et Mesures, on various properties of the nickel-

steel alloys, led to the discovery in 1896 that, as the proportion of nickel increases to about 36%, the value of the coefficient of expansion decreases to a minimum This least expansible alloy has been called *Invar*. It possesses the smallest coefficient of expansion of any metal or alloy known, and its application to precise linear measurement has greatly reduced errors arising from inexact knowledge of the temperature of base apparatus.

The coefficient of expansion of invar varies in different specimens. It depends not only upon the percentage of nickel but also upon the temperature, and is influenced by the proportions of carbon, manganese, silicon, etc. present, and by thermal and mechanical treatment. By suitable treatment, invar having a negative coefficient of expansion may be produced: the value of the coefficient rarely exceeds ·0000005 per 1° F.

Invar exhibits phenomena of change of volume, both of a permanent and a temporary character, the existence of which must be recognised in base apparatus The permanent change manifests itself in a slow increase with age which continues for years at a decreasing rate, the length of a tape or wire appearing to tend towards a definite limit This effect is considerably reduced, although not eliminated, by subjecting the tapes or wires to a process of artificial ageing (*étuvage*), which consists in annealing

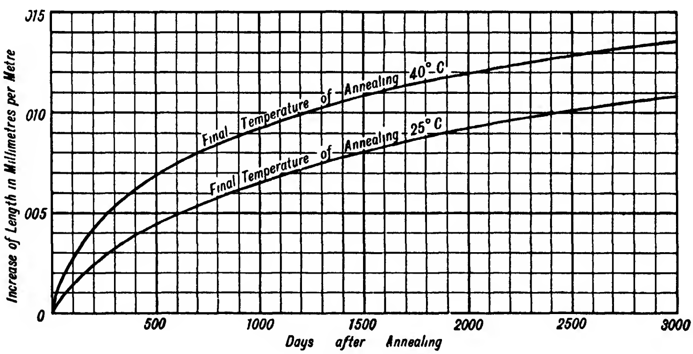


Fig. 51.

them by exposure for several days to the temperature of boiling water. The temperature is thereafter slowly lowered to reach 25° C. at the end of about three months. Fig. 51 shows results obtained at the Bureau International for the increase of length of invar bars of which the final temperature of annealing was 40° and 25° C. respectively, the bars being thereafter kept at the temperature of the laboratory. Because of the comparatively rapid growth at first, it is advisable to allow as long a period as possible to elapse between *étuvage* and standardisation of apparatus.

The temporary phenomena occur with change of temperature. When invar which has remained for some time at a certain temperature is subjected to a rise of temperature, it expands according to the coefficient of expansion, but, on continued exposure to the higher temperature, a slow residual contraction or creep is exhibited. When, on the other hand, the alloy is subjected to a fall of temperature. the residual change is an expansion. For temperatures between 0° and 100° C. the creep on a length L is given by Dr. Guillaume as

$$\delta L = 0{\cdot}00325{\cdot}10^{-6}\, L(T_1{}^2 - T_2{}^2),$$

where T_1 is the original, and T_2 the new temperature in degrees centigrade. At atmospheric temperatures creep develops very slowly, and several months may elapse before the alloy assumes equilibrium under the new temperature At high temperatures the movement is much more rapid

In general, the precision with which allowance can be made for these variations is sufficient for geodetic purposes Reference tapes or wires are standardised before and after use, and, if a considerable period elapses between the standardisations, the lengths at the time of base measurement are interpolated

Other properties of invar are not prejudicial to its employment in geodetic measurements. The tensile strength is 100,000 to 125,000 lbs. per sq in., the elastic limit about 75% of the ultimate strength, and the modulus of elasticity 21 to 23,000,000 lbs. per sq in. In tests made at the Bureau of Standards, Washington, no permanent set was detected under loads of at least 1/8th of the ultimate strength. The alloy is, however, much softer than steel, and is easily bent, so that tapes and wires must be handled with considerable care, and the drums on which they are wound must be of ample diameter to avoid set. While the resistance to oxidation is much greater than that of steel, it is not absolute, and occasional oiling is necessary.

Measurement by Wires or Tapes in Catenary.—In this method, introduced by Jaderin, the successive intervals between a series of movable marks are measured by a wire or tape freely suspended and subjected by means of straining tripods to a constant tension

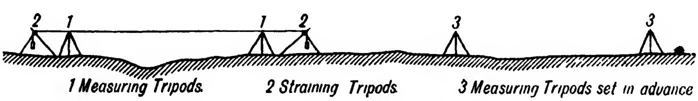


FIG. 52.

(Fig. 52). The movable marks are carried on tripods, which are set in alignment at approximately correct intervals in advance of the measurement, and the elevation of each is recorded for reduction of the measurement of each span to the horizontal.

As the employment of invar wires or tapes on this system yields the best results which can be attained by flexible apparatus, the following description relates to their use.

Apparatus.—*Wires and Tapes.*—Wires are employed in the apparatus, designed by Dr. Guillaume and M. Carpentier, which has been chiefly used since the introduction of invar. The wires are of 24 m. nominal length by 1·65 mm. diameter, and weigh 17·32 gm. per metre. They have attached at each end a *réglette* or scale, graduated to millimetres, the reading edge of which lies in the prolongation of the axis of the wire (Fig 53) Both scales read

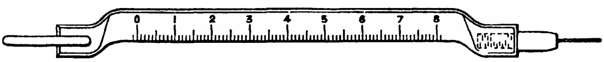
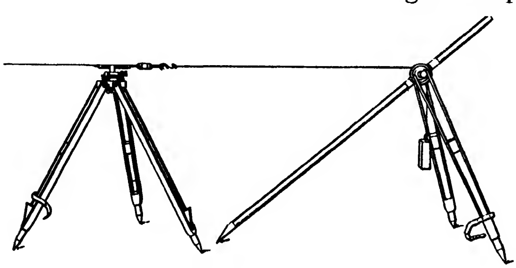


FIG 53 —*Réglette*

in one direction. When not in use, the wires are wound on a drum, of 50 cm. diameter, mounted in a box

Invar tapes for simple suspension are usually either 24 m. or 100 ft long with a cross section of 3 mm. by 0·5 mm The mètric tape is divided to millimetres for a length of 10 cm at each end, and in the case of the 100-ft tape the end scales are divided either to $\frac{1}{25}$ in. or $\frac{1}{100}$ ft Experiments made at the National Physical Laboratory indicate that a reel diameter of 15 in. is safe for tapes $\frac{1}{50}$ in. thick

To allow for the possibility of injury, as well as to afford checks on the work, two or three of these wires or tapes are required for use in the actual measurement As many more are reserved as reference wires exclusively for periodical standardisation of the field wires during the measurement. Whether wires or tapes are employed, a short, fully divided tape is required for measurement of the distance between the last tripod and the base terminal. It may also be necessary to provide one or more wires or tapes of twice or three times the normal length for spanning gorges, etc



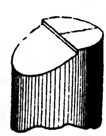


FIG 55. REFERENCE MARK

FIG. 54.—MEASURING AND STRAINING TRIPODS

Measuring Tripods.—The tripods (Fig. 54), usually ten in number, carry a small upright cylinder upon which is engraved the line to which measurement is made. The cylinder is set vertical by means

of levels and foot screws, and is capable of fine adjustment, relatively to the tripod, for alignment. Fig 55 illustrates the best form of cylinder head, half being cut away so that the surface of the tape or the face of the *réglette* may lie in the plane of the reference mark. A plumb line may be suspended for connecting the tripod mark to the ground when required, but it is preferable to employ two theodolites at right angles, the sights being taken on a fine needle held on the mark.

Straining Apparatus.—Straining of the wire or tape is effected by weights which are carried at each end by a cord attached through a swivel hook to the ring of the wire or tape They are suspended from two straining tripods (Fig. 54), each of which carries a pulley on ball-bearings The tension should be not less than 20 times the weight of the wire or tape. The Guillaume-Carpentier 24-m wires are strained by weights of 10 kgm In the measurement of the Lossiemouth base line by the Ordnance Survey the 100-ft tapes were given a tension of 20 lbs It is important that the same straining apparatus should be used in standardising as in the actual measurement because of uncertainty arising from friction in the pulleys

Miscellaneous—One theodolite is required for the alignment of the measuring tripods, and a second should be carried so that transfers between the tripod and ground marks may be effected independently of the plumb line In the Guillaume-Carpentier apparatus, alignment is performed by means of a telescope which fits on the measuring tripods.

The levelling observations are made either by a level and light staff, fitted with a rubber pad for contact with the tripod heads, or by the clinometric apparatus of the Guillaume-Carpentier equipment. The telescope of the latter fits on the measuring tripods, and carries at the focus of the object glass a scale of tangents against which the reading is observed The sights are taken on a target fitted on the adjacent tripod

Other items of equipment include a spacing tape or wire, usually of steel, for setting out the tripods, thermometers, reading glasses, and iron pickets with movable heads for marking section terminals.

Field Work.—*Preliminary Operations*—The ends of the base are defined in the usual manner by a sub-surface and a surface mark, as well as by reference points : the surface mark for base terminals is sometimes elevated on a pillar of concrete about 3 ft above the ground. The line of the base must be cleared of trees and bushes, and a series of line marks is accurately established at intervals of about half a mile to control the alignment of the tripods

Organisation of Field Parties.—The personnel is divided into two parties :

(1) A setting-out party, whose duty is to place the measuring

tripods in alignment in advance of the measurement, and at as nearly as possible the correct intervals. This party consists of two surveyors with a number of porters for carrying forward tripods.

(2) A measuring party of two observers, recorder, leveller, and staffman, with porters for the transport of apparatus. The duties of recorder and leveller are sometimes combined.

Setting Tripods.—If the base terminal is not elevated above the ground, the first tripod must be set with its fiducial mark vertically over the station mark Thereafter the tripods are aligned by a theodolite centered over one of the line marks previously established along the base, or by the aligning telescope of the Guillaume-Carpentier apparatus The setting out of the approximately correct intervals is performed by means of the spacing wire or tape, which is used at the tension adopted for the measuring wires or tapes, the pull being measured by a spring balance Each tripod when placed at the proper distance has the head set level and is finally adjusted into alignment

The setting of the tripods proceeds at about the same rate as the measurement Fresh tripods are carried forward from the rear, but one, at least, is always left behind the measuring party for reference in case of disturbance of the tripods under measurement.

Measurement.—The series of operations involved in measuring the distance between each pair of tripods must be executed on a uniform system to avoid delays. The first measurement in each bay is that of the difference of level between the tripod heads. If it is made by level, the instrument is set up equidistant from the tripods, and readings are estimated to ·001 ft With the clinometric telescope the slope is read to 0001, and is obtained as the mean of a forward and a back reading

While the levels are being observed, the wire, weights, and straining tripods are brought forward from the preceding span. The latter are placed at the proper distance from the measuring tripods and as nearly as possible in the correct line. The swivel hooks fastened to the cords over the pulleys are then handed to the observers, who attach them to the wire, and at the word of command the men carrying the weights simultaneously hook them to the cords as gently as possible To avoid lateral friction of the cords in the pulleys, the latter must lie in the vertical plane of the base. The straining trestles are finally adjusted for line and level to bring the scales into gentle contact with, and in the plane of, the reference marks on the measuring tripods. Before reading, an examination is made for twist of the wire.

The scale distance to be applied to the known length between the zeros is obtained as the mean of five to eight readings taken by the observers in simultaneous pairs, the wire being displaced

longitudinally between each. Readings are taken with the aid of magnifying glasses to one-tenth of a scale division, and are noted by the recorder, who at the same time tabulates the differences. The maximum range in the differences should not exceed 0·3 mm , otherwise further readings are required The record may be arranged as follows :

Span No	Wire No	Readings in mm		Difference	Mean	Temp Fahr
		Rear	Forward			
23	2	16 5 28 2 42 4 58 7 65 1 72 9	11 4 23 0 37 3 53 5 59 8 67 7	−5 1 −5 2 −5 1 −5 2 −5·3 −5·2	−5 18	53° 5

The temperature during the measurement of each bay is read by the recorder. If two or more field wires are used, the same procedure is repeated with another wire before lifting the straining tripods. When the measurement of a bay is completed, the weights are simultaneously unhooked, and the apparatus is carried forward for the next span The wires must, of course, not be allowed to drag on the ground, and should be carefully guarded against accidental overstrain, so that the observers should themselves take part in their transport.

To reduce errors of personal equation, the observers change ends after every tenth span, or oftener. Each day's work forms a section, the end of which is marked by transferring the last tripod mark to a picket set in the ground Each section must be measured at least once in each direction, the magnitude of the discrepancies indicating whether further measurements are required. Work must be suspended when wind causes appreciable vibration of the scales.

Standardisation of Field Wires and Tapes—The reference wires which are provided for comparison of the field wires are standardised before they go on service and again on their return The expansion coefficients of the field wires must also be known, but the lengths of the latter are determined in the field before and after the measurement of the base and at intervals during its progress. As a medium of comparison, a short reference base is laid out in the field in a convenient position. The length may be one or several wire lengths, and is marked by lines on metal plugs firmly bedded in concrete. A standardisation of the field wires consists in measuring this base three times with each reference and field wire alternately. The average of the results given by the reference wires represents the length of the reference base at the mean temperature during measurement, from which are derived the values of the field wires for that temperature.

Use of Long Tapes.—In the course of a measurement by the system just described, if roughness of the ground necessitates the use of a long wire or tape, a greater pull is required, and the consequent increase of friction in the straining pulleys is a source of uncertainty affecting the value of the actual tension. If possible, one or more intermediate supports should be provided

The system of using long tapes supported at intervals has been greatly used in American base measurements. The tapes are 50 m. and 100 m , or 300 ft , long by about 6 mm by 0·5 mm., and supports are provided at regular intervals of 10 m to 25 m A series of trestles or, more usually, firmly driven posts, about 4 in. by 4 in are set out in the line of the base and approximately a tape length apart On the top of each is nailed a plate of copper or zinc on which is to be marked the position reached by the forward end graduation of the tape Lighter pickets serve as the intermediate supports, the tape resting either on a nail in the side of each or in suspended wire loops The points of support are set on a uniform gradient between the mark posts when this proves inconvenient, their respective elevations are recorded, and must be such that the tape when stretched will not lift clear of any support.

The rear end of the tape is connected to a straining stake behind the mark post, and the tension is applied at the forward end by means of a tape stretcher A common and simple form consists of a wooden base to which an upright lever is hinged by a universal joint A spring balance, graduated to ounces, is connected to the lever through a gimbal, and to it the tape is attached Provision is made for adjusting the position of the spring balance and tape on the lever to suit the height of the mark post The operator, standing on the base plate, applies the tension by pulling the lever, and sees by the indication of the spring balance that it is maintained. When the tape is strained, the position of the forward end graduation is engraved on the plate by means of a sharp awl. After carrying forward the tape, the rear end is adjusted to coincide with the mark previously made, or its position is also marked on the plate, and the gaps or overlaps are measured by a finely divided scale

The rate of measurement on this system is roughly twice that with short, freely suspended wires or tapes, speeds of over a mile an hour being usual Although excellent results have been obtained, the error of marking the plates is naturally greater than can occur in the mean of repeated readings of scales against tripod marks, while the effect of friction at the supports is an additional source of uncertainty.

Temperature Measurement.—The measurement of the actual temperature of a wire or tape under field conditions is one of considerable difficulty. A close determination is essential when steel tapes are employed, but in the case of invar the influence of small errors of thermometry is rendered negligible by the small-

ness of the coefficient of expansion. The consistency attained in repeated invar measurements indicates that, when temperature conditions do not change rapidly, it is sufficient to record for each tape length the reading of a single thermometer suspended or whirled near the tape. In tropical climates two or three thermometers may be spaced along the span.

The coefficient of expansion of steel tapes is about ·0000063 per 1° F., or thirty times the mean value of the coefficient for invar, and an uncertainty of 1° Fahr in the tape temperature therefore corresponds to an uncertainty in distance of 1/160,000. In sunshiere the temperature indicated by a mercurial thermometer may diffn from that of the tape by several degrees, and to attain an accuracy of 1/500,000 or over, measurement by steel tape must be executed only when the air and the ground are at the same temperature, *i.e.* in densely cloudy weather or, preferably, at night

The thermometers used read to 0°·1 C or 0°·2 F, and are calibrated by comparison with a thermometer standardised with reference to the hydrogen scale The bulbs are sometimes encased in a thin sheath of steel or wound with steel wire to assist in reproducing the tape temperature. Three thermometers are placed at intervals along the tape, either suspended from poles, so that the bulbs are level with the tape, or attached to the tape itself, and they are read before and after the measurement of each tape length. The measurement of each section of the base should be divided equally between rising and falling temperatures to reduce the influence of lag in the thermometers caused by volume change in glass lagging behind temperature change.

Errors arising from uncertainty in temperature measurement may, in the future, be greatly reduced by standardising wires and tapes in terms of their electrical resistance instead of at a stated temperature, and by observing the resistance during the field measurement.

Measurement by Steel and Brass Wires.—In Jäderin's application of the principle of the bimetallic thermometer to flexible apparatus, the measurement is made by a steel wire and by one of brass. These are 25 m. long by 1·5 to 2·6 mm. in diameter, and have end scales 10 cm. long divided to millimetres. The wires used in combination are of similar diameter, and are nickel plated to ensure the same temperature conditions for both. They are freely suspended, one after the other, between movable tripods carrying a fine needle against which the scales are read. Tension is measured by a spring balance graduated to tenths of a kilogramme and read to hundredths by estimation. From the total lengths given by the steel and brass wires respectively the temperature effect is eliminated by the method of page 132.

Base measurement on this system can be executed in sunshine with as great precision as can be attained by the use of steel tapes

in densely cloudy weather or at night. The method has, however, been superseded by the employment of invar.

Calculation of Length of Base.—The final value of a base measurement is arrived at from the immediate results of the field work by the application of a series of corrections.

A number of corrections refer to the apparatus, and aim at the elimination of the effects produced by differences between its absolute and nominal length and between the field conditions and those of standardisation. In measurement by rigid bars, corrections of this class are required for absolute length and temperature. In the case of flexible apparatus, additional corrections are necessitated by any deviation from the standard conditions as to suspension and pull, which govern the form of the curve assumed by the wire or tape The application of these corrections yields the distance along the gradients in which the measurement has been made.

The remaining corrections depend upon the vertical and horizontal alignment and elevation of the base The length of each slope must be reduced to its horizontal equivalent. A correction for horizontal alignment is required in cases where the location of the base necessitates an alignment in plan which deviates from the straight line between the terminals Lastly, the length of the base is reduced to the value it would have if projected upon the mean sea level surface For uniformity, all stations in a geodetic survey are considered as projected normally on to this surface, and all linear distances are arcs upon it. To obtain the projected lengths of all the triangle sides, it is sufficient to reduce those of the measured bases.

Corrections are positive or negative according as the uncorrected distance is to be increased or decreased. Each section of the base is separately corrected, as the degree of consistency obtained in repeated measures can be gauged only from the final reduced lengths.

Correction for Absolute Length.—The absolute length of a base measuring instrument as obtained in standardising is usually expressed as its nominal length, l, plus or minus a small quantity, c, the value of which is given for the temperature of standardisation If, therefore, the instrument is laid down n times, the nominal distance, nl, is subject to a correction, nc, of the same sign as that of c in the expression for the absolute length of the instrument.

Correction for Temperature.—It is usually sufficient to deduce from the thermometer readings the average temperature prevailing during the measurement of each section and to apply a temperature correction for the whole section. If, however, the value of the coefficient of expansion of the apparatus varies under change of temperature, it is necessary to correct each bar or wire length. Thermometer readings are first corrected for scale errors,

and it is important in precise work that the coefficient of expansion should be known for the particular instruments used, instead of adopting average values.

Let T_m = mean temperature during measurement,
T_s = temperature of standardisation,
a = coefficient of expansion,
L = measured distance.

Correction for temperature difference $= +a(T_m - T_s)L$.

Temperature Correction for Bimetallic Apparatus.

Let L_s, L_b = distances as computed from the absolute lengths of the steel and brass components at standard temperature,
a_s, a_b = coefficients of expansion of the components,
D = corrected distance,
$T = T_m - T_s$.

Then $D = L_s(1+a_sT) = L_b(1+a_bT)$,

whence $T = \dfrac{L_s - L_b}{a_bL_b - a_sL_s}$,

and $D = \dfrac{L_sL_b(a_b - a_s)}{a_bL_b - a_sL_s}$,

so that correction to result given by steel component $= D - L_s$
$= +\dfrac{a_sL_s(L_s - L_b)}{a_bL_b - a_sL_s}$, or, with ample accuracy, $= +\dfrac{a_s(L_s - L_b)}{a_b - a_s}$.

Similarly, correction for brass component $= +\dfrac{a_b(L_s - L_b)}{a_b - a_s}$.

Correction for Change of Tension.—This correction is required when flexible apparatus is subjected in the measurement to a different tension from that of standardisation

Let F_m = tension applied during measurement,
F_s = tension applied during standardisation,
A = cross-sectional area of wire or tape,
E = modulus of elasticity of do.,
L = measured distance

Correction for change of tension $= +\dfrac{(F_m - F_s)L}{AE}$.

The cross-sectional area is most accurately obtained by weighing the wire or tape and dividing the weight by the length times the density. For geodetic work, in place of adopting a general value for the modulus of elasticity, it should be determined by subjecting the wire or tape to widely different tensions and measuring the change of length with change of tension.

Correction for Sag.—This correction represents the difference between the actual length of a suspended wire or tape and the

chord distance between the end graduations when these are at the same level. The correction is applicable when the wire or tape is standardised on the flat and suspended during measurement, in which case the value of the correction must be known with a precision equivalent to that of standardisation. Wires and tapes to be used in single catenary are standardised in terms of the chord distance obtained at standard tension, so that no correction for sag is necessary

Let w = weight of wire per unit length,
F = applied tension in same unit,
S = chord length between zeros of wire when suspended,
L = actual length of wire

Sag correction $= (S-L) = -\frac{S^3w^2}{24F^2}$, or $= -\frac{L^3w^2}{24F^2}$,

with sufficient approximation if F is not less than $20\,wL$

If the wire or tape is supported at intermediate points, forming n equal bights, the correction becomes $\frac{1}{n^2}$ times the above.

Correction for Index Error of Spring Balance.—When tension is measured by spring balance, the same instrument should be employed in the field measurement as is used in standardising If not, it is necessary to apply an index correction the value of which for horizontal tension differs from that for a vertical position of the balance A balance adjusted to read correctly in the vertical position indicates less than the true tension when used horizontally.

Let I = index error when balance is vertical, *i e* reading when suspended hook downwards, without load,
I' = reading with balance inverted and suspended from hook,
W = total weight of balance, found by weighing,

then index error for horizontal position $= \frac{W-I-I'}{2}$,

I being taken as negative when less than zero

The value of the correction for readings taken on a slope is investigated by Young in a paper to the Institution of Civil Engineers (*Min. Proc*, Vol CLXI)

The indications of spring balances are also influenced to some extent by temperature changes Balances should therefore be subjected at the standard tension to different temperatures, and a table of corrections prepared.

Correction for Change of Gravity.—When a wire or tape measurement is made in a different latitude or at a different elevation from that of the place of standardisation, allowance is made in geodetic work for the change in the value of g, the acceleration due to gravity. When tension is applied by means of suspended weights, the pull they exert and the weight of the wire are proportional to g. The form of the catenary remains constant, but the wire suffers

a small extension or shortening by the resulting increase or decrease from the standard tension. If the tension is measured by spring balance, it is unaffected by change of g, but the weight of the wire is altered, so that in this case increase of g causes increase of sag.

The various formulæ which have been proposed to express the variation of g over the earth give sufficiently accordant results that any of them can be used for the present purpose. The formula of Helmert is

$$g = g_o(1+0{\cdot}005310 \sin^2\phi),$$

where g = the acceleration due to gravity at sea level in latitude ϕ,
g_o = the value at sea level at the equator.

The above value of g is subject to various corrections depending upon the elevation of the place, the density of the underlying strata, and the conformation of the surrounding country The elevation corrections have the values :

(*a*) For height above mean sea level, $-g\dfrac{2h}{R}$.

(*b*) For the mass between sea level and the station, $+g\dfrac{3}{4}\dfrac{h}{R}$

where h = elevation of station above mean sea level in ft ,
R = mean radius of the earth = 21×10^6 ft.

In the case of straining by weights, let g_1 and g_2 be the respective values of gravity, in terms of g_o, at the places of standardisation and of measurement.

$$\text{Then } \frac{g_2-g_1}{g_1} = \frac{\delta F}{F},$$

where δF is the change produced on the nominal tension F The correction is then evaluated by the change of tension formula, and is positive or negative according as g_2 is larger or smaller than g_1.

Correction for Slope.

Let l_1, l_2, etc. = lengths of successive uniform gradients,
h_1, h_2, etc. = differences of elevation between the ends of each

For any gradient of length l and difference of elevation h,

$$\text{the slope correction} = -(l-\sqrt{l^2-h^2}),$$

$$= -\left(\frac{h^2}{2l}+\frac{h^4}{8l^3}+\text{etc.}\right).$$

The second term may safely be neglected for slopes flatter than about 1 in 25. With its omission,

$$\text{total correction} = -\tfrac{1}{2}\left(\frac{h_1{}^2}{l_1}+\frac{h_2{}^2}{l_2}+\ldots\right),$$

$= -\dfrac{\Sigma h^2}{2l}$, when the gradients are of uniform length l.

If the slope is measured in terms of θ, the angle of elevation or depression,

$$\text{correction} = -l\,(1-\cos\theta) = -2l\sin^2\tfrac{1}{2}\theta.$$

Wires and Tapes in Catenary.—The above slope corrections are strictly applicable only to measurements by rigid apparatus, but are also usually employed in the case of tapes with frequent supports. For a wire or tape suspended in catenary, account should be taken in precise work of the deformation of the catenary when the ends are not at the same elevation The development of formulæ for this case is too lengthy for inclusion, and the reader is referred to Prof Henrici's "Theory of Tapes in Catenary" in Ordnance Survey Professional Paper No 1. The formula obtained by Henrici is as follows.

Let S = nominal length of tape or wire, along chord between zeros,
h = difference of elevation between measuring tripods,
l = length along catenary between tripod marks,
a_o = standardisation correction to nominal length,
a_1 = algebraic sum of scale readings,
a = coefficient of expansion,
t = excess of field temperature over standard temperature,
$A = a_o + a_1 + Sat$,
λ = apparent shortening due to sag $= \dfrac{X_o^3 w^2}{24 F^2}$, as before,
X = required horizontal distance between tripod marks,
X_o = value of X when $h = 0$

Then $X_o = l - \lambda = S + A$,

$$\text{and } X = X_o\left(1 - \frac{h^2}{l^2}\right)^{\frac{1}{2}},$$

$$= (S+A)\left(1 - \frac{h^2}{(S+A+\lambda)^2}\right)^{\frac{1}{2}}.$$

Expanding, and retaining only the first powers of A and λ, this becomes

$$X = S - P + A + Q + R,$$

$$\text{where } P = S\left(\tfrac{1}{2}\frac{h^2}{S^2} + \tfrac{1}{8}\frac{h^4}{S^4} + \tfrac{1}{16}\frac{h^6}{S^6} + \tfrac{5}{128}\frac{h^8}{S^8} + \ldots\right),$$

$$Q = A\left(\tfrac{1}{2}\frac{h^2}{S^2} + \tfrac{3}{8}\frac{h^4}{S^4} + \tfrac{5}{16}\frac{h^6}{S^6} + \ldots\right),$$

$$R = \lambda\left(\frac{h^2}{S^2} + \tfrac{1}{2}\frac{h^4}{S^4} + \tfrac{3}{8}\frac{h^6}{S^6} + \ldots\right).$$

This formula for X gives a result accurate to 1/10,000,000 if $\frac{h}{S}$ is less than $\frac{1}{10}$, A is less than S/1000, and the tension applied exceeds 20 times the weight of the tape, and to 1/2,000,000 if $\frac{h}{S}$ is less than $\frac{1}{3}$, and A is less than S/1,000, provided the mean tension is constant.

Correction for Horizontal Alignment.—In the exceptional case where the base must be aligned as a series of two or more courses of

different bearings, the distance between the terminals falls to be computed as for a traverse survey. The deviation from straight alignment is usually small, and the formulæ for slope correction are applicable if l represents the length of any course, θ, the angle it makes with the straight line between terminals, or h, the difference between the offsets from that line to the ends of the course.

Reduction to Mean Sea Level.—The profile of the measurement must be referred to mean sea level, and the mean elevation of the base deduced.

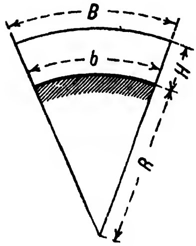


FIG. 56.

Let B = measured length of base,

b = length reduced to mean sea level,

H = average elevation of base,

R = radius of earth in latitude and azimuth of base

From Fig 56, $b = B\dfrac{R}{H+R}$,

or correction $(b-B) = -B\dfrac{H}{H+R}$ or $= -B\dfrac{H}{R}$, since H is small compared with R

R is commonly taken as the mean radius of the earth

Probable Error of Base Measurement.—The probable error (page 168) of the final result is the square root of the sum of the squares of the probable errors of the individual operations which can appreciably influence the result To take the case of a wire or tape measurement, the following errors require recognition

Errors in Standardisation (*a*) Probable error of prototype bar, as compared with the original international metre standard

(*b*) Probable error of mural base at standards laboratory, as derived from prototype

(*c*) Probable error of standardising reference wires on mural base.

(*d*) Probable error of values of coefficient of expansion of reference and field wires

(*e*) Probable error in estimation of length of reference wires at dates of comparison with field wires, arising, particularly in the case of invar, from molecular change.

(*f*) Probable error of comparison of field wires with reference wires.

Errors in Measurement: (*g*) Probable error of measurement of differences in level of supports

(*h*) Probable error in reading scales

(*i*) Probable errors of tension or of change in weight of wire due to adhesion of dirt, making the catenary assumed during measurement differ from that at comparison.

(*j*) Probable error of temperature measurement.

(*k*) Probable error in value of mean elevation of base, affecting reduction to mean sea level.

To estimate the final probable error of a measurement, that contributed by each of the above sources is separately worked out for the whole base. The probable errors of standardisation operate as constant errors, and are proportional to the length of the base. The data required for the evaluation of the total probable errors from a, b, c, and d are obtained from the standards office. The effect of d for the reference wires is proportional to the difference between the standard temperature and the mean temperature of the field comparisons, and in the case of the field wires depends upon the difference between the latter temperature and that during the base measurement. The value of e can only be roughly estimated, while that of f is obtained by the usual method of evaluating probable error from the results of repeated observations (page 1[illegible]).

The combined effect of all purely accidental errors of measurement, such as g and h, is obtained in the same manner for each section by its repeated measurement, and the probable error for the whole base from this source is the square root of the sum of the squares of the probable errors of the several sections. Error i is non-compensating, but allowance for it is unlikely to be required if the same straining apparatus is used for standardisations as in the field measurement. The value of j cannot be exactly assessed. Error k is obtained from the probable error of the value taken for the elevation of the reference datum and of the levelling between the bench mark and the base.

As indicating the relative magnitudes of the errors, those of the Semliki base in Uganda may be quoted.* This base was measured in 1908 with the Guillaume-Carpentier apparatus, and the reduced length is 16,532.37644 m. The individual probable errors for the whole base are in millimetres:— $a = \pm 1.65$, $b = \pm 6.89$, $c = \pm 3.59$, $d = \pm 8.06$, $e = \pm 6.365$, $f = \pm 1.878$, $g+h = \pm 4.511$, $j = \pm 3.746$, $k = \pm 4.03$. Errors c, d, and e are for the mean of three reference wires, and f is for three field wires used in combinations of two. The mean temperature during measurement happened to be the same as that of the comparisons, so that no account had to be taken in d of errors in the coefficient of expansion of the field wires.

The probable error of the result is therefore $= \pm\sqrt{1.65^2 + 6.89^2 + \text{etc.}}$

$$= \pm 14.92 \text{ mm} \quad \text{or} \quad \frac{1}{1,108,000}$$

ANGLE MEASUREMENT

Instruments.—Considerable variety exists in the forms of theodolites for precise work, but they do not differ materially in essentials from the patterns used in ordinary surveying. In the early days of geodetic surveying the required refinement of reading was secured by the use of large circles, the great theodolite of the Ordnance Survey having a diameter of 36 in. Modern improvements in graduating engines enable as good work to be done with a 12-in. circle, and for primary triangulation the usual sizes are

* "Report of the Measurement of an Arc of Meridian in Uganda," Vol. I. Colonial Survey Committee, 1913.

now 10 in. and 12 in., with 8-in to 5-in instruments for secondary and tertiary work.

A high standard of workmanship is required throughout primary instruments. The telescope must be of the best quality, particularly as regards definition, and should be of sufficient magnifying power that the refinement of pointing may not be inferior to that with which the circle can be read. Several eyepieces are provided, and the magnifying power usually ranges from about 30 to 80, the aperture of the objective being $2\frac{1}{2}$ in. to 3 in An eyepiece micrometer is commonly fitted for use in making signal bisections by means of the movable vertical hair. By turning the micrometer into the vertical position, it is made available for the measurement of small vertical angles. To illuminate the field for night work, a small electric light, the intensity of which is under control, is attached to one of the standards, and the rays are projected through a lens in the hollow trunnion axis on to a very small mirror which reflects them to the hairs.

The circle is usually divided to 5 min , and is read by equidistant micrometers to 0·1 sec by estimation. For reading the large circles formerly used, five micrometers were fitted, but three is the usual number on 8-in to 12-in theodolites, and two on smaller instruments. A pointer microscope is provided for reading the figures The vertical circle may be of the same diameter as the horizontal circle, or may be much smaller and intended merely for use in finding signals or for setting approximately on a known vertical angle. In some foreign patterns it is omitted altogether. When the vertical circle is read by micrometers with a refinement similar to that for horizontal angles, the instrument is adapted for astronomical observation and may be distinguished by the name, altazimuth instrument.

Plate levels are fitted as in smaller theodolites, but the final levelling is performed by means of a striding level placed on the horizontal axis The sensitiveness of this level is not less than 2 sec. per division, and should be ascertained either on a level trier (Vol. I, page 27) or by placing the level longitudinally on the telescope and measuring a small vertical angle Centering is performed either by plumb bob or by means of a nadiral or look-down telescope, which can be screwed into the horizontal plate. The latter is particularly useful on elevated scaffolds. The weight of large theodolites necessitates the provision of a stout lifting ring by which they can be handled without danger of overstrain.

The instrument illustrated in Fig. 57 is by Messrs. E R. Watts and Son, Ltd., London, and was constructed for the Ordnance Survey for use on the test triangulation in Scotland.

The horizontal circle is of 12 in. diameter, and is entirely protected from dust. It is graduated to 5 min., and is read by three micrometers to single seconds directly and to 0·1 sec. by estimation. The micrometers and a pointer microscope are carried by a single

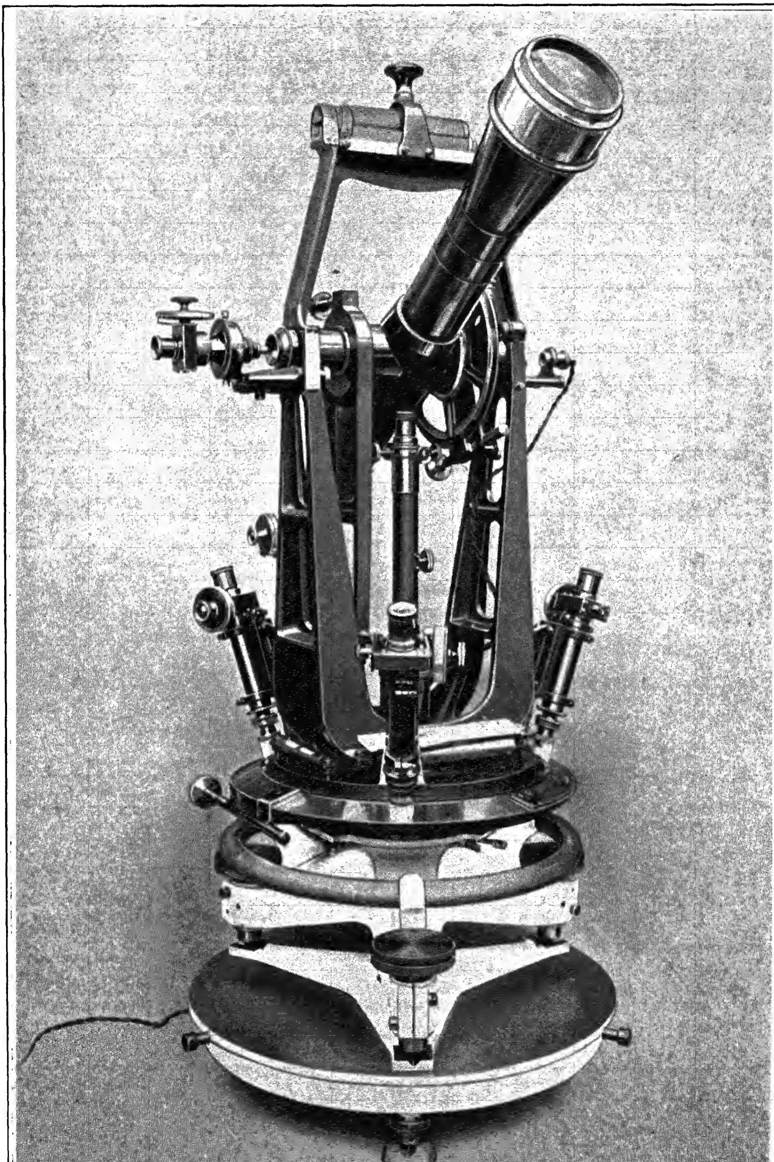


FIG. 57.—ORDNANCE SURVEY GEODETIC THEODOLITE.

casting The vertical circle is of 6 in diameter, reading to 1 min, and is used only for setting the line of sight approximately to a known vertical angle

The telescope, which transits both ways, has an objective of 24 in focal length and 3 in aperture It is fitted with an eyepiece micrometer, and the powers of the eyepieces range from 40 to 80. The field of view is illuminated electrically from either end of the trunnion axis The attachment shown at the left side is designed to verify the cylindrical form of the pivots The striding level has a sensitiveness of 1·25 sec per 0·1 in division.

For the accurate centering of the instrument over a station, a nadiral telescope, having an objective of about 8 in focal length, is screwed into the upper horizontal plate Centering is effected by bringing the line of sight of this telescope upon the station mark by means of three radial screws operating upon the lower base plate A strong circular leather-covered lifting handle is fitted to the tribrach plate

Fig 58 illustrates the instrument adopted by the Canadian Government Geodetic Survey and manufactured by Messrs Watts. The horizontal circle is of 12 in diameter, and is graduated and read in the same manner as in the foregoing example The setting circles are of 4 in diameter and read to 1 min. The telescope is non-transiting The objective has a focal length of 18½ in and an aperture of 3¼ in. An eyepiece micrometer is fitted, and the powers of the eyepieces range from 36 to 72

Adjustments of the Theodolite.—The adjustment of a precise theodolite does not differ materially from that of smaller instruments, but advantage is taken of the striding level for horizontal axis adjustment The adjustments pertaining to horizontal angle measurement are :

1 Plate level adjustment
2 Striding level adjustment
3 Horizontal axis adjustment
4. Collimation adjustment
5. Micrometer adjustments

Adjustment of the Plate Levels.—Unless the sights are nearly horizontal, these levels should be employed only in the approximate levelling of the instrument, and are subordinate to the striding level. Their adjustment is performed as described in Vol I, page 69.

Adjustment of the Striding Level.—*Object.*—To make the level axis parallel to its supports The V-shaped foot of either leg of the level affords two points of support, and the level axis is required to be parallel to the line joining the points midway between these two points in each V Two steps are necessary: *A*, to make the level axis coplanar with this line, *B*, to place them parallel.

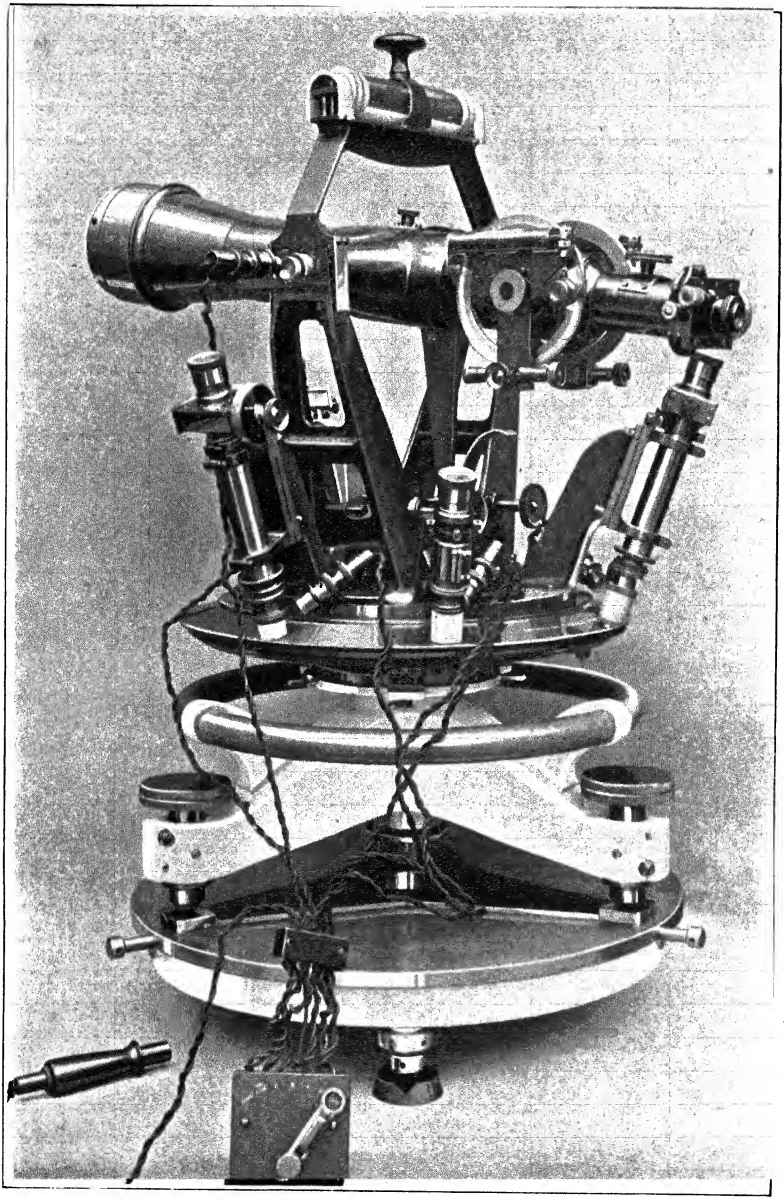


FIG. 58.—CANADIAN SURVEY GEODETIC THEODOLITE.

Necessity.—To enable the striding level to indicate the horizontality or otherwise of the horizontal axis

A : Test.—1. Set the striding level upon the horizontal axis, and level the instrument approximately.

2 Incline the level a little to one side or the other of the vertical plane. If the bubble remains in a constant position, the level is in lateral adjustment.

Adjustment.—If not, by means of the lateral controlling screw adjust the level until the test is fulfilled.

B : Test.—1. Level the instrument by reference to the plate levels.

2. Place the striding level on the horizontal axis, and centre the bubble exactly by the levelling screws

3. Remove the striding level, and carefully replace it end for end. If the bubble remains central, the level is in longitudinal adjustment

Adjustment.—1. If not, bring the bubble half-way back by means of the adjusting screws controlling the tube vertically

2. Relevel by the foot screws, and repeat until the test is satisfied.

Adjustment of the Horizontal Axis.—*Object and Necessity* —As for the engineer's transit theodolite (Vol. I, page 72).

Test.—1. Adjust the foot screws until the striding level maintains a constant position while the instrument is wheeled through 180° in azimuth. The vertical axis is now truly vertical.

2. If the bubble is central, the horizontal axis is correct, since the striding level is in adjustment

Adjustment.—1. If not, bring the bubble half-way back by means of the screws controlling the trunnion support in one standard.

2. Relevel, and repeat until the test is fulfilled.

Note.—If the error is small, it may be left, as its effect is eliminated by change of face, and in observations unbalanced as regards change of face the error can be corrected out from the reading of the striding level. Provision for making the adjustment is sometimes omitted in precise theodolites as in smaller instruments.

Adjustment of the Collimation Line.—This adjustment is performed as for the engineer's transit theodolite, except that the test sights should be as long as possible. If the telescope does not transit, reversal must be made by removing the axis from the standards and replacing with the telescope end for end.

Micrometers.—The essential features of the micrometer microscope have been described in Vol. I, page 56. Those fitted on precise theodolites are required to carry the subdivision to single seconds, and the magnifying power of the microscope must be sufficient to justify estimation of the readings to 0·1 sec. The circle is usually divided to 5 min., and the pitch of the micrometer screw

is such that five turns are required to move the hairs from one graduation to the next The micrometer drum is divided into sixty parts, which therefore represent single seconds For convenience in keeping count of the number of complete turns, there is provided a fixed comb scale, the teeth of which have the same pitch as the screw, so that one turn moves the hairs from the centre of one notch to that of the next. The centre notch is distinguished from the others, by greater depth or otherwise, and corresponds to the single notch in small instruments in approximately marking the position of the zero line

The appearance presented through the microscope is shown in Fig. 59. The hairs are shown centered over graduation 144° 10′, which is the approximate reading since the centre notch lies between 144° 10′ and 144° 15′. Assuming the hairs have been moved from the centre notch to this position, they have passed two notches, but have not reached the centre of the third The micrometer reading is therefore 2 min plus the number of seconds recorded on the drum, the complete reading being

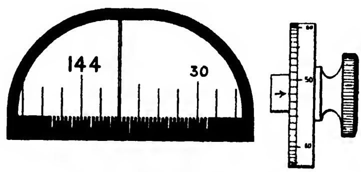


FIG. 59.

$$144° \, 10' + 2' + 48''{\cdot}0 = 144° \, 12' \, 48''{\cdot}0.$$

Notes —(1) In the reading of the comb scale, single minutes are reckoned by the number of spaces from hollow to hollow, and not from crest to crest of the notches The number of spaces to be counted is always that between the centre notch and the preceding graduation, representing the approximate reading, irrespective of the fact that the hairs may be centred over the following graduation. If the hairs appear to lie centrally in a notch, the drum reading shows whether they have reached the centre or passed it

(2) The utmost delicacy is required in manipulation, and strict attention must be paid to the precautions given in Vol I, page 58, notes 3, 4, and 5.

Run of the Micrometer.—The condition, hitherto assumed, that an exact number of turns of the screw should carry the hairs precisely from one graduation to the next, can be only approximately realised If, on moving the hairs across a circle space, the drum registers a different reading after the movement from that at starting, the discrepancy between the micrometer measurement of the space and its nominal value is termed the run of the micrometer. When run is present, the value of a fractional part of the circle division is not correctly given either by the back or the forward reading, *viz.* the respective readings obtained by setting the hairs on the graduation last passed by the zero line and on the graduation in advance of it. Since the drum reading decreases as the hairs are moved from left to right, the forward reading is less than the back reading when more than five turns are required to move the hairs across the space The micrometer is then said to overrun, or the run is positive otherwise it is negative.

Apart from errors of observation and manipulation, many factors contribute to the existence and variation of run. The circle spaces are not themselves equal on account of imperfect graduation, unequal temperature, or mechanical strain. Again, while the distances between the circle and the microscope objective and between the objective and the hairs should be such as to make the length of the real image of a circle division exactly five times the pitch of the screw, these distances are subject to variation under change of temperature, and the former is also affected by eccentricity Means are provided for adjusting the microscope to eliminate run, but with high power microscopes the perfecting of the adjustment is a delicate operation Since the run does not preserve a constant value, all that can be done is to limit the error to a few seconds The influence of this residual error on the measurement of a fractional part of a circle division is then eliminated by applying a correction.

Correction for Run.—If for each observation both the back and forward micrometer readings are noted, the value of the run is known for the particular division used and under the prevailing temperature conditions The amount of correction corresponding to the position of the zero line between the graduations can then be obtained as follows

Let D = the nominal value of the circle division, *i e* its mean value for the whole circle, = 300″ in the usual case ,

b = the back reading, *i e* the micrometer measurement of the arc between the zero line and the graduation last passed by it, as obtained by setting the hairs on that graduation ,

f = the forward reading, *i e* the micrometer measurement of the same arc, as obtained by setting the hairs on the graduation in front of the zero line ,

$m = \frac{b+f}{2}$;

r = the run developed in D, = $(b-f)$,

D_1 = the micrometer measurement of D, = $(D+r)$,

M = the corrected micrometer reading to be applied to the approximate reading.

It is to be assumed that the effect of all errors causing run is to displace the back and forward graduations by equal amounts and in contrary directions For positive run the image of a division is too long by $\frac{r}{2}$ on either side of its centre line, and the corrected micrometer reading must be less than b and more than f. Negative run produces the opposite effect Distances on the actual image bear the same ratio to D_1 as their correct equivalents should bear to D, so that

$$\frac{M}{b} = \frac{D}{D_1},$$

$$\text{whence } M = \frac{bD}{D_1},$$

$$\text{or correction to } b = -\frac{br}{D_1}.$$

M is commonly obtained from m,

$$\text{and since } m = \frac{b+f}{2} = b - \frac{r}{2},$$

$$M = \left(m + \frac{r}{2}\right)\frac{D}{D_1},$$

$$= m + \frac{r}{2} - \frac{mr}{D} \text{ with ample precision,}$$

$$\text{or correction to } m = \frac{r}{2} - \frac{mr}{D}$$

The correction has the same sign as the run for values of m up to 2′ 30″ and the opposite sign for values between 2′ 30″ and 5′. Its value may be tabulated* for various values of r and m. In making the reductions, it is unnecessary to correct each micrometer reading. The back and forward readings of all micrometers are noted and the mean back reading and the mean forward reading obtained From the mean of these means and the mean run the correction is evaluated for each pointing.

Adjustments of the Micrometer.—These are :

1. Adjustment of the entire microscope to place the image of the graduations across the middle of the field of view
2. Adjustment of the zero line.
3 Adjustment for run

There may be no means of making the first adjustment, but if the bracket carrying the microscope is controlled by adjusting screws, the method will be apparent.

Adjustment of the Zero Line.—*Object.*—To place the zero lines of the several microscopes at the desired intervals, and to make the hairs lie centrally in the middle notch when in the zero position.

Necessity.—It is not essential that the intervals between the microscopes should be exactly equal, but it is convenient in recording that the discrepancies should be small The adjustment is usually only necessary when new hairs have been fixed. The comb scale may then require adjustment to make the reference notch coincide with the hairs when they are brought to the zero position.

Test.—1. For equidistance, set the hairs of one micrometer in the zero position, and bring a circle graduation centrally between them.

* See Davidson, "The Run of the Micrometer," United States Coast and Geodetic Survey Report, 1884, Appendix No. 8.

2. See if the corresponding graduations fall on the zeros of the other micrometers.

Adjustment.—1. If not, by means of the screw at the side of the box opposite the drum (Vol I, Fig. 58) move the comb scale until the centre of the index notch coincides with the image of the graduation.

2. Centre the hairs on the graduation

3. Slacken the screw holding the milled head at the drum, and turn the drum until it reads zero, without rotating the micrometer screw. Finally, tighten the head.

Note.—In some instruments there is provision for moving bodily all but one of the micrometers in a circumferential direction The greater part of an error of spacing may be thus removed, and the remainder is eliminated as above.

Adjustment for Run.—*Object.*—To eliminate run as far as possible.

Necessity.—As the effect of run can be corrected out in the reduction, it is sufficient for the most refined work that its value for a 5′ space should not exceed 2″ or 3″, so that no appreciable error can be introduced in the correction by assuming that the error is developed at a uniform rate It is, however, convenient to have the run adjusted down so that the mean of the back and forward readings, or, in low grade work, the back or forward reading alone, may be used without correction.

Test.—1. Focus the eyepiece for distinct vision of the hairs.

2. Move the hairs across a division, and note the amount and sign of the run.

Adjustment —1. If the run is positive (negative), the image is too large (small) Release the clamping ring of the objective cell, and move the objective towards (away from) the hairs

2. The image is now decreased (increased), but does not lie in the plane of the hairs Eliminate the resulting parallax by moving the whole microscope in its collar away from (towards) the circle

3. Again take the run, and repeat the adjustment as often as necessary.

Notes.—(1) Since the micrometer reading for each pointing is obtained as the mean of the readings of all micrometers, it is usual to adjust for the mean run. Instead of adjusting each micrometer, the run of one of them is made equal and opposite to the sum of the runs of the others

(2) The elimination of parallax must be regarded as an essential feature of the adjustment, as its presence makes accurate reading impossible.

(3) The hairs must be left parallel to the graduation lines.

Stability of Instrument.—The instrument is supported on a scaffold, a masonry pier, or upon its own tripod set on the ground. In the latter case the tripod legs are supported on firmly driven stakes, the tops of which are level and at such a height that the observer can use the telescope without stooping.

In primary and secondary triangulation some form of observatory tent must be erected round the instrument to protect it against air

currents and to shade it from the sun. On observing scaffolds the tent must be entirely supported by the outer or observer's tower. It should be provided with means for lowering the walls sufficiently for sighting, or have removable flaps on each wall at the level of the telescope. For tertiary work an umbrella gives sufficient protection.

An instrument mounted on an elevated scaffold is exposed to the vibration of the instrument tower in wind, and the observations are also subject to error arising from twist of the tower under lateral heating. These effects are sometimes guarded against by wrapping the outer scaffold with canvas to shelter the inner.

Conditions Favourable for Observation.—Irregular atmospheric refraction forms a difficult source of error, particularly in the case of rays which are not greatly elevated above the ground. No uncertainty need be occasioned by visible phenomena, since work must be suspended when irregular refraction causes apparent trembling of the signals or when lateral refraction manifests itself by expanding a luminous signal into a wide vibrating sheet of light impossible of accurate bisection. Lateral refraction, however, also exercises the more dangerous effect of causing a slow swing of the image of the signal on either side of the vertical hair, although the light may appear of normal size and free from vibration.

Observations should therefore be made only under favourable atmospheric conditions. In densely cloudy weather observing on mast signals can be carried on all day The best results with heliotropes are obtained from about 4 p m till sunset, but an hour or two at sunrise usually permit of satisfactory work Observation on night signals is generally confined to the period between sunset and midnight. To minimise error due to lateral refraction, a rule followed in some surveys is that observations at each primary station should be distributed over at least two days.

Relative Merits of Day and Night Observations.—Except for short lines on which opaque signals can be satisfactorily employed, the usual practice is to observe either on heliotropes or lamps as the weather allows. The chief advantage gained by the addition of night work consists in its doubling the number o hours a day available for good observation, but in some respects observing can be carried on better at night than by day. With heliotrope signals serious delays are occasioned by cloudy weather. The atmosphere is clearer and refraction is more constant at night, and observation can be conducted with less interruption and with rather greater accuracy. Further, refraction is much greater at night than by day, and, in consequence, the line of sight is farther removed above intervening elevated ground, so that towers and signals erected exclusively for night observation may be rather lower than would be necessary for day work.

General Methods of Observation.—In precise angle measurement the routine of observation must be specially arranged to reduce to a minimum the effect of instrumental and observational errors. Half of the observations for each angle are made with the telescope direct and half with it reversed, to eliminate errors of collimation and horizontal axis If the telescope does not transit, it must be removed from the standards and replaced end for end with the trunnion pivots placed in the same supports as before reversal. Half of the observations are taken from left to right and half from right to left to eliminate errors arising in manipulation and due to twist of the instrument and its support caused by lateral heating Accidental errors of signal bisection and of reading are reduced to any extent by increasing the number of observations. Errors of eccentricity are rendered negligible by reading all the micrometers at each observation, and the effect of graduation errors is sufficiently reduced by using a different part of the circle for each measurement.

Observational programmes belong either to the *Direction* or the *Repetition* system In the direction, or series method the several angles at a station are measured in terms of the directions of their sides from that of an initial station. The signals are bisected successively, and a value is obtained for each direction at each of several rounds of observations The initial or reference station should be that one of the triangulation stations which is most likely to be always visible When all the sights are so long that one of the stations cannot be preferred for the purpose, a referring signal, of a form suitable for accurate bisection, should be established not less than a mile and a half away.

The distinguishing feature of the repetition method consists in measuring each angle independently by multiplying it mechanically on the circle, the result being obtained by dividing the multiple angle by the number of repetitions Theoretically, any desired refinement of reading can be obtained by sufficiently increasing the number of repetitions, the effect being to reduce the least count correspondingly Practically, however, owing chiefly to errors introduced in clamping, there exists for any instrument a limit beyond which the accuracy is not improved by increase of the number of repetitions The system is designed for vernier instruments, and must be adopted when these are employed for fine angle work

Although the repetition method has been successfully applied to primary observations, it is generally confined to secondary and tertiary work. The method of directions should be employed in primary triangulation. It is designed for micrometer instruments, and in British geodesy these have always been used for refined work to the exclusion of vernier instruments. Owing to the modern application of micrometer reading to all sizes of theodolites, the direction method is equally suitable for all grades of triangulation.

Programme of Measurement by Directions.—To measure the angles AOB, BOC, and COD (Fig 60), A being adopted as the initial station, first set one of the micrometers to about 0°, point on A with telescope direct, and read all micrometers Swing on to B, C, and D successively, booking the micrometer readings after each pointing Overshoot D, *i e* move the line of sight a little beyond D to the right Again point on D, and read the micrometers Swing on to C, B, and A successively, taking the readings at each. Overshoot A, reverse the telescope, setting to about 180° the microscope originally at 0°, and repeat the same observations to D and back to A This constitutes one series, and yields four measures of each angle The microscope originally at 0° is now brought to a new reading, and a second series is observed in the same manner on a different part of the circle The required number of series depends upon the quality of the graduation and the number of equidistant micrometers as well as the grade of the work Six or eight series as above, giving twenty-four to thirty-two measures, each derived from the mean of the micrometer readings, are sufficient for geodetic triangulation with modern instruments

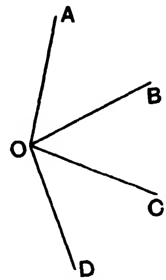


FIG 60

Instead of each series consisting of a swing right and a swing left on each face, as described, it is a common practice to use one face for the swing right and the other for the swing left. The number of pointings in a series is thus halved, so that to afford the same number of measures as before, twice as many zeros are required, and graduation error is likely to be reduced A second modification consists in closing the horizon by continuing each swing to finish on the intial station This ensures detection of any disturbance or twist of the instrument during the round, but it is generally considered that the alternation of right and left swings is sufficient precaution against error from this source.

Primary Direction Observations.—The average probable error of primary angles in modern triangulation is less than 0·4 sec., so that, to attain the necessary consistency between the several measures, the utmost delicacy is required in manipulating the instrument and, particularly, the micrometers. Clamping must be lightly performed to avoid stress : for the same reason the vertical circle is better left free. In making pointings, the line of sight should be brought on to all the signals in the direction of swing to avoid possible error due to friction. Care should therefore be exercised not to overshoot signals except in order to reverse the direction of swing at the end of a set. When a horizontal eyepiece micrometer is fitted, the signals are bisected by movement of the vertical hair and not by the tangent screw. Several bisections are made before and after reading the circle micrometers, the mean

reading of the eyepiece micrometer being applied to that of the circle. In making bisections, the observer should examine the image long enough to make sure that it is not swinging under the influence of lateral refraction.

For the elimination of periodic error of graduation, the change of zero between series must be such that the microscope readings for each signal are made on uniformly spaced points round the circle For a three-micrometer instrument the interval between these points should be about $\frac{60^\circ}{n}$, where n is the number of series. When one microscope is set at 0°, the others are at 120° and 240° respectively, and, after reversal, the readings are 180°, 300°, and 60°. By reversal of the telescope one zero therefore gives, for each signal, micrometer readings at 60° intervals round the circle. For a two-micrometer instrument the successive difference should be about $\frac{180^\circ}{n}$. The zero shift should not be an exact number of degrees in order better to distribute the microscope settings over the smallest division of the circle. The following are examples of the settings adopted in different surveys for primary triangulation.

Survey of India Three-micrometer theodolite: six series, each of three swings right and three swings left to each face.

Telescope Direct:

0° 0′	70° 1′	140° 2′	210° 3′	280° 4′	350° 5′

Telescope Reversed

180° 0′	250° 1′	320° 2′	30° 3′	100° 4′	170° 5′

Ordnance Survey, Test Triangulation in N.E. Scotland, 1910–12. Three-micrometer theodolite: eight series, partly of swing right and swing left on alternate faces, and partly of swing right and swing left to each face.

Telescope Direct:

0° 0′	45° 1′	90° 2′	135° 3′	200° 4′	245° 5′	290° 6′	335° 7′

Telescope Reversed.

180° 0′	225° 1′	270° 2′	315° 3′	20° 4′	65° 5′	110° 6′	155° 7′

United States Coast and Geodetic Survey Three-micrometer theodolite: sixteen series, each of swing right on one face and swing left on the other.

Telescope Direct:

0° 0′ 40″	195° 1′ 50″	30° 3′ 10″	225° 4′ 20″	64° 0′ 40″	.....	57° 4′ 20″

Telescope Reversed.

180° 0′ 40″	15° 1′ 50″	210° 3′ 10″	45° 4′ 20″	244° 0′ 40″	....	237° 4′ 20″,

the same intervals being repeated in groups of four, with a change of 18° 56′ 20″ between the groups

Note.—In the first example it will be seen that, neglecting minutes, readings for each signal are made by the different micrometers at 10° intervals round the circle. In the second the intervals are 5° and 10°, and in the third, 3° and 4°.

On completion of a series, the consistency between the measures may be found unsatisfactory, and additional swings are made if the

discrepancy between any two measures in the series exceeds about 3″. Discrepant measures are not to be omitted from the record, but the results of observations which are known to be not entirely satisfactory are noted as doubtful. A decision as to whether they will be retained or cancelled is made before adjusting the angles (page 171).

It sometimes happens in the course of a swing that one or more signals are invisible and must be omitted. When the opportunity occurs, such swings are completed, with the appropriate zeros, by sighting the omitted stations in conjunction with the initial station. If the latter is obscured, reference may be made to any other one station previously included.

Secondary and Tertiary Direction Observations.—Although the routine is much less elaborate for observations other than primary, measures should in all cases be equally divided between the two faces and directions of swing. While the programme to be adopted should depend largely upon the character of the instrument, it is generally sufficient in secondary triangulation to use two or three zeros with two faces to each and two swings on each face, or, alternatively, about five zeros each of one swing on each face. For tertiary work two zeros with swing right on one face and swing left on the other are all that are necessary. Directions taken merely to fix points by intersection require only one zero with a swing on each face.

Programme of Measurement by Repetition.—To measure angle AOB (Fig 60), set one of the verniers to about 0°, point on A with telescope direct, and book the readings of all the verniers Release the upper clamp, swing clockwise, and bisect B. Read a vernier to ascertain the approximate value of the angle. Slacken the lower clamp, turn *clockwise*, and again set on A Loosen the upper clamp, and bisect B Release the lower clamp, again turn clockwise, and point on A. Swing on to B as before. This constitutes three repetitions with telescope direct. Reverse the telescope, and, leaving the verniers unchanged, swing clockwise on to A. After making another three repetitions exactly as before, book the vernier readings. With the verniers unchanged, and the telescope still reversed, point on B, and, maintaining the clockwise direction of swing, make three repetitions on the exterior angle BOA, followed by three more with telescope direct Finally, note the readings of the verniers These six repetitions of the angle with six on its explement constitute one set, and additional sets to the number required are taken in the same manner from different initial readings

A programme for repetition observation must provide for the elimination of error introduced by repeated clamping. This may be accomplished either by making half the repetitions from right to left and half from left to right or by the above method of measuring the exterior angle in exactly the same manner as the interior. By

the latter routine the sign of the error is made the same for both angles, and, as its magnitude may be taken to be independent of their size, the mean of the measured value of the required angle and that given by subtracting the measured value of its explement from 360° is free from clamp error.

When the best possible results are required, all the angles at a station, including that required to close the horizon, are observed according to the above programme. Usually, however, the measurement of the explement of each angle is omitted, since, in closing the horizon, the explement of the sum of the required angles is measured. The amount by which the sum differs from 360° is equally divided among all the angles

Angles, beside being measured individually, are sometimes observed in various combinations, *e.g* AOC, AOD, BOD (Fig 60). While the additional data lead to a better determination of the required angles, the labour of station adjustment is considerably increased, and it is economical rather to amplify the programme for the measurement only of the angles to be used in calculating the triangulation system

In applying the repetition method to primary work, six sets of six repetitions each as described, have been used with 8-in. to 12-in theodolites For secondary triangulation with 7-in to 10-in instruments, two to four sets are sufficient in ordinary cases, and the same for tertiary work with 6-in. to 8-in instruments.

The Angle Book.—Readings must be registered in a permanent manner as soon as they are announced. The recorder should apply the necessary corrections and enter up the individual measures of the angles before the observations are completed, so that it may be decided whether further measures are required.

The tabular arrangement of the angle book may take various forms. That shown (Fig 61) is suitable for observations with a three-micrometer instrument without eyepiece readings, and is arranged for run correction The corrected directions or angles are transferred to an abstract in which the final average results are shown.

Miscellaneous Corrections.—Further corrections may have to be applied to give the final measured values which are to be subsequently adjusted. These include corrections for :

1. Horizontal Axis Dislevelment.
2. Bisections by Eyepiece Micrometer.
3. Phase of Signal.
4. Eccentricity of Instrument.
5. Eccentricity of Signal.
6. Reduction of Directions to Mean Sea Level.

1. Horizontal Axis Dislevelment.—Reversal of the telescope eliminates the effect of inclination of the horizontal axis only when the

Station . *Date* *Instrument* *Observer* *Recorder*

Series	Time	Station Observed	Face	° ′	Micr	″ Back	″ Forward	″ Mean	Run	Run Correction	Direction	Angle	Remarks
• 1	5 10 p m.	P	R.	00 00	A	15 2	16 4						Calm Cloudy
					B	18 3	16 6						
					C	17 8	17 0						
						17 10	16 67	16·88	+ 0 43	+ 0 19	00° 00′ 17″ 07		
		Q	R.	47 17	A	57 9	58 5						
					B	62 2	59 9						
					C	62 0	60 8						
						60 70	59 73	60 22	+ 0 97	− 0 10	47° 18′ 00″ 12	47° 17′ 43″ 05	

FIG. 61.

amount of dislevelment is the same for both sights. As a precaution against change between the observations, the striding level may be read at each and a correction applied in the case of sights of which the inclination exceeds, say, 2°.

Let e = dislevelment, in seconds, of the horizontal axis, as given by the readings of the striding level and the known value of its division (page 67).

a = angle of elevation, or depression, of the signal.

Then correction to observed direction = $e \tan a$ seconds.

The error causes an apparent displacement of signals towards the side of the higher pivot for angles of elevation and *vice versa* for angles of depression. The signs of the corrections to observed directions are therefore

	Elevation	Depression
Right pivot higher	−	+
Left pivot higher	+	−

2. Eyepiece Micrometer.—The mean of the micrometer readings for several bisections made by the movable hair is multiplied by the angular value of one division of the eyepiece micrometer, and the result is applied to the mean reading of the circle micrometers The telescope pointings are made sufficiently closely that the amount of correction need seldom exceed one or two seconds

The value of one division of the eyepiece micrometer must be determined from time to time It is most simply obtained by measuring a small angle both on the circle and with the micrometer Bisect a well-defined distant signal with the movable hair, and read the eyepiece micrometer and the circle micrometers. Move the hair from the signal by giving the micrometer screw, say, n turns Again bisect the signal by means of the upper tangent screw Read the circle micrometers, thus obtaining the angle through which the line of sight was turned, $1/n$ of which is the value of one turn of the eyepiece micrometer screw. A mean value is obtained by repeating the same process several times for each of several different parts of the screw.

3. Phase of Signal.—It has been explained (page 112) that the error of pointing caused by phase is a definite quantity for signals of cylindrical form. When the surface is sufficiently smooth to reflect sunlight in a bright line, as occurs with metal cylinders or wet surfaces, the pointing is made upon that line. With canvas-covered or whitewashed signals, the portion of the illuminated surface as seen from the instrument will be bisected. The correction is applied to convert the observed direction to that of the centre of the signal.

In Fig. 62, let A be the position of the observer, and C the centre of the signal, and let the direction of the sun make an angle a with AC. The visible part of the illuminated surface

extends from B to D. If the line of sight is directed along AE to bisect its projection, the value of the phase correction is

$$\mathrm{EAC} = c = \tfrac{1}{2}(b+d)$$

But, denoting the radius of the cylinder by r and the length of sight by D, since b and d are very small, we may put

$$b = \frac{r}{D} \text{ and } d = \frac{r \cos a}{D}, \text{ in circular measure,}$$

so that $c = \dfrac{r(1+\cos a)}{2D} = \dfrac{r \cos^2 \frac{1}{2}a}{D}$, or

correction, in seconds, to bisection of illuminated portion $= \dfrac{r \cos^2 \frac{1}{2}a}{D \sin 1''}$.

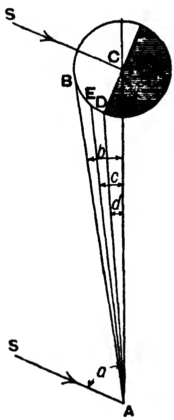


FIG. 62.

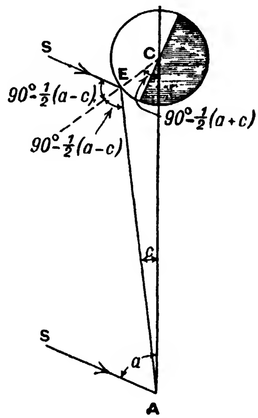


FIG. 63.

If the observation is made on the bright line formed by the reflected rays, then, in Fig 63, let SEA represent their path. The values of the marked angles are readily deduced, and, with the approximations allowable since c is small, it follows that

correction, in seconds, to pointing on bright line $= \dfrac{r \cos \frac{1}{2}a}{D \sin 1''}$.

The correction must be applied positively or negatively, according to the relative position of the sun and the signals

4. Eccentricity of Instrument.—When existing features, such as steeples, are adopted as triangulation points on account of their visibility, it frequently happens that the instrument cannot be centered over them. In such a case, a subsidiary instrument station, called a satellite station, is selected near the true station, and the values of the angles measured there are reduced to centre, *i.e.* corrected to the values they would have if measured at the triangulation point.

In Fig. 64, C represents the true station to which observations have been made from stations A and B, and S is the satellite station, at which angle s is measured with the same precision as if S were a triangulation point. The further measurements required for the reduction are the distance SC and the angle ASC $= d$. The unadjusted values of angles ABC and BAC are already known, and by solving triangle ABC the distances AC and BC are obtained with sufficient precision for the present purpose

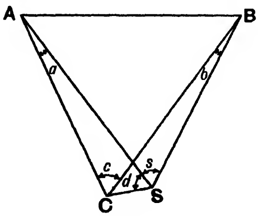


FIG 64

Now, in triangle ACS, $\sin a = \frac{\text{CS} \sin d}{\text{AC}}$, or, since a is usually very small, we may write

$$a, \text{ in seconds}, = \frac{\text{CS} \sin d}{\text{AC} \sin 1''}.$$

Similarly, from triangle BCS,

$$b, \text{ in seconds}, = \frac{\text{CS} \sin (d+s)}{\text{BC} \sin 1''}.$$

But a represents the difference in direction between CA and SA, and b that between CB and SB, so that, by applying a and b with the appropriate signs, the included angle c is readily deduced from s.

To ascertain the signs of the corrections, it is convenient to regard SC as an arbitrary meridian for the observations made at S. Bearing SA, situated in the first quadrant, would require the addition of the correction a to yield the bearing CA Similarly, SB in the second quadrant is converted to CB by addition of b, but lines in the third and fourth quadrants would have the corrections applied negatively. In the case illustrated, $(s-c) = (a-b)$, but when A and B are on opposite sides of CS, $(s-c) = (a+b)$

5. Eccentricity of Signal.—Correction is required when observations are made upon a signal which is found to be out of centre or is for special reasons placed so The value of the correction is obtained as in the previous case, the distance and direction between the signal and the centre being measured If, in Fig 64, the signal for station C is situated at S, the observed angles BAS and ABS are to be corrected by a and b respectively.

6. Reduction of Directions to Mean Sea Level.—This correction is made only in the most refined primary triangulation in districts at a considerable elevation above sea level and particularly in low latitudes. Owing to the spheroidal form of the earth the vertical through an observation station A is not coplanar with that through a signal B, except when A and B are in the same meridian or of the same latitude. When, therefore, A and B are projected normally to A_1 and B_1 upon the mean sea level surface, the plane AA_1B,

containing the observed direction, does not coincide with the plane AA_1B_1, containing its projection, and the direction AB must be corrected to yield that of A_1B_1.

The value of the correction, applicable to the observed direction, as given by Clarke in *Geodesy* is, in seconds,

$$c = \frac{e^2h}{2a} \sin 2A \cos^2\phi \operatorname{cosec} 1'',$$

where e = eccentricity of the earth (page 181),
a = major semi-axis of the earth,
h = elevation above mean sea level of the signal station B,
A = azimuth of AB, reckoned from north by east,
ϕ = mean latitude of AB

The sign of the correction is given by that of $\sin 2A$, *viz* positive for values of A in the first and third quadrants and negative for the second and fourth Its maximum value occurs in azimuth 45° at the equator, when it amounts to 0″0335 per 1000 ft elevation of the observed station

EXAMPLES

1 The elevation of an instrument at A is 210 3 ft Find the minimum height of signal required at B, 28 3 miles distant, the elevation of the ground at which is 296 0 ft The intervening ground may be assumed to have a uniform elevation of 150 ft, and the line of sight must nowhere be less than 6 ft above the surface Take $h = 5738D^2$.

2 The elevation of an instrument at A is 142 ft. Find the minimum elevation required for a signal at B, 54 miles distant, if the line crosses an arm of the sea The coefficient of refraction is to be taken as 0 08 and the mean radius of the earth as 3,960 miles

3 The altitudes of two proposed stations A and B, 80 miles apart, are respectively 2,066 ft and 3,487 ft The altitudes of two points C and D on the profile between them are respectively 1,803 ft and 2,216 ft, the distances being AC = 30 miles and AD = 55 miles Determine whether A and B are intervisible for $k = 0$ 07 and R 3,960 miles

4 Describe, in detail, the field operations necessary for measuring a long base-line with extreme accuracy by means of a steel tape or wire Enumerate the corrections that must be made

A line, 2 miles long, is measured with a tape of length 300 ft, which is standard under no pull at 60° F The tape in section is $\frac{1}{8}$ inch wide and $\frac{1}{20}$ inch thick. If one half of the line is measured at a temperature of 70° F., and the other half at 80° F, and the tape is stretched with a pull of 50 lbs, find the correction on the total length Coefficient of expansion = 0 0000065; weight of 1 cubic inch of steel = 0 28 lbs, E = 29,000,000 lbs per square inch. (Univ. of Lond, 1908)

5. A base line is simultaneously measured with a steel wire and a brass wire. The length given by the steel component is 31,342 622 ft., and that by the brass component is 31,339 144 ft, both referred to the absolute lengths of the wires at 32° F The coefficients of expansion of the steel and the brass components are respectively 0 0000063 and 0000100 per 1° F

Find the length of the base corrected for temperature

6. The horizontal distance between two points on a roadway which is on a slope is required very accurately.

The error of the standard tape at a known temperature and the coefficient of expansion of the standard tape and of the tape used to measure the line and the weight of the measuring tape are known.

Having measured the line at a known temperature, explain carefully how you obtain the correct horizontal distance between the ends of the line.

A line is measured on a uniform slope with a 100 ft. steel tape pulled with a force of 10 lb., and is found to be 1,725 ft long The temperature at the time of measurement is 34° F. The tape is correct at 62° F., when the pull on the tape is 10 lb. The difference of level of the two ends of the line is 30 ft. Determine the horizontal distance between the two ends of the line. Coefficient of expansion of the tape is 0 000006 per degree F. (Univ of Lond., 1915.)

7. The corrected measured length of the Semliki base in Uganda is 16,534·05438 m Its mean height above sea level is 645 4 m , and the radius of curvature of the earth computed for the latitude and azimuth of the base is 6,358,982 m.

Calculate the length of the base reduced to sea level.

8. Compute the value of the correction due to change of gravity in the case of the Lossiemouth base line, the length of which is 23,526 ft The 100-ft. tapes were standardised at Southampton, in latitude 50° 54′, at an elevation of 76 ft , the tension being applied by 20 lb weights The same weights were used in the field measurement in latitude 57° 42′ and at an elevation of 23 ft. The tapes stretch 00208 ft. per lb tension per tape length. Take the radius of the earth as 21×10^6 ft

9. The circle of a theodolite is graduated to 5′ spaces, and 5 turns of the micrometer screw are required to carry the hairs from one graduation to the next. The forward and back readings to be applied to the approximate reading of 97° 5′ are 3′ 39″ 4 and 3′ 41″ 2 respectively. What is the correct reading ?

10. The horizontal axis of a theodolite has an inclination of 5″·4 For a single observation in which the left hand pivot is the higher, compute the correction for dislevelment applicable to an angle the left hand station of which has an angle of elevation of 9° 35′ and the right hand station a depression of 4° 21′.

11. Compute the value of the correction to angle AOB for phase of cylindrical signals. The observed directions are OA = 39° 17′ 34″ 2, OB = 86° 52′ 07″ 4, and sun = 43° 25′ The diameter of the signal at A subtends 5″ at the instrument, and that at B subtends 3″, and the pointings are made upon the bright line.

12 Directions are observed from a satellite station, 204 ft. from station C, with the following results: A, zero , B, 71° 54′ 32″ 25 ; C, 296° 12′. The approximate lengths of AC and BC are respectively 54,072 ft and 71,283 ft. Compute the angle subtended at station C

CHAPTER IV

SURVEY ADJUSTMENT AND GEODETIC COMPUTATIONS

THIS chapter deals with the computing work which follows the derivation of the final field results of extended triangulation. The method of obtaining the final data from the field book figures by the application of corrections has already been given in the preceding chapter. Those results are subjected to a process of correction to eliminate inconsistencies, and finally the triangle sides and the geodetic co-ordinates of the stations are calculated.

THEORY OF ERRORS

Even with the most refined methods of angular or linear measurement, errors, however small, are unavoidable. These are evidenced by discrepancies between the results of repeated measurements of the same quantity and by non-fulfilment of geometrical relationships which should obtain between different quantities. Such inconsistencies must be eliminated by subjecting the field results to a process of adjustment designed to yield the most probable value of each measured quantity based upon the observed values and the conditions to be met.

Classification of Errors.—From whatever cause arising, errors can be classed as : (*a*) Mistakes ; (*b*) Systematic errors ; (*c*) Accidental errors.

Mistakes arise from carelessness or inattention on the part of the observer. While their occurrence is always possible, they should never influence the final result, since the system of checking required in every surveying operation must expose them.

Systematic or constant errors are those the effects of which are understood and can be eliminated, some by the adoption of a particular routine in measurement, and others by the application of computed corrections. Such errors do not contribute towards discrepancies between the results of repeated measurements of the same quantity. The methods of eliminating errors of this class have been sufficiently emphasised throughout the text.

Accidental errors include all unavoidable and unknown errors which are entirely beyond the control of the observer. Every

observation is subject to numerous accidental errors, and, as these happen by mere chance, their algebraic sum, representing the accidental error of the observation, follows no definite laws except the laws of probability.

Examples of Different Classes of Error —In the case of base measurement, mistakes might conceivably occur in reading the tape, counting tape lengths, or booking The effects of temperature, pull, sag, slope, etc , which are corrected out, constitute systematic errors. Accidental errors arise from, among other causes, lack of perfection of eyesight influencing the operations of standardising and using the tape, imperfections in thermometers and spring balances, and the effects of small and momentary changes of temperature and tension

In angle measurement, blunders, however unlikely, might occur by sighting the wrong signal, by repeating an erroneous micrometer or vernier reading, or by using the wrong tangent screw. Systematic errors are due to imperfect adjustment of the instrument and defects in the non-adjustable parts Accidental errors are due to such causes as limited refinement of reading, imperfect estimation of readings and signal bisections, the effects of irregular atmospheric refraction, imperfection of the observer's sight and touch in the levelling and manipulating of the instrument, etc

Definitions.—A *Direct Measurement* is one made directly on the quantity being determined, *e g* the measurement of a base, the single measurement of an angle

An *Indirect Measurement* is one in which the observed value is deduced from the measurement of some related quantity or quantities, *e g.* the sides of a triangulation, the measurement of an angle by repetition (a multiple of the angle being measured).

A *Conditioned Quantity*, as opposed to an independent quantity, is one whose value must bear a rigid relationship to some other quantity or quantities

The *Most Probable Value* of a quantity is that which is more likely than any other to be its true value, and is the best result which can be attained.

A *True Error* is the difference between the true and observed values of a quantity Since the true value of a quantity cannot be ascertained, the true error is never known. In the case, however, of the summation of the angles of triangles or polygons or of angles to close the horizon, the true error of their sum is known.

A *Residual Error* or *Residual* is the difference between an observed value and the most probable value of the quantity.

The *Weight* of an observation is a measure of its relative trustworthiness. In the course of a series of observations, the personal, instrumental, or atmospheric conditions may vary so that a uniform degree of reliability in the results would not be expected. The results are then said to be of unequal weight. Numerical values assigned as the weights of a series of observations are simply ratios indicating the relative precision of the observations. Weights are assigned by estimation, or in terms of the number of separate observations of equal reliability from which the result being weighted is derived, or by calculation from the probable error.

The Laws of Accidental Error.—After all known errors have been corrected out, the results of observations contain accidental errors only, and these are treated in accordance with the laws of probability. It must be understood that these laws are quite inapplicable to systematic errors, which are not considered further.

Experience shows that for a prolonged series of observations of the same quantity or quantities, under apparently constant conditions, accidental errors obey the following laws .

(1) Small errors are more frequent than large ones.
(2) Positive and negative errors are equally frequent.
(3) Very large errors do not occur.

The graphical representation of these laws is shown in Fig. 65, in which abscissæ represent magnitudes of errors and ordinates their respective frequencies or probabilities of occurrence The third law as stated does not lend itself to graphical interpretation with the others, but if it is expressed in the form that very large errors occur with very small frequency, it is represented by making the x axis an asymptote to the curve.

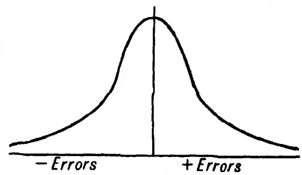


Fig. 65.

Equation of the Probability Curve.—It may be taken that the most probable value of a quantity subjected to a series of direct and equally reliable observations is the arithmetic mean of the several measurements, since there is no reason why one determination should influence the result more than another. Accepting this principle, the general equation of the probability curve is found to be

$$y = ke^{-c^2x^2},$$

where y is the probability of occurrence of an error x, k and c are constants depending upon the precision of the observations, and e is the base of Napierian logarithms.

General Principle of Least Squares.—When a quantity is being deduced from a series of observations, then to every value which may be adopted for the result there will correspond a series of errors or differences between adopted the value and the several observed results. The most probable series of errors is necessarily that derived from the adoption of the most probable value of the quantity. It is found from the probability equation that the most probable values of a series of errors arising from observations of equal weight are those the sum of the squares of which is a minimum. In this manner is derived the fundamental law of least squares, which states that, for observations of equal weight, the most probable value of an observed quantity is that which makes the sum of the squares of the residual errors a minimum. It may likewise be deduced that, for observations of unequal weight, the

most probable value of the observed quantity is that which makes the sum of the weighted squares of the residuals a minimum.

Most Probable Values of Directly Observed Independent Quantities. —It has been stated that the most probable value of a quantity subjected to a series of direct observations of equal weight is given by the arithmetic mean of the observed results This is obviously in accordance with the law of least squares. Similarly, when the observations are of unequal weight the most probable value is the weighted arithmetic mean, obtained by multiplying each result by its weight and dividing the sum of the products by the sum of the weights.

Example —Find the most probable value of an angle of which measurements under different conditions are 56° 12′ 3″ 2, weight 2, and 56° 12′ 8″ 0, weight 1.

	°	′	″		°	′	″
	56	12	3 2	× 2 =	112	24	6 4
	56	12	8 0	× 1 =	56	12	8 0
				3)	168	36	14 4
Most probable value =					56	12	4 8.

Most Probable Values of Indirectly Observed Independent Quantities. —In the general case of indirectly observed quantities the most probable values are determined by solution of normal equations compiled from observation equations expressing the observed results. These normal equations are formed for each quantity of which the most probable value is required, and the latter are obtained by simultaneous solution

To form the normal equation for any quantity, each observation equation is multiplied by the algebraic coefficient of the quantity in that observation equation, and the products are added When the observations are of unequal weight, the normal equation for any quantity is formed by multiplying each observation equation by the weight of that observation as well as by the algebraic coefficient of the quantity in that observation equation, and the products are added.

Example.—Find the most probable values of the angles a, b, and c at one station from the following observations :—

	°	′	″	
a =	40	13	28 7,	weight 1,
b =	34	46	15 4,	1,
$a+b$ =	74	59	43 0,	2,
$a+b+c$ =	132	31	07 2,	1,
$b+c$ =	92	17	42 2,	3.

Multiplying these observation equations by their weights, we have

	°	′	″
a =	40	13	28 7,
b =	34	46	15 4,
$2a+2b$ =	149	59	26·0,
$a+b+c$ =	132	31	07·2,
$3b+3c$ =	276	53	06·6.

To form the normal equation in a, multiply these equations by the algebraic coefficient of a in each, giving

$$\begin{array}{rcrrl} & & {}^{\circ} & {}' & {}'' \\ a & = & 40 & 13 & 28{\cdot}7, \\ 4a+4b & = & 299 & 58 & 52\ 0, \\ a+b+c & = & 132 & 31 & 07\ 2, \\ \hline 6a+5b+c & = & 472 & 43 & 27\ 9 \end{array} = \text{normal equation in } a.$$

By the same process,

$$5a+15b+10c = 1297\quad 55\quad 34\ 4 = \text{normal equation in } b,$$

and

$$a+10b+10c = 963\quad 10\quad 27\ 0 = \text{normal equation in } c.$$

On solving these equations, the most probable values are found to be

$$\begin{array}{rrrl} & {}^{\circ} & {}' & {}'' \\ a = & 40 & 13 & 27{\cdot}14, \\ b = & 34 & 46 & 15{\cdot}77, \\ c = & 57 & 31 & 26\ 22. \end{array}$$

Note.—As a check on the normal equations, it should be observed that the coefficients in the first equation are reproduced in the first row, those of the second equation in the second row, and so on

The labour of solution is diminished by assuming a set of values for the most probable values of the unknown quantities and determining by normal equations the most probable series of errors made in the assumptions The errors so found must then be added algebraically to the respective assumed values to yield the most probable values of the measured quantities.

Example.—For the previous case let it be assumed that the most probable values of a, b, and c are respectively $40^\circ 31' 28''\ 7$, $34^\circ 46' 15''{\cdot}4$, and $57^\circ 31' 26''\ 8$, taken from observation equations 1, 2, and 5, and let r_1, r_2, and r_3 represent the unknown residual errors made in the assumption.

The observation equations are to be reduced in terms of r_1, r_2, and r_3 to express discrepancies between the various observed results and those given by the assumed values, the latter being subtracted from the former The reduced observation equations then become

$$\begin{array}{rcrl} & & {}'' & \\ r_1 & = & 0\ , & \text{weight } 1, \\ r_2 & = & 0\ , & 1, \\ r_1+r_2 & = & -1\ 1, & 2, \\ r_1+r_2+r_3 & = & -3\ 7, & 1, \\ r_2+r_3 & = & 0\ , & 3. \end{array}$$

Multiplying by the weights, we have

$$\begin{array}{rcl} & & {}'' \\ r_1 & = & 0\ , \\ r_2 & = & 0\ , \\ 2r_1+2r_2 & = & -2{\cdot}2, \\ r_1+r_2+r_3 & = & -3{\cdot}7, \\ 3r_2+3r_3 & = & 0\ . \end{array}$$

Proceeding as before, the normal equations are found to be

$$\begin{array}{rcl} & & {}'' \\ 6r_1+5r_2+r_3 & = & -8{\cdot}1, \\ 5r_1+15r_2+10r_3 & = & -8{\cdot}1, \\ r_1+10r_2+10r_3 & = & -3\ 7, \end{array}$$

solution of which gives $r_1 = -1''\ 56$, $r_2 = +0''\ 37$, $r_3 = -0''\ 58$. By applying these to the assumed values, we have, as before, the most probable values—$a = 40^\circ 13' 27''{\cdot}14$, $b = 34^\circ 46' 15''{\cdot}77$, and $c = 57^\circ 31' 26''\ 22$.

Most Probable Values of Conditioned Quantities.—When the quantities of a set must fulfil rigorous geometrical relationships, the most probable value of any one quantity is influenced by the results of observation of the others. The problem is to ascribe a most probable set of values to the unknown quantities and not those which, if no conditions had to be met, would appear from the observation equations to be the most probable for each.

With conditioned quantities, therefore, there are, in addition to the observation equations, one or more equations of condition, which are always fewer in number than the unknowns, and which exhibit the conditions to be fulfilled. Thus, if a, b, c, and d are angles closing the horizon at a station, the equation of condition to be satisfied is $(a+b+c+d) = 360°$. If this is written as $d = 360°-(a+b+c)$, it is only necessary to find by the previous methods the most probable values of the independents a, b, and c, and then evaluate the dependent d. Generally, if there are n unknowns connected by m independent conditional equations, the number of independent quantities is $(n-m)$, the remaining unknowns being expressible in terms of those independent quantities and obtainable from them.

This method of dealing with conditioned quantities is suitable for the simple cases which may be encountered by the civil engineer, and is illustrated by the following examples.

Example 1.—The observations closing the horizon at a station are

	°	′	″	
$a =$	72	30	42·2,	weight 1,
$b =$	61	07	20·4,	2,
$c =$	75	45	12·8,	3,
$d =$	42	23	38·0,	2,
$e =$	108	13	08·9,	1,
$a+b =$	133	38	04·7,	1,
$c+d =$	118	08	48·9,	3,
$e+a =$	180	43	53·0,	1.

Conditional equation: $a+b+c+d+e = 360°$.

Find the most probable values.

The number of independent quantities is 4. Let e be the dependent quantity.

Rewriting the observation equations with e eliminated and multiplying by the weights, we have

	°	′	″
$a =$	72	30	42·2,
$2b =$	122	14	40·8,
$3c =$	227	15	38·4,
$2d =$	84	47	16·0,
$a+b+c+d =$	251	46	51·1,
$a+b =$	133	38	04·7,
$3c+3d =$	354	26	26·7,
$b+c+d =$	179	16	07·0,

giving the normal equations,

	°	′	″
$3a+2b+c+d =$	457	55	38·0,
$2a+7b+2c+2d =$	809	10	24·4,
$a+2b+20c+11d =$	2176	09	13·4,
$a+2b+11c+15d =$	1663	56	50·2,

solution of which yields the most probable values :

$$\begin{array}{rrrr} & ^{\circ} & ' & '' \\ a = & 72 & 30 & 42{\cdot}94, \\ b = & 61 & 07 & 20{\cdot}05, \\ c = & 75 & 45 & 12{\cdot}27, \\ d = & 42 & 23 & 36{\cdot}81, \\ e = 360^{\circ}-(a+b+c+d) = & 108 & 13 & 07{\cdot}93 \end{array}$$

The same result should be obtained by the student by reduced observation equations.

Example 2.—The angle observations of two triangles ABC and BCD, having a common side, gave the following results of uniform weight,

$$\begin{array}{llrrr} & & ^{\circ} & ' & '' \\ \mathrm{BAC} = a & = & 48 & 01 & 17{\cdot}0, \\ \mathrm{ABC} = b_1 & = & 56 & 34 & 02{\cdot}4, \\ \mathrm{CBD} = b_2 & = & 67 & 12 & 00{\cdot}8, \\ \mathrm{ABD} = b & = & 123 & 46 & 05{\cdot}6, \\ \mathrm{ACB} = c_1 & = & 75 & 24 & 42{\cdot}1, \\ \mathrm{BCD} = c_2 & = & 42 & 50 & 07{\cdot}0, \\ \mathrm{ACD} = c & = & 118 & 14 & 46{\cdot}2, \\ \mathrm{BDC} = d & = & 69 & 57 & 50{\cdot}1 \end{array}$$

The conditional equations are :

$$\begin{aligned} b_1+b_2 &= b, \\ c_1+c_2 &= c, \\ a+b_1+c_1 &= 180^{\circ}, \\ d+b_2+c_2 &= 180^{\circ}. \end{aligned}$$

The number of independent quantities is 4. Let these be b_1, b_2, c_1, and c_2.

Using reduced observation equations with assumed values for the independents as given in the 2nd, 3rd, 5th, and 6th observation equations, and letting r_1, r_2, r_3, and r_4 represent the residuals for b_1, b_2, c_1, and c_2 respectively, we have, on eliminating the dependents, the reduced observation equations,

$$\begin{aligned} r_1+r_3 &= -1{\cdot}5'', \\ r_1 &= 0\ , \\ r_2 &= 0\ , \\ r_1+r_2 &= +2{\cdot}4, \\ r_3 &= 0\ , \\ r_4 &= 0\ , \\ r_3+r_4 &= -2{\cdot}9, \\ r_2+r_4 &= +2{\cdot}1, \end{aligned}$$

yielding the normal equations,

$$\begin{aligned} 3r_1+r_2+r_3 &= +0{\cdot}9'', \\ r_1+3r_2+r_4 &= +4{\cdot}5, \\ r_1+3r_2+r_4 &= -4{\cdot}4, \\ r_2+r_3+3r_4 &= -0{\cdot}8, \end{aligned}$$

solution of which gives $r_1 = +0''{\cdot}29$, $r_2 = +1''{\cdot}49$, $r_3 = -1''{\cdot}47$, $r_4 = -0''{\cdot}27$.

On application of these, the most probable values are :

$$\begin{array}{lrrr} & ^{\circ} & ' & '' \\ a_1 = & 48 & 01 & 16{\cdot}68, \\ b_1 = & 56 & 34 & 02{\cdot}69, \\ b_2 = & 67 & 12 & 02{\cdot}29, \\ c_1 = & 75 & 24 & 40{\cdot}63, \\ c_2 = & 42 & 50 & 06{\cdot}73, \\ d = & 69 & 57 & 50{\cdot}98. \end{array}$$

Method of Correlates.—When there are several conditions to be fulfilled, the most probable values are usually obtained by the use of undetermined multipliers, called correlates or correlatives.

Taking the common case in which the observations are direct and equal in number to the unknowns, let there be n observed quantities, of which the most probable values are x_1, x_2, etc., and the observed values are o_1, o_2, etc.

The n observation equations are :

$$x_1 = o_1 + r_1, \text{ weight } w_1,$$
$$x_2 = o_2 + r_2, \text{ weight } w_2, \text{ etc.} \quad \ldots\ldots\ldots (1)$$

Let the m conditional equations be

$$a_1 x_1 + a_2 x_2 + \ldots a_n x_n = q_1,$$
$$b_1 x_1 + b_2 x_2 + \ldots b_n x_n = q_2, \text{ etc} \quad \ldots\ldots\ldots (2)$$

Substituting (1) in (2) we have

$$a_1 r_1 + a_2 r_2 + \ldots a_n r_n = q_1 - \Sigma ao = u_1,$$
$$b_1 r_1 + b_2 r_2 + \ldots b_n r_n = q_2 - \Sigma bo = u_2, \text{ etc} \quad \ldots\ldots\ldots (3)$$

For the most probable values of the residuals,

$$w_1 r_1^2 + w_2 r_2^2 + \ldots w_n r_n^2 = \text{a minimum},$$
$$\text{or } w_1 r_1 dr_1 + w_2 r_2 dr_2 + \ldots w_n r_n dr_n = 0 \quad \ldots\ldots\ldots (4)$$

But, differentiating (3),

$$a_1 dr_1 + a_2 dr_2 + \ldots a_n dr_n = 0,$$
$$b_1 dr_1 + b_2 dr_2 + \ldots b_n dr_n = 0, \text{ etc} \quad \ldots\ldots\ldots (5)$$

Forming the m equations (5) into a single equivalent equation, we have

$$k_1 (a_1 dr_1 + a_2 dr_2 + \ldots a_n dr_n) + k_2 (b_1 dr_1 + b_2 dr_2 + \ldots b_n dr_n) + \ldots = 0, \quad \ldots\ldots\ldots (6)$$

where k_1, k_2, . . . k_m are independent constants or correlates.

Equating (6) and (4), since the values of dr_1, dr_2, etc. are simultaneous, we obtain, on rearranging the terms,

$$(k_1 a_1 + k_2 b_1 + \ldots k_m m_1 - w_1 r_1)\, dr_1 + (k_1 a_2 + k_2 b_2 + \ldots k_m m_2 - w_2 r_2)\, dr_2 + \ldots = 0, \quad \ldots\ldots\ldots (7)$$

in which the coefficients of the differentials must each be zero so that

$$k_1 a_1 + k_2 b_1 + \ldots k_m m_1 = w_1 r_1,$$
$$k_1 a_2 + k_2 b_2 + \ldots k_m m_2 = w_2 r_2, \text{ etc} \quad \ldots\ldots\ldots (8)$$

Substituting these values of r_1, r_2, etc. in (3) we obtain

$$k_1 \Sigma \frac{a^2}{w} + k_2 \Sigma \frac{ab}{w} + \ldots k_m \Sigma \frac{am}{w} = u_1,$$
$$k_1 \Sigma \frac{ba}{w} + k_2 \Sigma \frac{b^2}{w} + \ldots k_m \Sigma \frac{bm}{w} = u_2, \text{ etc.}, \quad \ldots\ldots\ldots (9)$$

in which

$$\Sigma\frac{a^2}{w}=\frac{a_1{}^2}{w_1}+\frac{a_2{}^2}{w_2}+\ldots\frac{a_n{}^2}{w_n},$$

$$\Sigma\frac{ab}{w}=\frac{a_1b_1}{w_1}+\frac{a_2b_2}{w_2}+\ldots\frac{a_nb_n}{w_n}, \text{ etc.}$$

Equations (9) are solved simultaneously for $k_1, k_2, \ldots k_m$, and $r_1, r_2, \ldots r_n$ are then obtained from (8), whence $x_1, x_2, \ldots x_n$ from (1)

Example.—Find the most probable values of the angles of Ex. 2, page 165, by the method of correlates.

Let $r_1, r_2, \ldots r_8$ be the residuals for the angles $a, b_1, \ldots d$ in the order given.

The conditional equations (3) are

$$\begin{aligned} r_4-r_2-r_3 &= -2''.4,\\ r_7-r_5-r_6 &= +2.9,\\ r_1+r_2+r_5 &= -1.5,\\ r_8+r_3+r_6 &= +2.1 \end{aligned}$$

The coefficients in the correlate equations (9) are obtained by tabulating

	a²	ab	ac	ad	b²	bc	bd	c²	cd	d²
r_4	+1									
r_2	+1		−1					+1		
r_3	+1			−1						+1
r_7					+1					
r_5					+1	−1		+1		
r_6					+1		−1			+1
r_1								+1		
r_8										+1
Σ	+3	0	−1	−1	+3	−1	−1	+3	0	+3

The correlate equations are therefore

$$\begin{aligned} 3k_1+0-k_3-k_4 &= -2''.4,\\ 0+3k_2-k_3-k_4 &= +2.9,\\ -k_1-k_2+3k_3+0 &= -1.5,\\ -k_1-k_2+0+3k_4 &= +2.1, \end{aligned}$$

solution of which gives

$$\begin{aligned} k_1 &= -0''.613,\\ k_2 &= +1.153,\\ k_3 &= -0.320,\\ k_4 &= +0.880 \end{aligned}$$

In obtaining $r_1, r_2, \ldots r_8$, the coefficients in equations (8) are

	a	b	c	d
r_4	+1			
r_2	−1		+1	
r_3	−1			+1
r_7		+1		
r_5		−1	+1	
r_6		−1		+1
r_1			+1	
r_8				+1

$$
\begin{aligned}
\therefore\ r_1 &= +k_3 &&= -0''.32,\\
r_2 &= -k_1+k_3 &&= +0\cdot29,\\
r_3 &= -k_1+k_4 &&= +1\ 49,\\
r_4 &= +k_1 &&= -0\ 61,\\
r_5 &= -k_2+k_3 &&= -1\ 47,\\
r_6 &= -k_2+k_4 &&= -0\ 27,\\
r_7 &= +k_2 &&= +1\cdot15,\\
r_8 &= +k_4 &&= +0\ 88.
\end{aligned}
$$

These values will be found to fulfil the conditional equations (3), and, on applying them to the observed values, we have the most probable values,

$a = 48°\ 01'\ 17''\ 0 - 0''\ 32 = 48°\ 01'\ 16''\ 68$, etc , as before

Precision of the Most Probable Value.—When the most probable value of a quantity has been obtained, it is desirable to have some index of the precision of the result Thereby the relative accuracy of different series of observations may be ascertained, and, for purposes of adjustment, weights based on the value of the index may be ascribed to each. The number of individual measures of the quantity and their consistency afford data for estimating the probable precision of the result. Three criteria are employed, *viz.* probable error, mean square error, and mean error.

The *Probable Error* (p e) is of such magnitude that the probability of the true error being larger is equal to the probability of the true error being smaller than the probable error. In other words, in a large series of observations the probability is that there are as many errors numerically greater than the probable error as there are smaller.

The *Mean Square Error* (m s e) equals the square root of the arithmetic mean of the squares of the individual true errors.

The *Mean Error* (m e.) is the arithmetic mean of the individual true errors without regard to sign.

As the determination of the values of these errors is based on the theory of probabilities, it must again be emphasised that the employment of such standards presupposes the entire elimination of all but accidental errors.

When the number of observations is large, these errors have the relationship

$$\text{p.e.} = 0\cdot6745 \text{ m.s e.} = \cdot8453 \text{ m e.},$$

so that it is comparatively unimportant which is adopted.

Probable Error.—Probable error is the criterion employed in British and American practice, and the formulæ for its evaluation in the simplest cases are quoted below As the use of the word "probable" is confusing, the reader is cautioned at the outset against attaching any significance to the term other than that contained in the definition. The probable error is not the most probable error, as this is shown by the probability curve to be always zero.

In giving the result of a measurement from which constant errors are eliminated, the value of the probable error is written with a

positive and negative sign after the most probable value. Thus, the standardised length of a tape obtained from repeated comparisons might be stated as 100·00042±0 00005 ft., indicating that the true length is as likely to lie within the limits 100·00037 and 100·00047 ft. as to have any value outside them.

Formulæ for Probable Error.

Let n = the number of measures contributing to the most probable value,

Σr^2 = the sum of the squares of the residual errors,

w = the weight of a measure,

Σwr^2 = the sum of the weighted squares of the residuals.

For Direct Observations of Equal Weight on a Single Quantity.

$$\text{p e. of a single measure} = 0{\cdot}6745\sqrt{\frac{\Sigma r^2}{(n-1)}}.$$

$$\text{p.e. of the arithmetic mean} = 0\ 6745\sqrt{\frac{\Sigma r^2}{n(n-1)}}.$$

For Direct Observations of Unequal Weight on a Single Quantity.

$$\text{p.e. of a single measure of unit weight} = 0{\cdot}6745\sqrt{\frac{\Sigma wr^2}{(n-1)}}.$$

$$\text{p e. of the weighted mean} = 0{\cdot}6745\sqrt{\frac{\Sigma wr^2}{(n-1)\Sigma w}}.$$

These formulæ are strictly applicable only when n is indefinitely great, but are commonly used for cases when n is small In such cases the result does not really represent the probable error, but nevertheless serves to indicate the relative precision of similar sets of observations.

Example 1 —Eight measures of an angle are as tabulated Compute the p e. of the arithmetic mean, the observations being of uniform weight.

Measure.			r	r^2
°	′	″	″	″
67	34	14·2	−1 64	2 69
		18·9	+3·06	9 36
		13·2	−2 64	6 97
		17·8	+1 96	3 84
		16·9	+1 06	1 12
		15 5	−0 34	0 12
		14 0	−1·84	3 39
		16·2	+0 36	0·13
		8) 126 7	−0·02	Σr^2 = 27·62
		15·84		

$$\text{p e. of arithmetic mean} = 0\ 674\sqrt{\frac{27\ 62}{8\times 7}}$$

$$= \pm 0''{\cdot}47.$$

Example 2.—If the last four measures above are given a weight three times that of the first four, compute the p.e. of the weighted mean.

Measure	w	wm	r	r^2	wr^2
° ′ ″		″			
67 34 14·2	1	14·2	1·54	2 37	2·37
18 9	1	18 9	3 16	9 99	9 99
13·2	1	13 2	2 54	6·45	6 45
17 8	1	17 8	2 06	4 24	4 24
16·9	3	50 7	1 16	1·35	4 05
15·5	3	46 5	0·24	0 06	0·18
14·0	3	42 0	1·74	3·03	9·09
16·2	3	48·6	0·46	0 21	0·63
	$\Sigma w = 16$	) 251·9			$\Sigma wr^2 = 37{\cdot}00$
		15·74			

$$\text{p.e. of weighted mean} = 0\ 674\sqrt{\frac{37{\cdot}00}{7\times 16}}$$
$$= \pm 0\ 39''.$$

For Computed Quantities.

Let R = the result computed from the most probable values, m, n, etc., of one or more independent quantities $= f\,(m, n, \text{etc.})$,

e_1, e_2, etc. = the probable errors of m, n, etc.

$$\text{then p.e. of } R = \sqrt{\left(e_1\frac{dR}{dm}\right)^2 + \left(e_2\frac{dR}{dn}\right)^2 + \ldots}$$

In the common case where $f\,(m, n, \text{etc.})$ is linear, it follows that,

if R = the sum or difference of a constant and an observed quantity m, having a p.e. of e_1,

p.e. of $R = e_1$.

If R = the sum or difference of a number of observed quantities, m, n, etc.,

$$\text{p e of } R = \sqrt{e_1^{\,2}+e_2^{\,2}+\ldots}$$

Example 3.—If the most probable value of an angle is 48° 12′ 32″ 24 ± 0″·28, that of its explement is 311° 47′ 27″·76 ± 0″ 28

Example 4.—The most probable value of an angle AOC is 84° 40′ 21″ 20 ± 0″·90, and that of its part AOB is 36° 14′ 08″ 52 ± 1″ 30. Find the probable error of the value of BOC obtained by subtraction

$$\text{p e. of BOC} = \sqrt{(\ 90)^2+(1{\cdot}30)^2} = \pm 1''{\cdot}58$$

Example 5.—Find the probable error of the area of a rectangle the sides of which are in feet, 210 55 ± 0 01 and 372 40 ± 0·02.

$$R = mn.$$
$$\frac{dR}{dm} = n, \text{ and } \frac{dR}{dn} = m.$$
$$\text{p.e. of } R = \sqrt{\left(e_1\frac{dR}{dm}\right)^2 + \left(e_2\frac{dR}{dn}\right)^2} = \sqrt{(e_1\,n)^2+(e_2\,m)^2},$$
$$= \sqrt{(\cdot 01\times 372{\cdot}40)^2+(\cdot 02\times 210{\cdot}55)^2} = \pm 5{\cdot}62 \text{ sq. ft.}$$

Application of Probable Error to Weighting.—If the results of two or more series of observations of a quantity are available, and the probable error of each is known, the most probable value may be obtained from the individual results by giving to each a weight based on its probable error. It may be shown that weight is inversely proportional to the square of the probable error, and, in the absence of other data, the individual results are so weighted.

Example.—Telegraphic longitude differences between two stations gave the following results .—

h	m	s	s
0	4	10 48	±0 60,
		9 50	±0·30,
		8·56	±0 75,
		11 06	±0 50,
		9·12	±0·42,
		10 06	±0 72

Calculate the most probable value and its probable error

The weights are proportional to $\frac{1}{60^2}, \frac{1}{\cdot30^2}, \frac{1}{75^2}, \frac{1}{50^2}, \frac{1}{\cdot42^2}, \frac{1}{\cdot72^2}$,

or to 28, 111, 18, 40, 57, 19,

whence

m	w	wm	r	r^2	wr^2
10 48	28	293 4	0 75	0 563	15 76
9 50	111	1,054 5	0 23	0 053	5 88
8 56	18	154 1	1 17	1 369	24 64
11·06	40	442 4	1·33	1 769	70 76
9 12	57	519 8	0 61	0 372	21 20
10 06	19	191 1	0 33	0 109	2 07
	Σw = 273	) 2,655 3			Σwr^2 = 140 31
		9 73			

$$\therefore \text{p e of weighted mean} = 674 \sqrt{\frac{140\ 31}{273 \times 5}} = \pm 0\ 22 \text{ s},$$

and difference of longitude = 0 h. 4 m 9 73 s ±0 22 s

Rejection of Doubtful Observations.—In a series of observations it not infrequently happens that one or more measures differ considerably more than the others from the mean. This may be caused by a mistake or by some external influence which does not affect the other observations, and, if so, the discrepant measures should not be used in computing the final result. Since, however, each observation is subject to an indefinitely large number of very small accidental errors, it is in accordance with the laws of probability that these should occasionally combine to form a large accidental error, and this circumstance would not justify the rejection of the measure affected. It is therefore a matter of considerable difficulty to decide whether an observation at variance with the others should be discarded or retained.

In the field an observer should not cancel an observation merely because it differs rather widely from the others, unless the dis-

crepancy is so great that a mistake is obvious. When he is of opinion that an observation will not prove as trustworthy as usual, he should record the full circumstances affecting its accuracy. Such notes form a valuable guide to the computer, who may either use his own judgment as to whether any observations should be rejected or be guided by a rule based on mathematical principles. Many such criteria have been proposed That recommended by Wright and Hayford* is as follows :

> Reject each observation for which the residual exceeds five times the probable error of a single observation as derived from all the measures. Examine each observation for which the residual exceeds 3½ times the probable error of a single observation, and reject it if any of the conditions under which the observation was made were such as to produce any lack of confidence

Pierce's criterion has been largely used for the same purpose. The rule and the table of constants required in its application will be found in The Royal Geographical Society's *Hints to Travellers,* Chauvenet's *Astronomy,* Wilson's *Topographic Surveying,* etc.

ADJUSTMENT OF TRIANGULATION

Theoretically, all the angles of a triangulation system should be treated together by least squares to yield their simultaneous most probable values The labour of such a solution is so very great that even in primary work it is usual to divide the system into sections, which are separately adjusted. More generally, it is sufficient to adjust the angles of each triangle or simple system of triangles under the heads, (1) station adjustment, (2) figure adjustment.

Station Adjustment is directed to finding the most probable values of the angles at a station without reference to the results of observations at other stations. The only geometrical conditions which may have to be fulfilled occur when the horizon is closed, so that the angles must sum to 360°, and when angles are measured in combination, so that certain observed results should equal the sums of others.

Figure Adjustment involves the adjustment of the angles of each triangle or simple system of triangles so that the figures may be geometrically consistent. The conditions to be satisfied are (1) that the angles of each triangle or polygon should sum correctly, and (2) that the length of any side as computed from any other should, in a system of interlaced triangles, be independent of the route chosen.

* *Adjustment of Observations,* 2nd edition, p. 90.

Station Adjustment.—Any case may be solved by application of the method of normal equations, the conditional equations being used to reduce the number of independent unknowns (see *Ex.* 1, page 164). It is, however, unnecessary to have recourse to this method in simple cases. When the horizon is closed by measuring each angle independently, the error of closure is distributed to the angles by applying equal corrections if the weights of the observations are equal, and in amounts inversely proportional to the weights if these are unequal. In the case where two or more angles and their sum form the station observations, a discrepancy between the observed value of the total angle and the sum of the observed values of the parts is distributed to all the measurements in amounts inversely proportional to their respective weights, and with the opposite sign for the correction of the total angle to that of the parts.

Example 1.—Adjust the following angles closing the horizon at a station.

$$
\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
a = & 124 & 05 & 58{\cdot}6, & \text{weight } 2, \\
b = & 88 & 43 & 16{\cdot}3, & \qquad\quad 1, \\
c = & 70 & 52 & 31{\cdot}2, & \qquad\quad 3, \\
d = & 76 & 18 & 16{\cdot}7, & \qquad\quad 1 \\
\hline
 & 360 & 00 & 02{\cdot}8 &
\end{array}
$$

The excess of $2''{\cdot}8$ falls to be distributed in the ratio $\frac{1}{2} : 1 : \frac{1}{3} : 1$, or $3 : 6 : 2 : 6$, so that the negative corrections are :

$$\frac{3}{17}\times 2''{\cdot}8 = 0''{\cdot}5,\ \frac{6}{17}\times 2''{\cdot}8 = 1''{\cdot}0,\ \frac{2}{17}\times 2''{\cdot}8 = 0''{\cdot}3, \text{ and } \frac{6}{17}\times 2''{\cdot}8 = 1''{\cdot}0,$$

giving the adjusted values,

$$
\begin{array}{rrrr}
 & ^\circ & ' & '' \\
a = & 124 & 05 & 58{\cdot}1, \\
b = & 88 & 43 & 15{\cdot}3, \\
c = & 70 & 52 & 30{\cdot}9, \\
d = & 76 & 18 & 15{\cdot}7, \\
\hline
 & 360 & 00 & 00{\cdot}0
\end{array}
$$

Example 2.—Adjust the angles a and b, observations of which give

$$
\begin{array}{rrrrl}
 & ^\circ & ' & '' & \\
a = & 54 & 28 & 17{\cdot}4, & \text{weight } 1, \\
b = & 63 & 51 & 41{\cdot}3, & \qquad\quad 2, \\
a+b = & 118 & 19 & 55{\cdot}1, & \qquad\quad 2.
\end{array}
$$

The discrepancy of $3''{\cdot}6$ must be distributed in the ratio $1 : \frac{1}{2} : \frac{1}{2}$, or $2 : 1 : 1$, giving corrections of $-1''{\cdot}8$, $-0''{\cdot}9$, and $+0''{\cdot}9$, and yielding the adjusted values,

$$
\begin{array}{rrrr}
 & ^\circ & ' & '' \\
a = & 54 & 28 & 15{\cdot}6, \\
b = & 63 & 51 & 40{\cdot}4, \\
a+b = & 118 & 19 & 56{\cdot}0.
\end{array}
$$

Figure Adjustment—Spherical Excess.—In geodetic triangulation the triangles or polygons are recognised as spherical, and the sum of the angles exceeds that of the corresponding plane figure by an

amount termed the spherical excess On a true sphere spherical excess is given by

$$E = \frac{A}{R^2 \sin 1''},$$

where E = spherical excess in seconds,
A = area of figure,
R = radius of sphere.

For the terrestrial spheroid the excess for a given area varies with the latitude, decreasing from the equator to the poles, and for large figures the *spheroidal* excess is more precisely given by

$$E = \frac{A(1-e^2 \sin^2\phi)^2}{2a^2(1-e^2) \sin 1''},$$

where ϕ = mean of the latitudes of the bounding stations,
a = earth's equatorial semi-axis,
e = earth's eccentricity (page 181).

The value of the excess for triangles of moderate size being very small, the former expression is usually sufficient, R being taken as the mean radius of the earth, and A being computed from the unadjusted values of the angles and one side With sufficient accuracy for many purposes, spherical excess may be taken as 1″ for every 76 square miles of area.

Adjustment of Single Triangle.—The only requirement to be satisfied is the summation of the angles to $(180°+E)$ To obtain the angles required in computing the sides (page 178), the observed value of each angle is first reduced by $\frac{1}{3}E$ The difference between the sum of the resulting angles and 180° is then distributed as equal corrections if the observations are of uniform weight, or in amounts inversely proportional to the weights if these are unequal. The values required for the computation of azimuths are not reduced for spherical excess, and are similarly adjusted to sum to $(180°+E)$.

Adjustment of Quadrilateral.—If observations have been taken along one diagonal only, the figure consists of two triangles having a common side, and, after reducing, if necessary, for spherical excess, the conditions to be met and methods of solution have been shown in *Ex.* 2, page 165 and *Ex.* page 167.

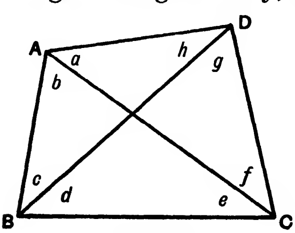


FIG. 66.

In the *geodetic* quadrilateral, observations are made along both diagonals, and the angles lettered in Fig. 66 are measured. The observed values are first reduced for spherical excess as required in computing the triangle sides. In the most refined work the excess is computed for each triangle formed by the intersection of the diagonals, and one-third is deducted from each of the two

appropriate angles: otherwise, all eight angles are reduced by one-eighth the excess for the whole figure.

The angle requirements are that. (1) the sum of the observed angles should be 360°, (2) the sum of the angles in any of the triangles formed should be 180°, (3) the opposite angles at the intersection of the diagonals should be equal. These conditions are not independent, and are fulfilled if three independent angle equations are satisfied. Several sets of three equations are available. That used below is

$$\begin{aligned} a+b+c+d+e+f+g+h &= 360^\circ, \\ a+h-d-e &= 0, \\ b+c-f-g &= 0. \end{aligned}$$

The requirement that the evaluation of one side from another should yield the same result through whatever triangles the calculation is carried gives rise to one side equation of condition. If, for example, CD is calculated from AB through triangles ABD and ADC, the result is given by

$$\mathrm{CD} = \frac{\sin a}{\sin f}\frac{\sin c}{\sin h}\mathrm{AB},$$

and if through triangles ABC and BCD,

$$\mathrm{CD} = \frac{\sin b}{\sin e}\frac{\sin d}{\sin g}\mathrm{AB}$$

For consistency, therefore,

$$\frac{\sin a}{\sin b}\frac{\sin c}{\sin d}\frac{\sin e}{\sin f}\frac{\sin g}{\sin h} = 1,$$

or, more conveniently,

$$(\log\sin a+\log\sin c+\log\sin e+\log\sin g)-(\log\sin b+\log\sin d+\log\sin f+\log\sin h) = 0,$$

which is the side equation.

Adjustment of Geodetic Quadrilateral.—This is best performed by means of correlates as illustrated below.

Example—Adjust the following observed angles lettered as in Fig 66. The observations are of equal weight, and are already corrected for spherical excess

	°	′	″		°	′	″
$a =$	30	27	07 2	$e =$	44	01	23 2
$b =$	37	10	32 6	$f =$	30	56	45 3
$c =$	48	26	09 0	$g =$	54	39	48 8
$d =$	50	21	54 6	$h =$	63	56	14 5

Denoting the residuals for $a, b, \ldots h$ by $r_1, r_2, \ldots r_8$, the angle conditional equations are:

$$\begin{aligned} r_1+r_2+r_3+r_4+r_5+r_6+r_7+r_8 &= +4{\cdot}8'', \\ r_1+r_8-r_4-r_5 &= -3{\cdot}9, \\ r_2+r_3-r_6-r_7 &= -7\ 5. \end{aligned}$$

The logarithmic form of the side equation in terms of the residuals becomes:

$$d_1r_1-d_2r_2+\ \ldots\ -d_8r_8+\log\sin o_1-\log\sin o_2+\ \ldots\ -\log\sin o_8 = 0,$$
$$= d_1r_1-d_2r_2+\ \ldots\ -d_8r_8 = u_4,$$

where d_1, d_2, etc., are the tabular differences for 1″ for the log sines of o_1, o_2, etc., and $-u_4$ is, as before, the amount by which the observed values fail to satisfy the side equation.

In this case, taking the unit as the 6th place of logarithms,

Angle	+ log sin	d	Angle	- log sin	d
a	9 7048506	3 58	b	9 7812249	2 78
c	9 8740253	1·87	d	9·8865616	1·74
e	9 8419526	2 18	f	9 7111563	3 51
g	9 9115677	1·49	h	9 9534283	1 03
	9 3323962			9 3323711	
	711				

$u_4 = -25\ 1$, and the side equation is

$$3\ 58r_1 - 2\ 78r_2 + 1{\cdot}87r_3 - 1\ 74r_4 + 2\ 18r_5 - 3\ 51r_6 + 1\ 49r_7 - 1\ 03r_8 = -25\ 1.$$

The coefficients in the correlate equations are

	a²	ab	ac	ad	b²	bc	bd	c²	cd	d²
r_1	+1	+1		+3 58	+1		+3 58			+12 816
r_2	+1		+1	−2 78				+1	−2 78	+ 7 728
r_3	+1		+1	+1 87				+1	+1 87	+ 3 497
r_3	+1	−1		−1 74	+1		+1 74			+ 3 028
r_5	+1	−1		+2 18	+1		−2 18			+ 4·752
r_6	+1		−1	−3 51				+1	+3 51	+12 320
r_7	+1		−1	+1·49				+1	−1 49	+ 2 220
r_8	+1	+1		−1·03	+1		−1 03			+ 1 061
Σ	+8	0	0	+0 06	+4	0	+2 11	+4	+1 11	+47 42

giving the correlate equations

$$\begin{aligned} 8k_1 \qquad\qquad\qquad\quad + 0\ 06k_4 &= +\ 4\ 8, \\ 4k_2 \qquad\quad + 2\ 11k_4 &= -\ 3{\cdot}9, \\ 4k_3 + 1{\cdot}11k_4 &= -\ 7\ 5, \\ 0{\cdot}06k_1 + 2\ 11k_2 + 1{\cdot}11k_3 + 47{\cdot}42k_4 &= -25\ 1, \end{aligned}$$

$$\text{whence } \begin{aligned} k_1 &= +0\ 6034, \\ k_2 &= -0\ 7342, \\ k_3 &= -1\ 7483, \\ k_4 &= -0\ 4565. \end{aligned}$$

In obtaining r_1, r_2, etc., the coefficients are

	a	b	c	d
r_1	+1	+1		+3·58
r_2	+1		+1	−2 78
r_3	+1		+1	+1·87
r_4	+1	−1		−1 74
r_5	+1	−1		+2·18
r_6	+1		−1	−3 51
r_7	+1		−1	+1·49
r_8	+1	+1		−1 03

so that

$$\begin{aligned}
r_1 &= k_1+k_2+3\ 58k_4 = -1\ 76,\\
r_2 &= k_1+k_3-2\ 78k_4 = +0\ 12,\\
r_3 &= k_1+k_3+1\ 87k_4 = -2\ 00,\\
r_4 &= k_1-k_2-1{\cdot}74k_4 = +2\ 13,\\
r_5 &= k_1-k_2+2\ 18k_4 = +0\ 34,\\
r_6 &= k_1-k_3-3\ 51k_4 = +3\ 95,\\
r_7 &= k_1-k_3+1{\cdot}49k_4 = +1\ 67,\\
r_8 &= k_1+k_2-1{\cdot}03k_4 = +0\ 34.
\end{aligned}$$

Application of these to the observed values gives the adjusted values,

	°	′	″		°	′	″
$a =$	30	27	05 44	$e =$	44	01	23 54
$b =$	37	10	32 72	$f =$	30	56	49 25
$c =$	48	26	07 00	$g =$	54	39	50 47
$d =$	50	21	56·73	$h =$	63	56	14 84

Approximate Adjustment of Geodetic Quadrilateral.—For observations of equal weight, an approximate adjustment, sufficient for minor work, may be performed as follows

(1) Satisfy the first angle equation (page 175) by distributing the error of summation of the observed values as equal corrections to all the angles

(2) Satisfy the second angle equation by distributing one-fourth of the discrepancy to each angle therein

(3) Satisfy the third angle equation in the same way.

(4) Adjust the values so obtained to suit the side equation by the following method.

Let a, b, etc. = values obtained after the angle conditions are met,
r_1, r_2, etc. = corrections to a, b, etc , required to satisfy the side equation,
d_1, d_2, etc. = tabular differences for 1″ for log sin a, log sin b, etc

As in the rigorous adjustment, the side equation may be expressed as

$$d_1r_1-d_2r_2+ \quad . \quad -d_8r_8 = u_4.$$

This equation is now independent of the angle equations, and the most probable set of values for r_1. r_2, etc is that in which r_1, r_2, etc. are numerically proportional to d_1, d_2, etc , or

$$\frac{r_1}{d_1} = \frac{-r_2}{d_2} = \frac{r_3}{d_3} = \text{etc.}$$

Dividing the side equation, term by term, by these ratios, we have

$$d_1{}^2+d_2{}^2+ \ . \quad . \ . \ +d_8{}^2 = \frac{u_4d_1}{r_1} = \frac{-u_4d_2}{r_2} = \text{etc.},$$

$$\text{or } r_1 = \frac{u_4d_1}{\Sigma d^2},\ r_2 = \frac{-u_4d_2}{\Sigma d^2}, \text{ etc}$$

Since these corrections have been determined without reference to the requirements of the angle equations, their application is likely to disturb the former adjustments. If the discrepancies thus caused are greater than is desirable, the results obtained may be

treated as observed values, and the complete adjustment is repeated by the same process.

Example—Apply the approximate method to the adjustment of the last case

(1) The observed angles sum to 4″ 8 short of 360°,

$$\therefore \text{correction to each} = +\frac{4''{\cdot}8}{8} = +0''\,6.$$

(2) $a+h$ exceeds $d+e$ by 3″ 9,

$$\therefore \text{correction to each} = \frac{3''\,9}{4} = 0''\,98, - \text{for } a \text{ and } h, + \text{for } d \text{ and } e$$

(3) $b+c$ exceeds $f+g$ by 7″ 5,

$$\therefore \text{correction to each} = \frac{7''\,5}{4} = 1''{\cdot}88, - \text{for } b \text{ and } c, + \text{for } f \text{ and } g.$$

The values obtained on application of those corrections are as tabulated.

(4) The algebraic sum of the log sines of the angles as adjusted is 21 0 in units of the 6th decimal place of logarithms,

$$u_4 = -21\,0,$$

and $r_1 = -\dfrac{21{\cdot}0\times 3\,58}{47\,42} = -1''{\cdot}59$, etc., as tabulated and applied on page 179

The final adjusted values in the table do not satisfy the angle equations a second adjustment gives results very nearly the same as those previously obtained by the rigorous method

General Figure Adjustment.—Any system of interlocking triangles can be adjusted by the method of correlates. The reader desiring further information is referred to the works on Geodesy and Least Squares cited on page 191

CALCULATION OF TRIANGULATION

Computing Triangle Sides.—Adjustment of the angles of the system is followed by the computation of the sides from the base net outwards Primary triangles are solved with ample accuracy by the aid of Legendre's theorem, which states that, if one-third of the spherical excess of the triangle is deducted from each of its angles, the solution can then be performed by the rules of plane trigonometry. Seven-figure logarithms are used in computing primary triangulation, and six-figure for minor systems.

Except in a chain of single triangles, each side may be computed in more than one way by using different triangles of the net. A valuable check is thus afforded by computing each side in two ways. If the conditional equations are not exactly fulfilled, so that the results differ, their mean is adopted.

Station Positions.—The calculation of the triangle sides gives the position of each station relatively to those adjacent. These data are now applied to computing the co-ordinate positions of the points in order to facilitate their plotting for mapping purposes and to form a record of the results for geodetic and other purposes.

APPROXIMATE ADJUSTMENT OF GEODETIC QUADRILATERAL

Angle	Observed Values			1st Corrn	2nd Corrn	Adjusted Values			Log Sines	d	d^2	Side Corrn	Adjusted Values			Check Log Sines
	°	′	″	″	″	°	′	″				″	°	′	″	
a	30	27	07 2	+0 6	−0 98	30	27	06 82	+9 7048492	3 58	12 816	−1 59	30	27	05 23	+9 7048435
b	37	10	32·6	+0 6	−1 88	37	10	31 32	−9 7812214	2 78	7 728	+1 23	37	10	32 55	−9 7812248
c	48	26	09 0	+0 6	−1 88	48	26	07 72	+9 8740229	1 87	3 497	−0 83	48	26	06 89	+9·8740214
d	50	21	54·6	+0 6	+0·98	50	21	56 18	−9 8865643	1 74	3 028	+0 77	50	21	56 95	−9·8865656
e	44	01	23 2	+0 ,	+0 98	44	01	24 78	+9 8419561	2 18	4 752	−0 97	44	01	23 81	+9 8419540
f	30	56	45 3	+0 6	+1 88	30	56	47 78	−9 7111650	3 51	12 320	+1 55	30	56	49 33	−9 7111704
g	54	39	48·8	+0 6	+1 88	54	39	51 28	+9 9115714	1 49	2 220	−0 66	54	39	50 62	+9·9115704
h	63	56	14 5	+0 6	−0 98	63	56	14 12	−9 9534279	1 03	1 061	+0 46	63	56	14 58	−9·9534284
	359	59	55 2			360	00	00 00	$u_4 = -21\ 0$	$\Sigma d^2 = 47\ 42$			359	59	59·96	·1

In large surveys the station positions are expressed as geodetic or absolute positions in terms of the co-ordinates, latitude and longitude, as well as the azimuths of the lines joining them. The minimum data required, in addition to the relative positions, are the latitude and longitude of one station, the azimuth of one line from that station, and the dimensions of the earth By proceeding from the known station, and computing from station to station by means of the formulæ to be given, the required quantities are obtained for the whole system.

If the survey forms the control for large scale mapping, it is almost essential to have the positions expressed in terms of linear co-ordinates, and rectangular spherical co-ordinates (page 187) are applied for this purpose. These may be employed, to the exclusion of the previous method, for isolated surveys extending to several thousand square miles.

Since the calculation of station positions involves a knowledge of the form and dimensions of the earth, the principal facts relating thereto will first be given.

The Figure of the Earth.—By the figure of the earth is meant the form of the surface corresponding to mean sea level. This is nearly, but not quite, a sphere, the principal departure from an exactly spherical form occurring in a flattening at the poles This circumstance was deduced by Newton, and announced in the *Principia* in 1687. Verification of Newton's hypothesis by measurement was first achieved by the French Academy, who despatched two expeditions, one to Peru in 1735, and the other to Lapland in 1736, to determine the length of a degree of latitude near the equator and the arctic circle respectively. The results showed that the northern degree was the greater. It was not, however, until the close of the eighteenth century that instruments and methods attained a degree of refinement sufficent for the needs of figure determination. Since then much geodetic work has been accomplished in Europe, America, India, and Africa, and knowledge of the dimensions of the earth has steadily grown.

The results of geodetic measurements show that the earth very closely approximates to an oblate spheroid, which is the solid generated by rotation of an ellipse about its minor axis. The actual figure deviates slightly and irregularly from a true spheroid, and this is recognised by giving it the name, "geoid." Since, however, geodetic computions can be made with sufficient precision on the assumption of a spheroidal form, figure determinations are directed to ascertaining the dimensions of the spheroid which most nearly coincides with the actual figure.

Properties of the Spheroid.—Let the ellipse of Fig. 67 represent a meridian section of the earth, and let A be a station on the meridian PQP′E. The major axis, or equatorial diameter $(2a)$ is represented by EQ, and the minor or polar axis $(2b)$ by PP′. The

ratio $\frac{a-b}{a}$ is called the compression c. ABD is the normal to the ellipse at A, and coincides with the direction of the plumb line at A, if the latter is free from local deviation. The angle ϕ between the

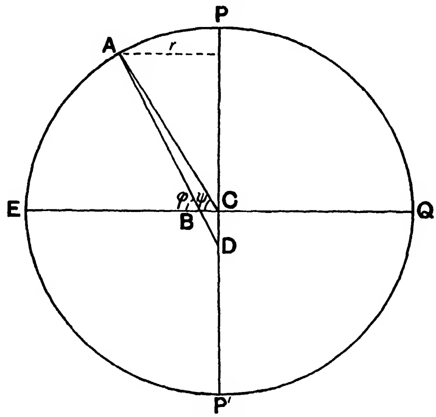


FIG 67.

normal and the equator is the geodetic latitude of A, and its geocentric latitude is represented by the angle ψ between the radius vector CA and the equator The radius of curvature R of the ellipse at A lies along AD, the centre of curvature being situated between B and D.

Let e = the eccentricity $= \sqrt{\frac{a^2-b^2}{a^2}}$,
N = AD,
n = AB,
r = the radius of the parallel of latitude through A,

then for a meridian section,

$$R = \frac{a(1-e^2)}{(1-e^2 \sin^2\phi)^{3/2}};$$

$$N = \frac{a}{(1-e^2 \sin^2\phi)^{1/2}};$$

$$n = (1-e^2)N,$$

$$r = N \cos \phi,$$

$$\psi = \tan^{-1}(1-e^2) \tan \phi.$$

If any section of the spheroid, other than the meridian section, be made by a vertical plane passing through A, the radius of curvature at A for the given azimuth is given by

$$R_{\prime} = \frac{NR}{N \cos^2 A + R \sin^2 A},$$

where R and N = the quantities formulated above,
A = the azimuth of the vertical plane through A.

In particular, if $A = 90°$ or $270°$, $R_A = N =$ the radius of curvature of the prime vertical section. The mean radius of curvature at A, or the mean of the radii for all azimuths, is $\sqrt{NR}$.

Figure Determination.—The results of geodetic surveys are applied to determining the dimensions of the spheroid (1) by comparing the linear and angular values of arcs, (2) by discussing areas. The form, but not the dimensions, of the spheroid may also be deduced by studying the variation of gravity by means of pendulum observations.

Determinations have usually been made from arcs. The length of meridian between two stations of a chain of triangles lying approximately north and south may be computed by projecting the appropriate intervening triangle sides on to the meridian. The corresponding angular distance is represented by the difference in latitude of the stations. On dividing the linear distance by the angular, in circular measure, the radius of curvature for the mean latitude of the stations is obtained, or

$$\frac{l}{d\phi \sin 1''} = \frac{a(1-e^2)}{(1-e^2 \sin^2\phi_{1/})^{3/2}}, \text{ for short arcs,}$$

where $l =$ the linear distance between the stations,
$d\phi =$ their difference of latitude in seconds,
$\phi_{1/} =$ their mean latitude.

The dimensions of the spheroid are known if a and e are obtained. A second meridian arc, preferably in a considerably different latitude, must therefore be treated in the same way, and a and e are derived by simultaneous solution. The method is extended to utilise chains of triangulation oblique to the meridian.

In place of relying upon two arcs, several should be discussed by least squares to afford the most probable values of a and e. It is found that the results derived from different arcs are much more discrepant than can be accounted for by the small errors in the triangulation or the astronomical observations and, further, that the curvature of arcs of parallel is not constant. The disagreements are evidence that the actual figure is not exactly a spheroid, and indicate local deviations of the plumb line from the normal to the mathematical figure. These deviations are due to the irregular distribution of mass, not only at the surface of the earth but in the crust.

The results of astronomical observations are dependent upon the direction of the plumb line and therefore of the horizon at the place. By regarding the errors of primary triangulation and of astronomical observation as negligible, the discrepancies are utilised in discussing the form of the geoid and in determining the spheriod which most nearly resembles it over any portion of the earth.

In utilising the data of a survey for this purpose, a tentative spheroid is adopted for reference. Starting from the position of

one of the stations as obtained astronomically, the geodetic positions of the other stations on the adopted spheroid are computed The astronomical positions of several of these stations will have been observed, and a list of differences between astronomical and geodetic positions, or station errors, as well as those between observed and computed azimuths can be drawn up The effects which would be produced upon these differences by applying corrections to the elements of the assumed spheroid and to the assumed position of the initial station are computed, and the most probable values of the corrections are deduced by least squares The spheroid which best fits the area under discussion and the positions of the stations on that spheroid have now been determined. The station errors remaining after adjustment are the local deflections of the plumb line They serve for the determination of the geoid, the surface of which is everywhere normal to gravity, and measure the inclination of its surface to that of the spheroid In modern geodetic survey this investigation is facilitated by pendulum or torsion balance observations

Results of Figure Determinations.—As geodetic surveys have been extended, the amount of data available for figure determination has increased, and the earlier figures are subject to revision In consequence, the more recent determinations are probably the more reliable, but different national surveys use those figures which best conform to the part of the surface with which they are concerned

The results of some of the more important determinations of the spheroid are, in metres,

			a	*b*	*c*
1	Everest,	1830	6,377,304	6,356,103	1/300 8
2	Bessel,	1841	6,377,397	6,356,079	1/299 2
3	Clarke,	1866	6,378,206	6,356,584	1/295 0
4	Clarke,	1880	6,378,249	6,356,515	1/293 5
5	Helmert,	1906	6,378,200	6,356,818	1/298 3
6	Hayford,	1910	6,378,388	6,356,909	1/297 0

1 is still employed by the Survey of India It was based only on an Indian arc and the French arc, and is known to be erroneous because of large irregularities of gravity in India. 2 has been extensively used in Europe. 3 has been employed in the United States Survey since about 1880, and has recently been adopted by Canada. 5 is based on European, Indian, and African surveys , and 6 on work in the United States only Their similarity shows that there is no great difference in the form of the eastern and western hemispheres

Calculation of Geodetic Positions.—From the known latitude and longitude of a station A and the azimuth and distance from A to B, the co-ordinates of B are obtained by computing the differences of latitude and longitude to be applied to those of A.

The calculation of reverse azimuth is similarly made by computing the difference of azimuth to be applied to the azimuth from

A to B to give that from B to A. This azimuth difference represents the convergence of the meridians or the angle between the meridian at A and that at B. Any two meridians are parallel at the equator, and, as they are traced towards the poles, the angle between them increases until at the poles it equals their difference of longitude. Because of convergence, the azimuth of any great arc other than the equator or a meridian varies throughout its length.

In computing positions, each should be obtained from two others as a check on the calculation. The computed azimuths of two lines from a station should differ by the angle between the lines, and the azimuths of any other lines from the station are obtained by application of the included angles. These angles are the spherical angles and not those used in computing the sides

The notation to be used is as follows :

ϕ, L = known latitude and longitude of A,
ϕ', L' = latitude and longitude of B,
$d\phi = \phi' - \phi$,
$\phi_{M} = \dfrac{\phi+\phi'}{2}$,
$k = 90° - \phi$,
$dL = L' - L$,
A = known azimuth from A to B, reckoned clockwise from south,
A' = azimuth from B to A, ,, ,, ,,
$dA = A' - (A+180°)$.
l = linear distance from A to B,
R = radius of meridian section at A,
R_M = ,, ,, ,, at latitude ϕ_{M},
N = normal at A,
N' = normal at B

Clarke's Formulæ.—The formulæ given by Col. Clarke* are suitable for computing positions over the longest lines it is possible to observe, and were used for the larger triangles of the Ordnance and other surveys They are .

$$\tan \tfrac{1}{2}(A'+\zeta-dL) = \frac{\sin \frac{1}{2}(k-\theta)}{\sin \frac{1}{2}(k+\theta)} \cot \frac{A}{2},$$

$$\tan \tfrac{1}{2}(A'+\zeta+dL) = \frac{\cos \frac{1}{2}(k-\theta)}{\cos \frac{1}{2}(k+\theta)} \cot \frac{A}{2},$$

$$d\phi = \frac{l \sin \frac{1}{2}(A'+\zeta-A)}{R_M \sin 1'' \sin \frac{1}{2}(A'+\zeta+A)} \left[1 + \frac{\theta^2 \sin^2 1''}{12} \cos^2 \tfrac{1}{2}(A'-A) \right] \text{ in sec.},$$

where θ = the angle, in sec , subtended by l at D (Fig 67)

$$= \frac{l}{N \sin 1''} + \frac{e^2 \theta^3 \sin^2 1'' \cos^2\phi \cos^2 A}{6(1-e^2)},$$

and ζ = a small correction $= \dfrac{e^2\theta^2 \sin 1'' \cos^2\phi \sin 2A}{4(1-e^2)}$, in sec .

* *Geodesy*, 1880.

Since the mid-latitude is unknown, R_M is obtained from a preliminary solution.

Puissant's Formulæ.—The formulæ given by Puissant in his *Traité de Géodésie*, Vol I, have been expressed in different ways, and appropriate factors, depending upon the figure of the earth adopted and the known latitude, are tabulated by various surveys

It may be shown that

$$-d\phi = \frac{l\cos A}{R_M} + \frac{l^2\sin^2 A\tan\phi}{2NR_M} - \frac{l^3\sin^2 A\cos A(1+3\tan^2\phi)}{6N^2R_M} + \ldots,$$

$$\sin dL = \sin\frac{l}{N'}\cdot\frac{\sin A}{\cos\phi'},$$

$$-dA = dL\sin\phi_M\sec\tfrac{1}{2}d\phi + \tfrac{1}{12}dL^3(\sin\phi_M\sec\tfrac{1}{2}d\phi - \sin^3\phi_M\sec^3\tfrac{1}{2}d\phi).$$

These formulæ are used by the United States Coast and Geodetic Survey for distances up to about 70 miles in the following modified form :

$$-d\phi'' = l\cos A\cdot\mathrm{B} + l^2\sin^2 A\cdot\mathrm{C} + (\delta\phi'')^2\cdot\mathrm{D} - hl^2\sin^2 A\cdot\mathrm{E},$$

$dL'' = l\sin A\sec\phi'\cdot\mathrm{A}$ (subject to a correction for the difference between the arc and sine of dL and of the angular value of l),

$$-dA'' = dL''\sin\phi_M\sec\tfrac{1}{2}d\phi + (dL'')^3\cdot\mathrm{F},$$

where
$$\mathrm{A} = \frac{1}{N'\sin 1''},\quad \mathrm{B} = \frac{1}{R\sin 1''},\quad \mathrm{C} = \frac{\tan\phi}{2NR\sin 1''},$$

$$-\delta\phi'' = l\cos A\cdot\mathrm{B} + l^2\sin^2 A\cdot\mathrm{C} - hl^2\sin^2 A\cdot\mathrm{E},$$

$$\mathrm{D} = \frac{3e^2\sin\phi\cos\phi\sin 1''}{2(1-e^2\sin^2\phi)},\quad h = l\cos A\cdot\mathrm{B},$$

$$\mathrm{E} = \frac{1+3\tan^2\phi}{6N^2},\quad \mathrm{F} = \tfrac{1}{12}\sin\phi_M\cos^2\phi_M\sin^2 1''.$$

The logarithms of A, B, C, D, E, and F, based on the 1866 Clarke spheroid and metric units, are published* for latitudes 0° to 72°.

In performing the computation, $d\phi$ is first obtained, B, C, D, and E being taken from the tables with ϕ as argument. The algebraic sum of the 1st, 2nd, and 4th terms gives $\delta\phi''$, an approximate value of $d\phi''$. $(\delta\phi'')^2\cdot\mathrm{D}$ is a corrective term which allows for the difference between R_M and R. In computing dL, the value of A is obtained with ϕ' as argument To the resulting value of dL is applied the algebraic sum of the tabulated corrections for log l and log dL. The evaluation of dA is straightforward In all cases care must be exercised with the signs of the functions of A.

When l is less than about 12 miles, and in minor triangulation, the formulæ may be simplified to

$$-d\phi'' = l\cos A\cdot\mathrm{B} + l^2\sin^2 A\cdot\mathrm{C} + h^2\cdot\mathrm{D},$$
$$dL'' = l\sin A\sec\phi'\cdot\mathrm{A},$$
$$-dA'' = dL''\sin\phi_M.$$

* United States Coast and Geodetic Survey Report, 1894, Appendix No. 9, and 1901, Appendix No. 4.

In the method of expressing Puissant's formulæ devised by Col. Everest, each term is derived from those preceding. Everest employed four terms of the series for primary work. The constants, published in the Survey of India Auxiliary Tables were, however, computed for Everest's figure, which is unsuitable for general use. The logarithms of the factors P, Q, R, S, T, for the first two terms, based on Clarke's 1858 figure, are given in Sir C F Close's *Text Book of Topographical and Geographical Surveying.* These are sufficient for all minor work.

The two-term formulæ are :

$$d\phi = d_1\phi + d_2\phi,$$
$$dL = d_1L + d_2L,$$
$$dA = d_1A + d_2A,$$

where $d_1\phi = \mathrm{P}\,l \cos A,\qquad d_2\phi = d_1A\ \mathrm{R}\,l \sin A,$

$d_1L = d_1\phi\ \mathrm{Q} \sec \phi \tan A,\ d_2L = d_2\phi\ \mathrm{S} \cot A,$

$d_1A = d_1L \sin \phi,\qquad d_2A = d_2L\ \mathrm{T},$

the tabulated quantities for seconds units being,

$$\mathrm{P} = \frac{1}{R \sin 1''},\quad \mathrm{Q} = \frac{R}{N},\quad \mathrm{R} = \frac{1}{2R},\quad \mathrm{S} = \frac{2R \sec \phi}{N},$$
$$\mathrm{T} = \tfrac{1}{2}\left(2 \tan^2\phi + \frac{N}{R}\right) \cot \phi \cos \phi.$$

Taking north latitudes and east longitudes as positive, and reckoning azimuth clockwise from south, the signs of the terms are as follows, when the known station A is in north latitude.

Term	*A*			
	0–90°	90°–180°	180°–270°	270°–360°
$d_1\phi$	–	+	+	–
d_1L	–	–	+	+
d_1A	–	–	+	+
$d_2\phi$	–	–	–	–
d_2L	+	–	+	–
d_2A	+	–	+	–

When A is in south latitude, south latitude and east longitude are considered positive, and the signs of $d_1\phi$, d_1A, and d_2L are reversed.

Inverse Cases.—(1) It is frequently required to determine l, A, and A' from the given quantities ϕ, ϕ', L, and L'.

For the most precise results Clarke's formulæ may be used indirectly. Close approximations to l and A are first obtained from the Puissant formulæ by the method below. These values are substituted in the Clarke formulæ, and ϕ' and L' are computed. The small discrepancies between the results and the known values of ϕ' and L' indicate the nature of the errors in the assumed values of l and A, and by successive approximations these are eliminated

In using the Puissant formulæ, the equations for $d\phi$ and dL are combined to yield l and A. In the American form the expression for dL gives

$$l \sin A = \frac{dL \cos \phi'}{\mathrm{A}}.$$

Substituting this value for $l \sin A$ in the formula for $d\phi$, we have

$$l \cos A = -\frac{1}{\mathrm{B}}\left[d\phi + \left(\frac{dL \cos \phi'}{\mathrm{A}}\right)^2 \cdot \mathrm{C} + d\phi^2 \cdot \mathrm{D} - h\left(\frac{dL \cos \phi'}{\mathrm{A}}\right)^2 \cdot \mathrm{E}\right],$$

of which h is not exactly known. It is usually taken as $-d\phi$, but a closer approximation may be made by applying the known small terms to $-d\phi$.

Since $l \sin A$ and $l \cos A$ are both known, A is obtained from $\frac{l \sin A}{l \cos A} = \tan A$, and l can then be derived from either of the above equations The Everest formulæ may be used in the same way.

Sufficiently accurate results for mapping are obtained by the use of fewer terms in the evaluation of $l \cos A$. Other forms of approximate solutions are given by Close.

(2) A similar case occurs when ϕ, ϕ', L, and A are known, and l, A', L' are required.

The Puissant expression for $d\phi$ is a quadratic in l, on solving which dL can be evaluated

In applying the calculation to latitude and azimuth traverse (page 228) refined computation is not required, and, since such traverses should run nearly north and south, it is usually sufficient to employ only the first terms of the formulæ, so that

$$-d\phi'' = \frac{l \cos A}{R \sin 1''},$$

$$\text{or } l = d\phi R \sin 1'' \sec A,$$

$\frac{1}{R \sin 1''}$ being the constant B of the American tables, or P of the Indian.

Rectangular Co-ordinates.—In the rectangular system of co-ordinates the position of any station is referred to that of an initial station O, the co-ordinates being distances along and perpendicular to the meridian of O. The directions of survey lines are derived from the initial azimuth and the observed angles of the triangulation, and are therefore expressed as bearings from the reference meridian. Thus, in Fig. 68, AK and BK′ are parallels to the reference meridian through stations A and B respectively, and α is the bearing of AB. AM is the meridian of A; and the azimuth of AB, if required, is obtained by application of the convergence correction

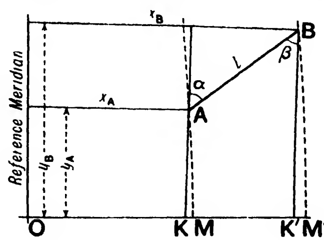


FIG 68.

Starting from O, the co-ordinates of successive stations are computed as in finding the total latitudes and departures of traverse stations. In obtaining the co-ordinates of B from those of A, we have

$$x_B = x_A + l \sin \alpha, \quad y_B = y_A + l \cos \alpha,$$

as in a plane system. Although not precise, these formulæ are nearly so for considerable distances, and are usually sufficient. Their use does not imply that the survey is treated as plane, since the triangle sides are spherical distances. With greater precision,

$$x_B = x_A + n - \frac{m^2 x_A}{2RN} - \frac{m^2 n}{6RN},$$

$$y_B = y_A + m + \frac{m x_B^2}{2RN} - \frac{mn^2}{6RN},$$

$$\beta'' = \alpha'' - \frac{m x_A}{RN \sin 1''} - \frac{mn}{RN \sin 1''},$$

where $n = l \sin \alpha$, and $m = l \cos \alpha$ R and N may be taken out for the latitude of the initial station

Location of Boundaries.—Geodetic computations are required in the operation of tracing out long lines, as in marking interstate boundaries situated along meridians, parallels of latitude, or great arcs in any direction.

The best method consists in first running a triangulation chain along the general line and computing the geodetic positions of its stations. The distance and azimuth from those stations to any number of points on the line are then calculated, and the latter are located by traversing or by intersection from the triangulation stations. If the expense of triangulation cannot be incurred, or the distance is comparatively short, the procedure is as follows.

Meridians—To trace a meridian, the terminal stations are approximately located, and their longitudes are determined by means of telegraph signals from some point. The errors in longitude of the assumed positions are eliminated by moving the stations by the linear equivalents of the longitude errors At the initial station the direction of the meridian is obtained by azimuth observations, and a forward instrument station is established on it. The line is extended by backsighting and foresighting at successive instrument stations, and the distances of intermediate points are directly measured. Observations for azimuth are made at intervals The error discovered on reaching the terminal station is proportioned to the intermediate points.

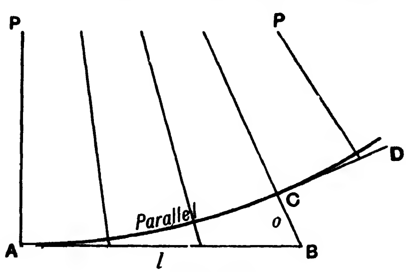


FIG 69.

Arcs of Parallel—In locating

a parallel of latitude, the position of the initial point A (Fig. 69) is established by astronomical observations. The direction AP of the meridian is obtained, and a perpendicular AB is set out. The parallel AC is located from this tangent.

Since the azimuth of AB is 90° or 270°, the change of latitude in a distance l along AB is, by Puissant's formula,

$$d\phi = -\frac{l^2 \tan \phi}{2NR_M} = -\frac{l^2 \tan \phi}{2NR}, \text{ with ample precision.}$$

The offset o is the linear equivalent of $d\phi$, so that

$$o = d\phi R = \frac{l^2 \tan \phi}{2N}.$$

The direction of the offset differs from that of a perpendicular to AB by dA, the convergence of the meridians, and

$$dA = -dL \sin \phi, \text{ with sufficient accuracy,}$$

$$\text{where } dL = \frac{l \sec \phi'}{N}.$$

To keep the offsets short, l should not exceed 50 miles, a new tangent CD being then set out The work is controlled by latitude observations at intervals, and will be found to require some adjustment because of local deflections

Oblique Arcs.—In locating an oblique arc, the positions of the terminals are first found by astronomical observations The azimuth between these points is computed, and a line is run on this azimuth from the initial station Intermediate points are marked at known distances, and the azimuth is checked at intervals, observed azimuths being corrected for convergence The discrepancy found on reaching the end of the line is distributed to the intermediate points.

EXAMPLES

Note —Where necessary, take the earth's major and minor semi-axes as 6,378,300 m. and 6,356,860 m respectively.

1. Observations of three angles and their sum give

	°	′	″	
$a =$	40	32	12 4, weight	1,
$b =$	49	07	50 6,	1,
$c =$	60	22	36 3,	2,
$a+b+c =$	150	02	38 0,	2

Adjust the angles.

2 Adjust the following station observations closing the horizon :

	°	′	″	
$a =$	65	18	30 2, weight	2,
$b =$	78	37	12·3,	1,
$c =$	72	48	02 5,	1,
$d =$	59	00	49 2,	3,
$e =$	84	15	22 7,	1.

3. At a station O in a triangulation survey the following results were obtained :

Angle	Observed Value °	′	″	Weight
AOB	67	14	32·4	1·1
BOC	75	36	21 5	1·0
COD	59	56	02 0	·7
DOE	83	24	17 1	·8
EOA	73	48	45 0	1 4

The weights are proportional to the reciprocals of the squares of the probable errors Adjust the angles (R T.C , 1915)

4. Adjust the following station observations :

	°	′	″	weight
$a =$	54	12	40 7,	weight 2,
$b =$	46	31	15 4,	2,
$a+b =$	100	43	53 8,	1,
$c =$	69	22	31 2,	1,
$b+c =$	115	53	49 0,	1

5. Find the most probable values of the angles a, b, and c from the following observations of equal weight :

	°	′	″
$a =$	48	27	11 04,
$b =$	56	40	30 22,
$a+b =$	105	07	40 65,
$b+c =$	96	36	59 87,
$c =$	39	56	28 24,
$a+b+c =$	145	04	11 33

6 Neglecting spherical excess, adjust the angles of a triangle of which the observed values are

	°	′	″	weight
$a =$	49	17	23 2,	weight 3,
$b =$	75	32	46 7,	1,
$c =$	55	09	53 1,	3

7 Eight measures of an angle give, in the seconds, 10 33, 5 10, 7 23, 9 90, 6 17, 10 07, 6 43, 9 97. Compute the probable error of the arithmetic mean and of a single measure

8 From the following results, obtained in the measurement of a base line six times by means of a standardised steel tape on the prepared surface of the ground, calculate the probable error of the mean

3050 53, 3050 26, 3050 48, 3050 19, 3050 22, 3050 36 feet

State the precautions which would have to be taken to ensure this degree of consistency in the results (R T C , 1914)

9. A secondary angle is measured by A and B each eight times. A's observations in the seconds are 2 4, 5 0, 3 6, 0 8, 7 5, 8 7, 3 2, 9·6 ; and B obtains 6 8, 3 0, 5 4, 9 6, 2 8, 10 4, 7 6, 8 0 Compare the weights of their results, and compute the most probable value.

10. Latitude observations are taken at intervals in a long route traverse running approximately north and south If the linear error of the traverse is of the order $\frac{1}{400}$, at what intervals should a check be made by a latitude observation if the probable error of the latter is $\pm 3''$?

1″ of meridian arc may be taken as equivalent to 101 ft.

11. An observer A measures an angle 32 times with a mean result of 54° 27′ 41″·62, and B measures it 16 times with the result 54° 27′ 40″ 05. The sums of the squares of their residual errors are respectively 331·50 and 125·35 in seconds units Find the most probable value of the angle.

12. In measuring a base line two field tapes were used. No. 1 was suspended 105 times and No. 2 86 times. The probable error in length of No. 1 tape when reduced to the field temperature was 00012 ft, and that of No. 2 was ·00015 ft. Calculate the probable error in the length of the base due to uncertainty in the length of the tapes.

13. Measurement of the angles of two triangles having a common side BC gives

	°	′	″		°	′	″
$A =$	54	17	28·2,	$C_2 =$	71	25	14·1,
$B_1 =$	65	02	36·0,	$B =$	111	14	38·2,
$C_1 =$	60	39	57 2,	$C =$	132	05	13·4,
$B_2 =$	46	12	03 8,	$D =$	62	22	44·0.

Adjust the angles.

14. Adjust by the method of correlates the following observed values of the angles of a geodetic quadrilateral (Fig 66).

	°	′	″		°	′	″
$a =$	54	30	02 7,	$e =$	60	19	22·8,
$b =$	42	23	34 2,	$f =$	48	11	17 8,
$c =$	37	40	12 5,	$g =$	31	52	31 5,
$d =$	39	36	46 6,	$h =$	45	26	08 3

15 Make an approximate adjustment of the preceding case.

16 Compute the unknown sides of the triangle ABC of which the side AC = 41,345 30 m and the observed values of the angles are A = 51° 17′ 11″ 4, B = 58° 50′ 20″ 6, C = 69° 52′ 35″ 9 Take the weight of the measure of C as twice that of the others

17 Compute the mean radius of curvature of the earth at a point in latitude 50° N.

18. A line AB from the above point has a length of 48,532 metres and an azimuth of 164° 17′. Compute the latitude of B

19 Calculate the distance and azimuth from a point in latitude 56° 6′ 21″ N, longitude 4° 57′ 52″ W to one in latitude 56° 20′ 17″ N., longitude 4° 58′ 31″ W

20 Find the lengths of 1 second of meridian and of parallel at the equator and in latitude 50°

REFERENCES ON GEODETIC SURVEYING

"An Account of the Measurement of a Geodetic Base Line at Lossiemouth, in 1909, together with a Discussion on the Theory of Measurement by Metal Tapes and Wires in Catenary." Ordnance Survey, Prof Paper No. 1, 1912.

ADAMS, O S "Application of the Theory of Least Squares to the Adjustment of Triangulation." U S. Coast and Geodetic Survey, Special Publication No. 28, 1915.

ATHERTON, J. W. "Tables for Determining Geodetic Positions, Latitudes 0° to 65°." Admiralty, London, 1912.

Auxiliary Tables. *Survey of India Hand-book, No. 4,* 1906.

BAUGH, J. A. A "Invar Tapes." *The Geographical Journal,* Vol. 39, No. 1, 1912.

BENOIT, J. R., and GUILLAUME, C. E. *La Mesure Rapide des Bases Géodésiques.* Paris, 1917.

BLACK, C. E. D. *A Memoir on the Indian Survey* London, 1891.

BOUTELLE, C. O. "Report on Geodesic Night Signals" U.S. Coast and Geodetic Survey, Report for 1880, Appendix No 8.

— "Geodetic Reconnaissance" U.S Coast and Geodetic Survey, Report for 1885, Appendix No. 10.

BOWIE, W. "The Texas-California Arc of Primary Triangulation" U.S Coast and Geodetic Survey, Special Publication No. 11, 1912

— "Report on the Connection of the Arcs of Primary Triangulation along the Ninety-Eighth Meridian in the United States and in Mexico and on Triangulation in Southern Texas." U.S Coast and Geodetic Survey, Special Publication No. 54, 1919.

BRIGGS, H. *The Effects of Errors in Surveying.* London, 1912.

BURRARD, S. G. "An Account of the Scientific Work of the Survey of India and a Comparison of its Progress with that of Foreign Surveys" Survey of India, Prof Paper No 9, 1905.

CARSON, P.A "Precise Measuring with Invar Wires and the Measurement of Kootenay Base" Department of the Interior, Ottawa, 1911.

CARY, E. H. *Geodetic Surveying.* New York, 1916.

CLARKE, A R. *Ordnance Trigonometrical Survey of Great Britain and Ireland Account of the Observations and Calculations of the Principal Triangulation* London, 1858

— *Geodesy* Oxford, 1880.

CLARKE, A R, and HELMERT, F R "Figure of the Earth" *Encyclopedia Britannica,* 11th Edition, 1911.

CLOSE, SIR C. F. "Notes on the Geodesy of the British Isles" Ordnance Survey, Prof Paper No 3, 1914

CRANDALL, C L *Text-book on Geodesy and Least Squares* New York, 1914.

DAVIDSON, G "The Run of the Micrometer." U S Coast and Geodetic Survey, Report for 1884, Appendix No 8

ECCLES, J. A. "Notes on the Theory of Errors of Observation," Survey of India, Miscellaneous Paper No 11, 1903

— "On a Simplification of the Computations relating to Rectangular Co-ordinates" Survey of India, Miscellaneous Publication No 13, 1911.

EIMBECK, W. "The Duplex Base Apparatus and Directions for its Use in the Field" U S Coast and Geodetic Survey, Report for 1897, Appendix No 11.

— "Report on the Measurement of the Salt Lake Base Line in Utah" U S Coast and Geodetic Survey, Report for 1897, Appendix No. 12

"Formulæ and Tables for the Computation of Geodetic Positions" U S Coast and Geodetic Survey, Special Publication No 8, 1919

FRENCH, O B. "Invar Tapes on the Measurement of Six Primary Base Lines." *Trans Am. Soc C E*, Vol LX

"Fundamental Standards of Length and Mass." U S Coast and Geodetic Survey, Report for 1893, Appendix No 6.

"Geodetic Accuracy—Old and New" *Engineering,* Vol 97, 1914

GILL, SIR D. "Base-measuring Apparatus." *The Geographical Journal,* Vol. 39, No. 1, 1912.

GILL, SIR D., etc. Reports on the Geodetic Survey of South Africa.

GORE, J. H. "A Bibliography of Geodesy." U S. Coast and Geodetic Survey, Report for 1902, Appendix No 8.

GUILLAUME, C. E. "Les Aciers au Nickel et leurs Applications à la Géodesie." International Geodetic Conference Report, 1901. (Translated in Survey of India Prof. Paper No. 7, 1903.)

Hand-book of Professional Instructions for the Trigonometrical Branch, Survey of India Department, 1902

HAYFORD, J F. "The Figure of the Earth and Isostasy from Measurements in the United States" U S Coast and Geodetic Survey, 1909.

— "Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy" U S. Coast and Geodetic Survey, 1910.

HINKS, A R "Recent Progress in Geodesy." *The Geographical Journal*, Vol. 38, No. 2, 1911.

HOSMER, G L. *Geodesy*. New York, 1919.

HUNTER, J de G "The Earth's Axes and Triangulation" Survey of India, Prof Paper No. 16, 1918.

INGRAM, E L. *Geodetic Surveying and the Adjustment of Observations*. New York, 1911

"Invar and Related Nickel Steels" U S Bureau of Standards, Circular No. 58, 1916.

JADERIN, E "On the Measurement of Base Lines with Steel Tapes and with Steel and Brass Wires" U S Coast and Geodetic Survey, Report for 1893, Appendix No 5

MACDONALD, J A. "Testing Surveyors' Tapes by the Canadian Government" *Eng News*, Vol 74, No 9, 1915

MERRIMAN, M *A Text-book on the Method of Least Squares* New York, 1900

— *Elements of Precise Surveying and Geodesy* New York, 1904

NELLES, D H "The Projection and Survey of the 141st Meridian Boundary Line" *Trans Can Soc C E*, Vol XXVII

PARRY, H. "Report of the Triangulation of the Railway Belt of British Columbia" Department of the Interior, Ottawa, 1915

"Report of the Measurement of an Arc of Meridian in Uganda," Vol I. Colonial Office, 1913

SCHOTT, C A "The Transcontinental Triangulation and the American Arc of the Parallel" U S Coast and Geodetic Survey, Special Publication No 4, 1900

TURNER, H H "Report on the Treatment and Use of Invar in Measuring Geodetic Bases." Survey of India, Miscellaneous Paper No 13, 1907.

WADE, H T "Testing Steel Tapes at the National Bureau of Standards" *Eng. News*, Vol 60, No. 7, 1908.

WALKER, J T *Account of the Operations of the Great Trigonometrical Survey of India*. Vol II, *History and General Description of the Principal Triangulation and of its Reduction*. Dehra Dūn, 1879.

WELD, L D. *Theory of Errors and Least Squares* New York, 1916

WINTERBOTHAM, H ST J L. "An Investigation into the Accuracy of the Principal Triangulation of the United Kingdom" Ordnance Survey, Prof. Paper No. 2, 1913

WOODWARD, R S. "The Iced Bar and Long Tape Apparatus." U S Coast and Geodetic Survey, Report for 1892, Appendix No. 8.

WRIGHT, T. W, and HAYFORD, J. F. *The Adjustment of Observations*. New York, 1906

YOUNG, A. E. "Corrections to be applied to the Readings of a Spring-Balance when used Horizontally or on a Slope." *Min Proc Inst. C.E.*, Vol. CLXI.

CHAPTER V

GEODETIC LEVELLING

THE operations of geodetic levelling are directed to the determination of the elevations of a number of vertical control points from which those of any other points throughout the survey may be obtained. The elevations required are absolute elevations from the datum of mean sea level, and, unless the levelling can be connected to that of an adjoining system, it will be necessary to establish the datum by tidal observations

Methods.—The elevations of control points are determined either by precise spirit levelling or trigonometrical levelling, or by both.

Precise spirit levelling represents the highest development of the methods of ordinary levelling, every precaution being taken in the construction of the instruments and in the field work to reduce errors to a minimum By running lines of precise levels along the most favourable routes through the survey, primary bench marks are established for its ultimate vertical control. From these points subsidiary lines are run and bench marks established for the control of detail and, in State surveys, for the information of the public. Precise levelling is also required to provide data for geodetic research relating to gravity, secular upheaval or depression, and the comparison of ocean levels Investigations in connection with certain classes of engineering work, particularly those relating to the flow of water over considerable distances, also demand levelling of a high order of accuracy, and the engineer is sometimes required to produce work comparable to the best precise levelling

In trigonometrical levelling, differences of elevation are determined from vertical angles and geodetic or horizontal distances. The measurement of vertical angles between triangulation stations is usually undertaken at the same time as that of the horizontal angles. The same stations therefore control the survey both horizontally and vertically, and the elevations can be determined economically. In point of accuracy trigonometrical levelling is much inferior to precise spirit levelling, particularly in flat districts. In rugged country the method is of great value, and the results are comparatively more accurate.

Determination of Mean Sea Level.—In order to obtain a measurement of the average elevation of the sea, a continuous record of

its varying level is required for as long a time as possible. The period of observation should cover an integral number of lunations, and unless it extends to at least a year the effect of the solar components will not be truly represented. The employment of a self-registering tide gauge is almost essential, and its zero must be connected and periodically checked by precise levelling to a permanent bench mark near at hand.

PRECISE SPIRIT LEVELLING

There is no difference in principle between precise and ordinary levelling. In the latter, the distances run between checks are relatively short, and, with the usual precautions, the results are sufficiently accurate for everyday purposes Since very small errors cannot be detected, the relative coarseness of the determinations may, through compensation of errors, give a fictitious idea of accuracy. In precise levelling, on the other hand, the circuits may be of considerable length, and the operations must be conducted so that the uncertainty of each individual determination as well as the actual closing error is reduced to a minimum. This necessitates the employment of high-class instruments of superior sensitiveness and their manipulation in such a manner that instrumental and observational errors are eliminated as far as possible. Success in precise levelling depends upon a due appreciation of the nature and relative importance of those errors and their treatment.

Precise Levels.—Precise levels belong in their essentials either to the wye or the dumpy type. Many different designs have been employed, and constructional details vary considerably.

Stability is an important requirement, and is promoted by the provision of a broad levelling base and by having the height above the base and the exposed area as small as practicable. Fittings subject to wear are formed of hard cast steel or iron, and adequate protection must be given to parts exposed to unequal temperature changes.

The telescope is an important feature, and it must afford superior definition, illumination, and flatness of field The focal length of the objective ranges from 11 in. to over 20 in , and the effective aperture should be at least $1\frac{1}{2}$ in. The eyepiece is non-erecting, and the magnifying powers range from 25 to 50 diameters Stadia hairs are fitted. Focussing is performed either by movement of the eyepiece end or by an internal lens, and the motion must be truly axial.

The level tube is of very uniform curvature, and is furnished with an air chamber (Vol. I, page 28), so that a fairly constant length of bubble can be maintained. The liquid should be sulphuric ether in preference to alcohol on account of its superior mobility. In

the more recent instruments of the highest class the angular value of a 2-mm. division of the level tube ranges from about 1·2 sec to 3 sec The tube is either attached to the telescope, as in ordinary levelling instruments, or is arranged as a striding level to permit of its reversal without moving the telescope

The three foot screws are used only in the preliminary approximate levelling of the instrument, for which a circular spirit level or two small bubble tubes at right angles to each other are fitted. An important feature of precise levels is the provision under the telescope of a micrometer levelling screw whereby the telescope can be tilted about a horizontal axis. By rotation of this screw the main bubble is brought to the centre of its run for each observation, the verticality of the vertical axis being neglected. To enable the operator to maintain the bubble central while he is observing, or to inform him of its position at the instant of reading the staff, a reflecting device is essential This may consist of a mirror or an arrangement of prisms.

Two modern designs of precise level are selected for illustration and description.

The Binocular Precise Level.—The type of level designed by the United States Coast and Geodetic Survey embodies several admirable features, and marked a decided advance in precise levelling instruments when it was introduced in 1900

The instrument (Figs 70 and 71) is of the dumpy type with non-reversible bubble tube As made by various firms in America and England, the telescope objective has a focal length of 16 in to 18 in , with an aperture of about 1·7 in Two orthoscopic eyepieces are provided, the powers ranging from 25 to 32 and 40 to 50 respectively The level tube has a sensitiveness of from 1·2 to 2 sec per 2-mm division

To minimise errors arising from unequal temperature of the instrument, all parts influencing the constancy of the relationship between the line of sight and the level axis are made of nickel iron and nickel steel For the same reason, the bubble tube is placed as near as possible to the line of sight. The vertical axis is long, and carries the cylindrical tube *4*, in which the telescope is supported by the micrometer screw *5* and two nickel steel points *6* diametrically opposite each other. The telescope is pivoted about the latter by rotation of the micrometer screw. The annular spaces between the telescope and the ends of the supporting tube are closed by leather hoods *7*, which shut out air currents without affecting the action of the micrometer. The upper portion of the telescope and encasement is cut away opposite the level, and the outer tube is provided with a glass cover *9* for its protection.

To present to the observer a view of both ends of the bubble with the graduations in their vicinity, the mirror *10* reflects their images into two prisms fitted in a tube *11* alongside the telescope.

The distance between these prisms is adjustable to suit the distance between the ends of the bubble. The images are reflected along the tube by the prisms, and the optical arrangement is such that they are presented to the left eye of the observer at the normal

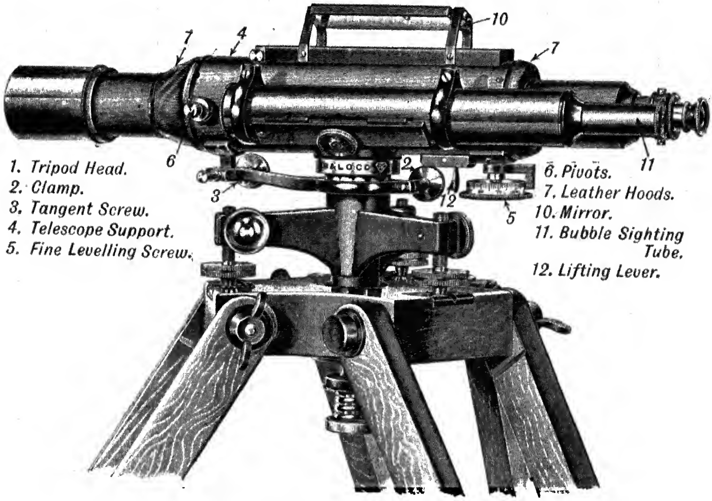


FIG. 70.—BINOCULAR PRECISE LEVEL.

distance of distinct vision. A lens suited to the requirements of an observer may be fitted at the eye end of the tube to adapt the normal distance to abnormal vision. The distance between the axes of the telescope and of the bubble-sighting tube is adjustable to suit the pupillary distance between the observer's eyes. A tall tripod is used so that the observer can stand upright, and, while observing,

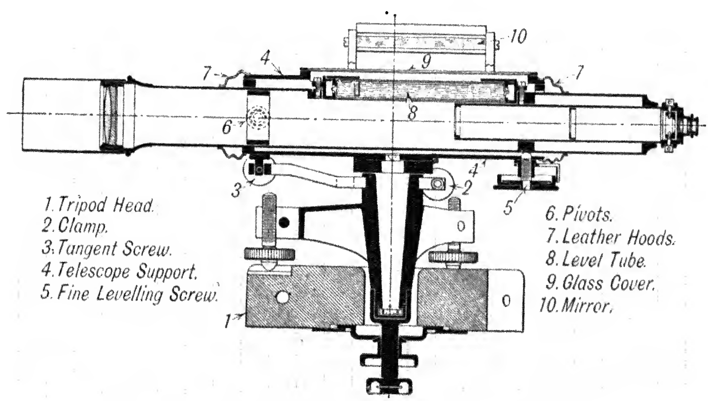


FIG. 71.—BINOCULAR PRECISE LEVEL—SECTION.

he has merely to examine the staff and bubble alternately without moving any part of the body.

The preliminary levelling is performed with reference to a circular level fitted on the right-hand side of the telescope, and the final levelling by means of the micrometer screw *5*, which has a pitch of 1/100 in. A cam with a lever handle is fitted at *12* for lifting the telescope off the micrometer in order to avoid possible disturbance of the adjustment during transport of the instrument

Adjustment of the Binocular Level.—The intersection of the vertical hair with the middle horizontal hair having been placed by the maker in the optical axis, the user has to attend to one adjustment only, *viz* to make the level tube axis parallel to the line of sight. The adjustment is made in the same way as for the engineer's dumpy level (Vol. I, page 95), the level tube having a vertical adjusting screw at the end next the eyepiece.

Exact adjustment is difficult, and, instead of attempting to eliminate very small errors, the error of parallelism may be determined daily and corrections applied to observed differences of elevation The United States Coast and Geodetic Survey routine in testing the instrument consists in driving two pins about 100 m apart and setting the instrument about 10 m beyond one The average of the readings of the three hairs is taken on a staff held on each point, and the instrument is then moved to about the same distance beyond the second point, and the readings are repeated. A bubble error constant, C, is evaluated from

$$C = \frac{\text{sum of near staff readings} - \text{sum of distant staff readings}}{\text{sum of distant staff intercepts} - \text{sum of near staff intercepts}}$$

For the best determinations the two distant staff readings are corrected for curvature and refraction. The level is not adjusted if C is less than ·005, and it is not advisable to adjust unless C exceeds ·01, the stadia ratio being 1/333 After an adjustment the value of C is at once redetermined If C is positive (negative), the line of sight is depressed (elevated) from the horizontal In deriving one elevation from another, the correction applicable to the observed result is C times the difference between the sum of the backsight staff intercepts and that of the foresights between them The correction is of the same sign as C if the backsight intercepts are in excess and of opposite sign if the foresights are in excess.

The Zeiss Level.—This modern example of a European level (Fig. 72) is that employed by the Ordnance Survey As in the pattern used for ordinary levelling (Vol. I, page 87), the most distinctive features are the method of mounting the telescope and level tube, the design of the telescope, and the arrangement for showing the observer whether the bubble is central while he is sighting

The level tube *5* is fixed on one side of the telescope. The metal tube carrying it is cut away both on top and bottom, and the whole is protected by being encased in a glass cylinder. The bubble is illuminated by means of the reflector *4*. The telescope with the

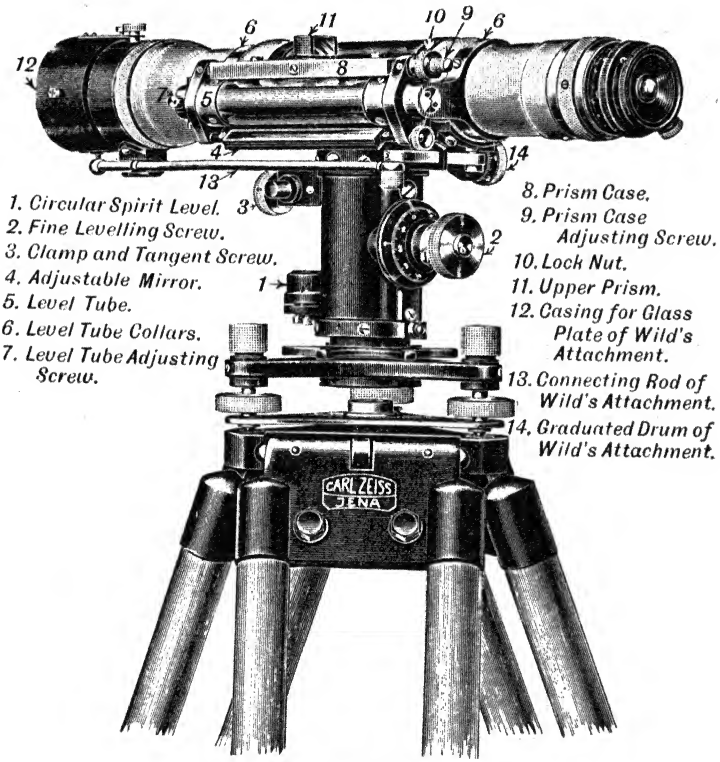


FIG. 72.—ZEISS PRECISE LEVEL.

level tube is capable of rotation through 180° about its own axis, so that the level can lie either on the left- or right-hand side. The level tube is not graduated, but the observer can tell when the bubble is central by means of a combination of prisms contained within the casing *8*, the view presented in the upper prism *11* being as illustrated in Vol. I, Fig. 83.

In place of estimating the position of the hairs between marks on the staff, the nearest graduation may be bisected by means of a device designed by Wild. It consists of a plate of parallel glass mounted on a horizontal axis and fitted in front of the object glass. The level bubble being maintained in the central position, inclination of the glass plate from the vertical displaces the image of the

staff by an amount depending upon the angle of tilt In taking an observation, the plate is turned by the screw *14* until the nearest graduation is brought to bisection by the hair, and the fractional part of a staff division is read on the drum The arrangement is successful in reducing errors of estimation of readings

Adjustment of the Zeiss Level.—In addition to the means for reversal of the telescope about its own axis, the telescope fittings are such that the ends can be reversed, the eyepiece being inserted in the objective cap. Observations can therefore be taken in four positions, *viz*

1, 2 Eyepiece direct, bubble tube on left (right) of telescope,
3, 4. Eyepiece reversed, bubble tube on left (right) of telescope.

The prism is reversed for positions 3 and 4

If from one position of the instrument a staff is read in each of these four ways, the mean of the readings is free from instrumental error The angular error for each position can therefore be determined, as well as the combined error of positions 1 and 2, which may be used in conjunction with each other for precise work To adjust for any position, it is only necessary to manipulate the fine levelling screw *2* to bring the horizontal hair to the mean of the four readings The images of the ends of the bubble are then made to coincide by releasing the locknut *10* and displacing the prism case *8* by the screw *9* To adjust for positions 1 and 2 together, the mean of the readings in those positions must be made to agree with the mean of the four test readings.

The bubble has provision for lateral adjustment to secure parallelism between the vertical planes containing its axis and that of the telescope respectively. The test and adjustment are performed as for the wye level (Vol I, page 98), and any error should be eliminated before proceeding to the principal adjustment.

Precise Staffs.—The various forms of staff employed are of the self-reading type, and are made in a single length of from 10 to 14 ft. To afford the necessary rigidity, the cross section is either rectangular with stiffening side pieces, tee-shaped, cruciform, or a hollow triangle The staff is usually made of strips of well-seasoned yellow pine, and, before being painted, is subjected to immersion in boiling paraffin or to other similar treatment in order to minimise the variation in length caused by change of humidity. Small metal plugs are sometimes inserted at intervals for use in determining the actual length Handles are usually provided, and to ensure steadiness it is desirable to have the staff fitted with two back legs hinged either at its top or at an intermediate point, so that in use the whole forms a rigid tripod Most types of staff have the base ending in a cylindrical pin of hard metal, the centre of which should lie in the plane of the graduation. Verticality is

determined by means of a circular spirit level, or two level tubes, attached at the back. The level adjustment is tested at frequent intervals by a plumb bob, for which a fitting is provided Temperature is measured on an attached thermometer

Metric graduation is almost universally adopted. The division must be sufficiently fine to enable readings to be made to the nearest millimetre, but the pattern must be bold enough to facilitate rapid reading and prevent mistakes The smallest division ranges from 1 mm to 1 cm

The staff recently adopted by the Ordnance Survey, and made by the Cambridge and Paul Instrument Co , is illustrated in Fig 73

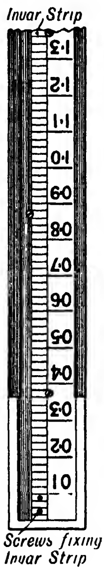


Fig 73 —Ordnance Survey Staff

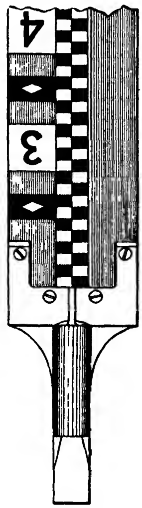


Fig 74 —U S Coast and Geodetic Survey Rod

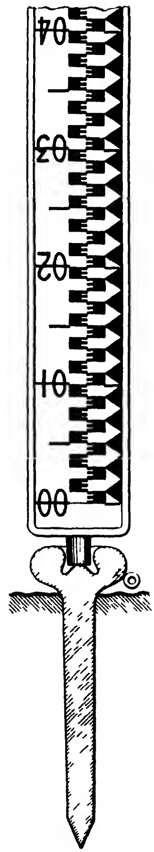


Fig. 75.—Prof. Molitor's Rod.

The length is 10 ft , and, to eliminate uncertainties arising from temperature and humidity changes, the graduation, to ·02 ft , is cut on a strip of invar This is fixed to the foot of the staff, but is otherwise free to slide in a groove in the woodwork without side play.

American designs are illustrated in Figs. 74 and 75 The former represents the U S Coast and Geodetic Survey rod, which is cruciform in section and carries a strip of nickel steel 10 ft 2 in. long. The rod is divided to cm , the graduation being marked on the edge of the cross. Fig. 75 shows the

rod designed by Prof. Molitor. It is tee-shaped in section, has a length of 3·5 m, and is divided to 2 mm

For the rigid support of a staff at change points either a foot-plate or pin is used. The former is a cast-iron disc of about 6 in diameter with spikes on the underside. A steel pin driven nearly flush with the ground is found to be more generally satisfactory, and a suitable form is that designed by Molitor and illustrated in Fig. 75.

General Methods of Field Work.—For the purpose of providing a check on all bench marks and of ascertaining the quality of every portion of a line of precise levels, the work is invariably duplicated. For the best results, the two lines must be run in opposite directions between the bench marks and preferably at different times of the day. If the discrepancy between the resulting differences of elevation exceeds a fixed amount, both lines are repeated until the required consistency is obtained. In the best work the allowable discrepancy varies from less than 3 mm $\sqrt{K}$ to about 4 mm $\sqrt{K}$, or ·012 ft $\sqrt{M}$ to ·017 ft $\sqrt{M}$, where K and M represent the distance between the bench marks in kilometres and miles respectively.

In place of running two entirely separate lines, simultaneous duplicate lines have been much used. These are less accurate, but can be usefully applied to refined engineering levelling when speed is important. The usual procedure consists in employing two staffs and having two sets of change points for each position of the instrument. The two lines are affected nearly equally by settlement or heaving of the instrument and by errors due to natural causes, so that the degree of consistency between them is not a good index of their real precision. A simultaneous duplicate line may with advantage be divided into sections run alternately in opposite directions.

Bench Marks.—The lines of a large system of precise levelling are of at least two classes—primary and subsidiary—and the bench marks established on them are similarly distinguished. Primary bench marks form the fundamental control points They are comparatively widely spaced, and are marked in a thoroughly permanent manner. In the revision of the primary levelling of Great Britain, commenced in 1911 by the Ordnance Survey, these fundamental marks are established at intervals of about 25 miles. Their sites are selected with reference to the geological structure so that they may be placed on sound strata clear of mining areas. They are placed at about 3 ft below the surface, and are embedded in the floor of a covered concrete chamber formed in the solid rock. The new secondary bench marks, about 1 mile apart, are on plates, which are fixed flush on vertical surfaces of walls, etc. with solid foundations. The tertiary marks, averaging about 400 yards apart, consist of copper plugs inserted in horizontal surfaces.

The areas within and between lines of primary levels are dealt with by subsidiary lines closing within shorter distances. The allowable discrepancy between forward and back lines may be greater than in the case of primary lines The general methods, however, are the same, since the instruments and methods of observation used in primary work are not prejudicial to good progress The interval between subsidiary bench marks depends upon local circumstances, and should be suited to the needs of the detail surveys A complete record must be made of the position and description of all bench marks.

Precautions in Running Lines.—In running a line of precise levels, the surveyor must consistently adhere to a definite routine of observation The routine depends upon the instrument used, and should be designed to reduce all possible errors to a minimum

To limit the effect of instrumental errors, the requirement of equality of backsight and foresight distances is all important These distances are given by the stadia hair readings, and a limit, of from 1 m to 10 m, is set to the maximum allowable difference between a backsight and the associated foresight It is especially necessary that the sum of the backsight distances should balance that of the foresights between benches The observed staff intercepts should therefore be kept summed by the recorder, and the total difference between the backsight and foresight intercepts should be maintained throughout within narrow limits and be balanced out on reaching a bench mark Error in the ascertained value of the sensitiveness of the level tube is eliminated by observing with the bubble central in preference to reading with the bubble off centre and applying corrections To minimise staff error, the length in terms of standard and the coefficient of expansion must be known. The former should be determined at least once a month and also after considerable changes of humidity. In some surveys the temperature of the staff is read at every change point The adjustment of the staff level must be tested daily.

Errors of reading are kept low by limiting the length of sight. The allowable distance depends upon the atmospheric conditions as well as the quality of the telescope and the sensitiveness of the level tube. On the Ordnance Survey the length of sight rarely exceeds 40 yd and never 50 In some surveys it exceeds 100 m under favourable conditions Readings are taken against the three horizontal hairs and averaged. Errors of estimation are thereby reduced, and comparison of the stadia hair readings with that of the centre hair affords a useful check on the sighting and booking.

To minimise errors caused by settlement or heaving of the instrument between observations, as little time as possible should elapse between them, and from every second station the foresight should be read before the backsight. Two staffs should be used to economise time, and therefore the same staff is always observed first after

setting up the instrument The precaution of alternately reading the backsight and foresight first would eliminate the effect of settlement or heaving if the same change always occurred between observations Errors caused by change of elevation of a change point, although they are unlikely to be serious when pins are used, are reduced by running the duplicate lines in opposite directions and distributing the discrepancy

Errors from natural causes may contribute largely to the closing error The instrument must be shielded from wind, and work is suspended when the vibration of the staff interferes with the estimation of readings Sources of error due to temperature are threefold . (*a*) variable atmospheric refraction , (*b*) irregular refraction near ground level , (*c*) unequal expansion and contraction of the instrument. The effect of change in the coefficient of refraction occurring between backsights and foresights is reduced by taking them in rapid succession Refraction changes most rapidly in the morning and evening, and is steadiest during the middle of the day (page 209), but, unfortunately, irregular refraction, which causes an apparent trembling of the staff, is then at its worst This atmospheric disturbance is frequently such as to necessitate a suspension of work during the midday hours, but its effects can be greatly reduced by shortening the length of sight and by avoiding readings near the ground Unequal heating of the instrument may give rise to serious error, and in sunshine it is essential to shade the instrument during both observation and transport

The effect of curvature and refraction is generally very small because of the balancing of the backsight and foresight distances When an unusually long sight is unavoidable, as at a river crossing, reciprocal levelling (Vol I, page 199) is preferable to equalising the sum of the backsight and foresight distances later, probably under different atmospheric conditions In the most refined method of reciprocal levelling, possible error due to change of refraction while the instrument is being taken across the river is eliminated by simultaneous observations from both banks Two instruments are used, and the two staffs are read from each, the long sights being simultaneous. The positions of the instruments are then interchanged, and the observations are repeated The mean of the four differences of level so obtained is the required difference

Note-keeping.—The columnar arrangement of the field book may take various forms, but at least the following quantities are tabulated in five columns for the backsights and five for the foresights.

1. The readings of the three hairs.
2. Their mean.
3. The two partial staff intercepts and their total.
4. The sum of the whole intercepts up to each observation.
5. The number of the staff observed.

The record also includes, either in tabular form or as remarks, a note of the bench marks between which levelling is being conducted, whether the line is the forward or backward one, the hour of day, the staff temperatures, the state of the weather as regards sunshine, wind, and clearness of atmosphere, and the direction of the line relatively to the sun and wind The respective sums of the backsight and foresight mean readings between bench marks are brought out, their difference being the observed difference of elevation.

Computation of Levels.—Differences of elevation shown in the field book are only approximate since they are subject to various corrections. To facilitate checking, corrections are best set out and applied in tabular form in a book into which are transferred from the field book the quantities required for computation These include, for both the forward and backward lines, sums of backsights and foresights between bench marks, sums of staff intercepts for backsights and foresights, average staff temperatures, state of weather, and direction of line The corrections to the approximate difference of elevation are for (1) instrumental error, (2) staff length and temperature, (3) curvature and refraction, (4) orthometric elevation.

(1) Constant instrumental error, consisting of collimation error and other errors peculiar to particular types of instrument, is corrected by multiplying the difference between the sums of the backsight and foresight staff intercepts, or the corresponding actual distance, by the appropriate constant

(2) The staff correction to standard and for temperature is proportional to the observed difference of elevation, the temperature correction being derived with sufficient accuracy from the average temperature during the observations between bench marks

(3) The curvature and refraction correction is applicable only to the difference between the sums of the backsight and foresight distances, and this should be so small that an average value of the refraction coefficient may safely be adopted.

(4) The orthometric elevation of a point above mean sea level is the length of the vertical between that surface and the point. Its dynamic elevation is measured by the work required to raise unit mass from mean sea level to the point. A line of constant orthometric elevation is parallel to the mean sea level surface, but, because of the variation of gravity with latitude (page 134), a line of constant dynamic elevation is parallel to mean sea level only when it lies along a parallel of latitude.

The free surface of a still liquid is dynamically level Elevations obtained by spirit levelling are therefore dependent upon gravity, and require correction to transform them to orthometric heights. The amount of the correction becomes appreciable for lines at a

considerable height above sea level and lying roughly north and south. Its value in millimetres is approximately given by

$$c = 0.000832\, HK \sin 2\phi,$$

where H = mean elevation in metres of the two points above mean sea level,
K = meridional distance between them in kilometres,
ϕ = mid-latitude of the two points.

The correction is additive (subtractive) to the dynamic elevation of a point north (south) of that from which its elevation is derived. This correction takes no account of local deviations in the direction of gravity caused by irregularity of the distribution of mass in the earth's crust (page 182).

Adjustment of Observations.—Comparison of the results given by the forward and the backward lines shows whether the discrepancy is within the allowable limit. If so, the mean of the two values for a bench mark elevation is accepted, subject to adjustment for the closing error of the line on which it is situated.

The final check is furnished by having the lines of levels arranged to form circuits closing on themselves or on different tide gauges, or by running the levels between points on different circuits. The closing error of a circuit includes both cumulative and compensating errors, and is eliminated by distributing corrections to the intermediate bench marks proportional to their distances along it. If the precision of the work is considered unequal over the circuit, weights are assigned to its different parts, and the discrepancy is then distributed proportionately to the weighted distances from the origin. When more than two similar lines of levels connect two points, the elevation of the second point as derived from the first is given by the weighted mean of the results, each result being weighted inversely as the length of the route followed in obtaining it.

In the usual case, level circuits interlock with each other, and form a network. Large nets are adjusted by the method of least squares, but the existence of constant errors must be recognised. In most cases it is quite sufficient to adjust by distributing the closing error in each circuit separately on the distance basis with or without the application of weights. The circuits containing the largest closing errors are first adjusted, and points which have undergone correction are not altered in adjusting the remaining circuits. When the most probable differences of elevation along the lines have been thus determined, the elevations are obtained from that of the origin.

Accuracy of Precise Levelling.—A satisfactory criterion of the accuracy of precise levelling is difficult to obtain, since constant errors are likely to be as serious as accidental errors. The closing error of large circuits, considered in conjunction with the limiting discrepancy allowed between forward and back lines, forms a good

index, but is not generally suitable for comparisons. A standard of comparison is usually based on the amount of accidental error only, and consists either of the limit of discrepancy allowed between duplicate lines or the probable error derived from the discrepancies observed.

A statement of the value of the probable error or discrepancy is useless unless the distance is known. Since a difference of elevation is the algebraic sum of the differences between successive change points, its probable error equals the square root of the sum of the squares of the probable errors of these individual differences. The distance between change points being sensibly uniform, it follows that the probable error of levelling is proportional to the square root of the distance, or

$$\text{p e.} = u\sqrt{L},$$

where u is the p e in unit distance (kilometre or mile) By means of this relationship the quality of a particular piece of work may be assessed in terms of u

In general, only the backward and forward measurements of a difference of elevation are available, and these are insufficient to indicate the probable error of the mean The formula for probable error (page 169) is sometimes conventionally applied, but since the residuals have each the value $\frac{1}{2}d$, where d is the discrepancy, the probable error of the arithmetic mean reduces to 0 337d, and does not afford any better index of the precision than is given by the discrepancy itself

Probable error can, however, be derived either from numerous measurements of the same difference of elevation or from the observed discrepancies of several duplicate lines, not necessarily of the same length The latter method is the more useful, since the several duplicate lines are those between adjacent benches and no additional field work is required, while a measure of the quality of the actual work is obtained. In this case,

let d_1, d_2, etc. = individual discrepancies of duplicate lines,
l_1, l_2, etc. = their lengths one way,

$$\Sigma wd^2 = \frac{d_1{}^2}{l_1} + \frac{d_2{}^2}{l_2} + \ldots,$$

n = number of lines.

Then the p e of the arithmetic mean of the two results given by a duplicate line of unit length

$$= u = 0{\cdot}6745\sqrt{\frac{\Sigma wd^2}{4n}}.$$

If the lines are of constant length, l, then

$$u = 0{\cdot}6745\sqrt{\frac{\Sigma d^2}{4nl}} = 0{\cdot}6745\sqrt{\frac{\Sigma d^2}{4L}}.$$

The latter formula is commonly used, and is quite accurate enough when the discrepancies are those developed between benches at roughly equal intervals.

Example—The discrepancies observed between the forward and back lines between benches established at the given distances apart are as tabulated. Find the probable error of the duplicate line per kilometre and in the total length of the line.

Discrepancy mm	Distance between Marks kilom.	w.	d^2.	ud^2.
3 1	76	$\frac{1}{76}$	9 61	12 64
2 8	·80	$\frac{1}{80}$	7 84	9 80
2 7	70	$\frac{1}{70}$	7 29	10 41
3 5	82	$\frac{1}{82}$	12 25	14 94
1 0	·58	$\frac{1}{58}$	1 00	1 72
3 2	74	$\frac{1}{74}$	10 24	13 84
2 4	·73	$\frac{1}{73}$	5·76	7 89
2 9	69	$\frac{1}{69}$	8 41	12 19
$n=8$	$L=5$ 82		$\Sigma d^2=62$ 40	$\Sigma ud^2=83$ 43

Allowing for variation of length of sections, the p e of mean per kilometre,

$$u = 0\ 67\sqrt{\frac{\Sigma wd^2}{4n}} = \pm 1\ 1 \text{ mm}$$

Neglecting do ,

$$u = 0\ 67\sqrt{\frac{\Sigma d^2}{4L}} = \pm 1{\cdot}1 \text{ mm}$$

$$\text{p e of line} = 1\ 1\sqrt{L} = \pm 1\ 1\sqrt{5\ 82} = \pm 2\ 7 \text{ mm}$$

The standard fixed by the International Geodetic Association for levelling of high precision is

$$\text{p e} = 1 \text{ mm. } \sqrt{K},$$

which is equivalent to ·004 ft $\sqrt{M}$, where K and M represent the distance in kilometres and miles respectively This degree of accuracy is frequently exceeded in modern work In the case of the revision of the primary levelling of Great Britain, the probable error is less than half the above.

TRIGONOMETRICAL LEVELLING

Angle Measurement.—Refinement in angular observation is necessary in order to obtain satisfactory results with long sights For the best work the theodolite should have the vertical circle fitted with micrometers reading to single seconds and by estimation to tenths,

and the angular value of a division of the bubble tube attached to the micrometer arm should not exceed 2 sec. The instrument known as the vertical circle has been extensively used in trigonometrical levelling It possesses all the features of the vertical angle measuring part of the theodolite without those for horizontal measurement, and may have the circle of repeating or non-repeating pattern

The record of an observation must include the height of the instrument axis above the ground point and the height of the observed point above the distant station, in order that, by application of the eye and object correction, the difference of elevation between the station marks may be determined. Each observation is made face right and face left, and is accompanied by readings of the position of the bubble for application of the level correction. At least two such observations constitute a measurement. The pointings may be made by means of the eyepiece micrometer. If the several angles to be measured at a station are so nearly equal that their differences are within the range of the eyepiece micrometer, these differences may be observed micrometrically with reference to one or more points, the elevation of which is either known or is measured in the usual way. In the case of important determinations, one or more measurements are made each day during the tenancy of the instrument station for triangulation. For points of minor importance, a single observation, face right and face left, is all that is required. Each elevation should be determined by measurements from at least two points, and if observations have to be made from a satellite station, the results must be corrected for eccentricity.*

Refraction.—Owing to the curvature of the line of sight caused by atmospheric refraction, an observed vertical angle differs from the true angle which the straight line joining the instrument to the signal makes with the horizontal through the instrument. The chief source of error in trigonometrical levelling is that arising from uncertainty regarding the amount of refraction. The uncertainty is greater in the case of terrestrial refraction, with which we are here concerned, than for celestial or astronomical refraction, since the sights of trigonometrical levelling are never greatly inclined to the horizontal

The usual way of dealing with the question is by use of the coefficient of refraction. This is defined as the ratio between the refraction angle, or angle at the instrument between the ideal straight line of sight and the actual curved line of sight, and the angle which the two stations subtend at the centre of the earth. The line of sight is assumed to be a circular arc, and the coefficient of refraction to be independent of its elevation or inclination, but these assumptions are not strictly correct Mr. Hunter, of the

* See Gillespie, *Surveying*, Part II.

Survey of India,* has shown that refraction depends very largely on the rate at which the temperature changes with height and with the change of this rate, and that it also depends on the height above the horizontal plane through the observing station to which the ray extends. Thus, the refraction on a ray of given length differs according as the ray is ascending or descending.

It is well known that refraction varies in amount throughout the day. It is greatest in the morning and evening, and least and steadiest during the middle of the day. Its variation from day to day is found to be less during the period of minimum refraction than at other hours Vertical angle observations should be confined to this period, the duration of which, however, varies in different parts of the earth, and should be investigated in important surveys by preliminary reciprocal observations Generally speaking, the period from noon to 3 p m is the best, but in some cases good observations are possible from about 10 a m to 4 p m

The value of the coefficient of refraction to be used in reducing observations is its average minimum value for the district, and should, if possible, be determined by simultaneous observation of reciprocal angles. Otherwise its value must be estimated from the results obtained under similar conditions For the British Isles, Col. Clarke obtained mean values of 0 0750 and 0 0809 for sights over land and sea respectively † The average values obtained by the United States Coast and Geodetic Survey range from 0·065 for the interior to 0·078 for lines crossing parts of the sea ‡ A mean value of 0·065 was found in the measurement of the Uganda arc, and of 0·060 for observations to the highest peaks §

Methods of Trigonometrical Levelling.—Two general methods are employed to obtain the difference of elevation of two points of known distance apart In the first, it is determined from vertical angles observed from each station to the other. The object of such reciprocal observations is to remove the effect of uncertainty regarding the value of the coefficient of refraction. The angle of refraction is taken as being the same for both stations, and is eliminated in the reduction. For the best results the reciprocal observations should be taken simultaneously. Owing to the difficulty of arranging for simultaneous observations, the measurements are sometimes made at the time of minimum refraction on different days, but with less accurate results.

In the second method, the difference of elevation is computed from the vertical angle measured at one of the stations only, and a

* See "Formulæ for Atmospheric Refraction and their Application to Terrestrial Refraction and Geodesy," by J. de Graaf Hunter Survey of India, Professional Paper, No 14, 1913

† Ordnance Survey, "Account of the Principal Triangulation," 1858.

‡ United States Coast and Geodetic Survey Report, 1882, Appendix No 9.

§ "Report of the Measurement of an Arc of Meridian in Uganda" Colonial Survey Committee, 1913.

knowledge of the value of the refraction coefficient is required. This method is available for determining the elevation of the station occupied by observation to a point of known elevation. It must be used for determining the elevations of inaccessible and intersected points, and is the method employed in the bulk of trigonometrical levelling.

Notation.—In developing reduction formulæ, it will be supposed that it is required to determine the elevation of a station B from that of a station A, assumed known The notation adopted is as follows :

h_1 = elevation of A above M.S L.,
h_2 = ,, B ,, ,, ;
D = geodetic or M S L distance between A and B,
R = radius of the earth at the mid-latitude of A and B,
θ = angle subtended at the centre of the earth by D.
a = observed vertical angle from A to B, corrected in reciprocal observations for the difference in height of the instrument and the signal above the ground,
b = ,, from B to A ,, ,
k = coefficient of refraction, $k\theta$ being the angle of refraction,
c = angular eye and object correction for reciprocal observations,
i = height of the instrument above the ground,
s = height of the point sighted above the ground.

Reciprocal Observations. Eye and Object Correction.—In trigonometrical levelling generally, it usually happens that the height above the ground, or above the station mark, at which the angle is observed is different from that of the signal The difference of elevation obtained from an observation at one end of the line is that between the instrument and the point sighted, and is finally corrected for their difference of height above the ground. In the case of reciprocal observations, however, when the point observed to at each station is not that from which the observations are made, the observed angles must be reduced to true reciprocity before they are used in computing the difference of elevation The amount of this angular correction to the observed angles may be deduced as follows.

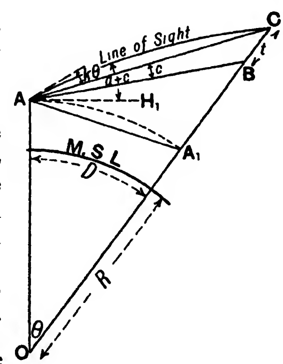


FIG. 76.

In Fig. 76, let A represent the centre of the instrument at one station. Let B at the other station represent a point at the same height above the ground point as A, and let C, at a distance t, = $(s-i)$, from B, be the point sighted. It is required to obtain the value of the angle c

subtended by t at A. Mark A_1 on OC at the same elevation above mean sea level as A, and join AA_1. AH_1 is the horizontal at A, and $(a+c)$ is the angle of elevation actually observed at A.

In triangle ABC, $\dfrac{\sin c}{\sin ABC} = \dfrac{t}{AC}$,

but $ABC = AOB + OAB = 90° + a + \theta(1-k)$,

and $AC = AA_1 \dfrac{\sin AA_1C}{\sin ACA_1} = \dfrac{AA_1 \sin (90° + \frac{1}{2}\theta)}{\sin (90° - a - c - \theta(1-k))}$,

$$\therefore \sin c = \frac{t \sin (90° + a + \theta(1-k)) \cdot \sin (90° - a - c - \theta(1-k))}{AA_1 \sin (90° + \frac{1}{2}\theta)}$$

Since c is a very small angle, we may write

$$\sin c = \frac{t \cos^2 (a + c + \theta(1-k))}{AA_1 \cos\frac{1}{2}\theta},$$

where $AA_1 = 2(R + h_1) \sin\frac{1}{2}\theta$.

But, with sufficient accuracy for the purpose, D may be substituted for AA_1, and since θ is small, we have, with a smaller error than that of refraction,

$$\sin c = \frac{t \cos^2 (a+c)}{D},$$

$$\text{or } c'' = \frac{t \cos^2 (a+c)}{D \sin 1''} \quad \ldots\ldots \qquad (1)$$

When the observed angle $(a+c)$ is very small, it is sufficient to take

$$c'' = \frac{t}{D \sin 1''}, \quad \ldots\ldots \qquad (2)$$

which is the more commonly used formula.

The angle observed at B is similarly corrected for the corresponding value of t, attention being paid to the signs of the corrections.

Difference of Elevation from Reciprocal Observations.—Two cases may occur: (1) the difference in elevation of the two points may be sufficiently great that one of the observed angles is an elevation and the other a depression, (2) the difference in elevation may be small enough in relation to the distance between the points that both angles are depressions.

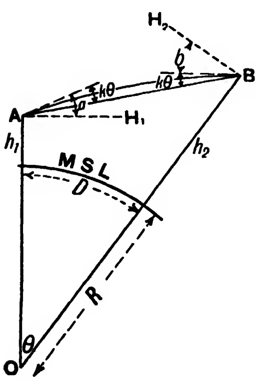


FIG. 77.

Case (1).—Fig. 77 illustrates the case in which B, the point of unknown elevation, is higher than A. The angles a and b are the observed angles corrected for difference in height of eye and object, and may be treated as the angles which would be observed at and to A and B, the respective ground points.

The required difference of elevation $(h_2 - h_1)$ is obtained by solving triangle AOB, of which

the known quantities are OA $= (R+h_1)$, AOB $= \theta = D/R$ in circular measure, and (OAB$-$OBA) $= (90°+a-k\theta)-(90°-b-k\theta) = (a+b)$. Solving, we have

$$\tan\frac{a+b}{2} = \frac{(R+h_2)-(R+h_1)}{(R+h_2)+(R+h_1)}\cot\frac{\theta}{2},$$

$$\text{or } (h_2-h_1) = \tan\frac{a+b}{2}\cdot\tan\frac{\theta}{2}\cdot(2R+h_1+h_2)$$

Substituting for $\tan\frac{\theta}{2}$ its expansion, $\frac{D}{2R}+\frac{D^3}{24R^3}-\quad\ldots,$

$$(h_2-h_1) = D\tan\frac{a+b}{2}\left(1+\frac{(h_1+h_2)}{2R}+\frac{D^2}{12R^2}\right) \qquad (3)$$

In applying this formula, a first approximation is derived from $(h_2-h_1) = D\tan\frac{a+b}{2}$, and the value of h_2 so obtained is used for a second approximation by the formula

An alternative method of solution consists in first obtaining the true vertical angles H_1AB and H_2BA by eliminating the refraction angles

$H_1AB = (a-k\theta)$, and $H_2BA = (b+k\theta)$,
but $H_2BA - H_1AB = \theta$,

$$k\theta = \frac{\theta-b+a}{2} \qquad (4)$$

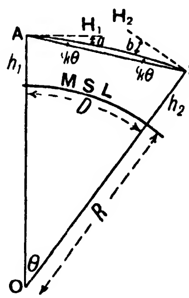


FIG 78.

Let the values of H_1AB and H_2BA so obtained be denoted by a' and b'. Then, solving triangle AOB, we have

$$\frac{R+h_2}{R+h_1} = \frac{\sin(90°+a')}{\sin(90°-b')},$$

$$\text{whence } (h_2-h_1) = (R+h_1)\frac{\cos a'-\cos b'}{\cos b'},$$

$$= (R_1+h_1)\left(\frac{\cos a'}{\cos b'}-1\right) \quad \ldots\ldots \ldots\ldots \ldots (5)$$

Case (2).—(Fig 78) For the first method of solution, (OAB$-$OBA) is now $= (b-a)$, and

$$(h_2-h_1) = D\tan\frac{b-a}{2}\left(1+\frac{(h_1+h_2)}{2}+\frac{D^2}{12R^2}\right) \quad \ldots\ldots\ldots (6)$$

In the second method, $(H_1AB+H_2BA) = \theta$, so that

$$k\theta = \frac{\theta-a-b}{2}, \quad \ldots\ldots\ldots\ldots\ldots (7)$$

and the expression for (h_2-h_1) is the same as before.

Difference of Elevation from Observations at One Station.—In the case where observations are taken at one end of the line only, a value must be assigned to k. A likely value may be assumed, but

it is preferable to determine it by means of equation (4) or (7) from simultaneous reciprocal observations taken in the same locality. Otherwise the coefficient may be deduced from an observation to a point of known elevation.

Let A (Fig. 77) be the station occupied, and let a be the uncorrected angle of elevation observed to B The previous formulæ are applicable if the corresponding angle b at B is deduced. From (4) or (7),

$$b=a+\theta-2k\theta,$$

a being positive for an elevation and negative for a depression: a negative value of b indicates an angle of elevation at B. The value of b so deduced includes all errors of observation and in the assumed value for k. On substituting this value of b, we have, corresponding to (3) and (6),

$$(h_2-h_1) = D \tan \left(a+(\tfrac{1}{2}-k)\theta\right) \left(1+\frac{(h_1+h_2)}{2R}+\frac{D^2}{12R^2}\right), \quad \ldots\ldots \ldots\ldots(8)$$

in which a is given a negative sign if a depression. The $\frac{D^2}{12R^2}$ term may safely be omitted, and $\frac{(h_1+h_2)}{2R}$ may be treated as negligible, in rough determinations.

For the second method of solution we have $b' = (a'+\theta)$, a' being given the negative sign for a depression, and

$$\ldots (h_2-h_1) = (R+h_1)\left(\frac{\cos a'}{\cos (a'+\theta)}-1\right) \quad \ldots\ldots\ldots\ldots \quad \ldots (9)$$

Eye and Object Correction.—When reciprocal observations are not taken, (h_2-h_1) represents the difference of elevation between the instrument axis and the point sighted, and must be corrected to give the difference of elevation between the ground points. If i and s denote the respective heights of instrument and signal (if any) above the ground, the required difference of elevation

$$= (h_2-s)-(h_1-i) = (h_2-h_1)+(i-s).$$

Rough Determinations.—When, as in plane tabling, the angular observations are not made with a high degree of precision, the reductions are commonly made with the aid of a table of curvature and refraction (page 108) The value of the correction is $\frac{D^2}{2R}(1-2k) = \cdot574D^2$ ft. for $k = 0\cdot07$ and D in miles. A rough value for (h_2-h_1) in ft. is therefore given by

$$(h_2-h_1) = 5280\ D \tan a+\cdot574D^2, \text{ where } D \text{ is in miles};$$

or, when a is small, $\tan a = a'' \tan 1'' = \cdot000004848\ a''$, and

$$(h_2-h_1) = \cdot0256Da''+\cdot574D^2.$$

Accuracy of Trigonometrical Levelling.—The probable error computed from the differences of elevation given by repeated vertical angle observations is not likely to be a trustworthy index

of the precision of a result Even in simultaneous reciprocal observations, constant errors are introduced in the assumptions made regarding the effect of refraction. The quality of a system of trigonometrical levelling is ascertained by connecting it at intervals to lines of precise levelling or by reference to its own errors of closure In the usual case where the stations of the main system of trigonometrical levelling are those of the triangulation, triangular circuits are formed, the closing error of each of which is known. These elementary triangles, however, depend upon each other, and are combined to form quadrilaterals, etc , so that the most probable corrections to the observed differences of elevation must be such as to satisfy the conditional equations for closure of each part. Further conditions may arise if the elevations of some of the points have been determined by precise levelling Observed differences of elevation are weighted inversely as the length of sight.

The computed probable error of a difference of elevation, measured under favourable conditions by means of reciprocal, but not necessarily simultaneous, observations, should not exceed about $0{\cdot}1$ to $0{\cdot}3\sqrt{D}$ ft. for sights of from 5 to 20 miles and 0 2 to $0{\cdot}6\sqrt{D}$ ft for sights of over 20 miles, where D is in miles.

REFERENCES ON GEODETIC LEVELLING

AVERS, H. G , and COWIE, G. D "Precise Leveling from Reno to Las Vegas, Nev , and from Tonopah Junction, Nev , to Laws, Cal " U S Coast and Geodetic Survey, Special Publication No 39, 1916.

BAGNALL, G " Long Distance Levelling " *Min Proc Inst C E* , Vol CXXI

" Bench Levelling for the Board of Water Supply, New York " *Eng Record,* Vol 58, No 14, 1908

BLEICH, S D " Adjustment of Bench Levels " *Eng Record,* Vol 54, No 18, 1906.

BOOCK, C F V "The Main Levelling of Copenhagen, 1907-8 " *The Surveyor and Municipal and County Engineer,* Vol XLI, 1912.

BOWIE, W "Precise Leveling from Brigham, Utah, to San Francisco, California." U S Coast and Geodetic Survey, Special Publication No 22, 1914.

—" Innovations in Precise Leveling Methods in Coast Survey." *Eng. News,* Vol 76, No 2, 1916

BURRARD, S G. *Account of the Operations of the Great Trigonometrical Survey of India,* Vol XIX, *Levelling of Precision in India* (1858–1909). Dehra Dūn, 1910.

COLE, J H "Systematic Error in Spirit Levelling" Survey of Egypt, Departmental Paper No 35, 1919

CROSTHWAIT, H. L. " Levelling Across the Ganges at Damukdia " Survey of India, Prof Paper No 7, 1903.

DOUGLAS, E. M " Experience with the Prism Level on the U S. Geological Survey." *Eng News,* Vol 55, No 20, 1906

ECCLES, J. " Experiment to test the Changes in the Length of a Levelling Staff due to Moisture and Temperature." Survey of India, Prof. Paper No. 7, 1903.

FISCHER, E. G "Description of Precise Levels Nos 7 and 8" U.S Coast and Geodetic Survey, Report for 1900, Appendix No 6

— "Improved Level Rod." *Eng News*, Vol 76, No 19, 1916.

HARDESTY, W. P. "Precise-Level Survey of the City of Portland, Ore." *Eng. News*, Vol. 76, No 2, 1916

HENRICI, E O. "Precise Levelling" *Min Proc Inst C E* Vol CCIX.

HUNTER, J. DE G "Formulæ for Atmospheric Refraction and their Application to Terrestrial Refraction and Geodesy." Survey of India, Prof Paper No. 14, 1913

HUSBAND, J. "The Zeiss Level" *Trans Inst. M.E*, Vol. XLVI, Part 2

KOOP, F. W. "Precise Leveling in New York City." *Eng News*, Vol 70, No 10, 1913.

MITCHELL, H. C. "Recent Work with the Precise Level Instrument of the U S. Coast and Geodetic Survey" *Eng News*, Vol 65, No 12, 1911.

MOLITOR, D. A. "The Theory and Practice of Precise Spirit Levelling." *Trans Am Soc. C E*, Vol XLV

"Report of the Measurement of an Arc of Meridian in Uganda," Vol I. Colonial Office, 1913.

Reports of the Progress of the Ordnance Survey (Annual), from 1912

SHAW, H. G. "Observation of Atmospheric Refraction, 1905–09." Survey of India, Prof Paper No 11, 1911

WALLACE, J N. "Report on Levelling Operations from their Inauguration in 1908 to October 31, 1914." Department of the Interior, Ottawa, 1915

CHAPTER VI

TOPOGRAPHICAL AND RECONNAISSANCE SURVEYING

Topographical Surveys.—Topographical surveys have for their object the preparation of maps as complete in detail as the scale will allow They are distinct from cadastral surveys, the primary object of which is the location and representation of boundaries and the measurement of areas of land National topographical surveys are undertaken for the production of maps which, while on smaller scales than those of cadastral surveys, will fulfil the various purposes, civil and military, for which reference to general maps is required

The representation of relief is an essential feature of a topographical map, and to the engineer such maps are of the greatest utility as an aid to the location of railways, roads, canals, reservoirs, pipe lines, etc In the absence of existing maps he must employ the methods of topographical surveying in location surveys Methods applicable to the production of large scale contoured plans of small areas have been dealt with in Vol. I, Chap VI In the following pages are considered the field operations required for the preparation of maps of large areas on smaller scales.

General Features of Topographical Surveys.—To prevent excessive accumulation of error, a topographical survey must be based upon a system of horizontal control points located with such precision that their errors of position cannot be detected on the scale of the map. Between these points detail surveys are conducted by methods suited to the character of the country and with a standard of accuracy so related to the distance between their control points that the errors in location of detail may also be negligible The most rigid framework is available if the country has been covered by triangulation of geodetic standard Triangulation affords much the best control, but the principal system need not necessarily attain primary precision, and may be of a much lower standard when the mapping is to be executed only on a small scale. In flat and densely wooded country, for which triangulation is unsuitable, a framework of primary traverse must take its place.

In a similar manner, the survey of relief should be based upon a framework of precise spirit levelling. The distances between the bench marks established must be such that the less refined depen-

dent work may not develop errors sufficient to affect appreciably the location of the contours. The methods to be adopted both in the precise and the subsidiary levelling depend not only upon the scale but also upon the character of the country. In regions characterised by flat slopes a given error of elevation will produce a more serious displacement of the contours than in rugged country, and a higher grade of work is required.

The scale adopted for a general topographical map depends not only upon the physical character of the country, but in greater degree upon its state of development and wealth. The purposes for which the map will be used are, however, so varied that it is impossible to select a scale to suit them all, and a wide range—between 1/25,000 and about 1/125,000—has been adopted for what may properly be classed as topographical maps The more important nations possess topographical maps on more than one scale: those of the Ordnance Survey are 1/63,360 and 1/126,720, or one inch and half-inch to a mile For topographical surveys in connection with engineering schemes the requirements as to scale depend upon the nature of the work. The scale adopted is commonly larger than the above owing to the necessity for the inclusion of what would in general mapping be regarded as minor topographic features.

Geographical Surveys.—Where the expense of a deliberate topographical survey is not warranted, the survey may be executed with a view to reproduction on a smaller scale Surveys on scales of less than about half-inch to one mile are sometimes distinguished as geographical, but no hard and fast distinction can be drawn between topographical and geographical surveys, since both aim at the production of maps as accurate as the scale will allow.

Exploratory and Reconnaissance Surveys.—As the name implies, an exploratory survey is one made to record the geography of the country passed through by an exploring party. The same methods are applicable to rapid reconnaissance executed as an aid to the planning of deliberate surveys, as well as to military reconnaissance conducted under active service conditions. They are employed by the civil engineer for preliminary location surveys and in connection with reports on proposed schemes A high grade of work is not expected, since only the most rapid methods are available and the most portable instruments are used. The scale is usually small, and may be as low as 1/1,000,000.

For locating a system of points to control the sketching, triangulation is to be preferred, even although it is of low grade, and time-saving methods are freely used. If the survey can proceed from or terminate on stations of an existing survey, difficulties of base measurement are removed, but, even so, triangulation may be impracticable on account of the limited time available or the nature of the ground. In this case, a system of points may be located by

astronomical observations, or a rough traverse is conducted along the route followed. The latter method has been greatly favoured by explorers because of its rapidity Route traversing, however, is of low precision, and errors must be kept in check and adjusted at intervals by means of observations for latitude and azimuth, and, if the route lies east and west, for longitude difference. A sufficient number of elevations to control the sketching of relief are obtained by trigonometrical levelling if a theodolite is carried. Otherwise, the barometer or the boiling-point thermometer, or both, are employed.

Methods.—In the absence of an existing framework for the ultimate control of a topographical or geographical survey, the principal factors which influence the planning of the work are the nature of the country and the accuracy required Reconnaissance will reveal whether triangulation can be economically conducted over the whole area or whether primary traverse may have to be substituted in parts. The substitution of traversing for triangulation as primary control is justifiable only in densely wooded or flat country, where the latter would involve unreasonable delay and expense It is decidedly inferior to good triangulation because of the greater number of directions observed and the limitation in the choice of ground over which linear measurements are made. The question of accuracy is, however, largely dependent upon the scale of the mapping No greater refinement is warranted than is necessitated by the requirement that the errors will not show in scaling

There is a considerable choice of methods for the survey of detail, and experience is required to enable the topographer to select that best suited to a particular case. On the whole, the plane table is the most useful instrument, the advantage of being able to sketch the topography with the ground in view being universally recognised. Whatever system is adopted for the survey of relief, the sketching of topographic form is a highly important feature of the work, and the topographer must be qualified to produce a good representation of the characteristic physical features of the country and to eliminate the unessential. In exploratory surveys only the most prominent features can be recorded, and the sketching is controlled by fewer points.

The methods in use for the survey of the framework and detail may be summarised as follows :

Class of Survey	Topographical and Geographical	Exploratory and Reconnaissance
Horizontal Control	Triangulation Primary, Secondary, Tertiary, or Topographical Triangulation of Different Grades Primary Traverse	Rapid Theodolite Triangulation Plane Table Triangulation Latitude and Azimuth Traverse Astronomical Positions
Horizontal Detail	Plane Tabling Resection, Intersection, Traversing Secondary Theodolite Traverse Tacheometry Compass Traverse Photographic Surveying Sketching	Plane Tabling Resection Route Traverse Sketching
Vertical Control	Precise Spirit Levelling Trigonometrical Levelling	Trigonometrical Levelling Barometric Levelling
Vertical Detail	Spirit Levelling Trigonometrical Levelling by Theodolite, Tacheometer, or Plane Table Clinometer Barometric Levelling Photographic Surveying Sketching	Barometric Levelling Sketching

HORIZONTAL CONTROL

Topographical and Geographical Triangulation.—Although the refinements of primary triangulation are not required when the results are to be applied only to mapping, the methods employed for the principal control of deliberate topographical surveys of extended areas are very similar, particularly if it is necessary to produce maps of certain districts on larger scales than the general scale adopted On the other hand, considerable deviation from the rigorous methods of primary triangulation is allowable in work to be mapped on a geographical scale only. For any survey, standards of accuracy must be set for the various operations, and rules are formulated to ensure consistency in the quality of the work of different field parties.

In the highest class of topographical survey, the conduct of the preliminary operations of reconnaissance, signal building, and station marking is the same as that described under geodetic survey-

ing.' The erection of high towers is, however, avoided, even at the expense of introducing figures not as well-conditioned as would be required in primary triangulation. If the work is not connected to an existing triangulation system of suitable precision, a base, at least a mile long, must be measured with an accuracy represented by a probable error of 1/100,000 to 1/250,000, the latter for the most extended work. This degree of precision may be attained with less labour by the use of invar tapes or wires than by steel tapes, but either are used. If the catenary apparatus (page 125) is not employed, the tape is suspended in catenary between mark posts with copper or zinc strips to receive the scratches, a constant tension being applied by a tape stretcher through a spring balance The base must be measured at least once in each direction The methods of horizontal angle measurement must be such as will yield a triangular error not exceeding 3″ to 5″ in the principal system, 10″ to 15″ in the second, and 20″ to 30″ in the third.

In the majority of topographical surveys and in geographical surveys, the standard of accuracy may be considerably lower, and progress is more rapid. Although the time available for the field work may be limited, a preliminary reconnaissance by plane table can never be dispensed with, unless existing maps sufficient for the purpose are available During reconnaissance as many stations as possible are beaconed by cairns of stones or other simple means, but artificial signals are frequently dispensed with in the triangulation proper if well-defined hilltops, isolated trees, rocks, etc can be used as station points Bases necessary should be at least half a mile long, special attention being paid to securing a well-conditioned expansion The precision of the base measurement is of the order 1/10,000 to 1/50,000 Invar or steel tapes are used suspended between mark posts, or, alternatively, the tape may be laid on the ground, which is first graded between the posts The allowable triangular errors of the angular measurements may be increased to twice those cited above.

Computation of Topographical and Geographical Triangulation.—In the highest class of extended topographical triangulation the methods of adjusting the angles and calculating the sides and station positions follow those given in Chap IV, the approximate methods and formulæ being sufficient. In topographical surveying generally and in geographical surveying no attempt at elaborate adjustment is required. Spherical excess can generally be neglected, and the angles of each triangle are adjusted for its total error without reference to side conditions, but the check provided by computing each side in two ways should not be omitted.

Primary Traverse.—The methods described in Vol. I, Chap. III, are applicable to the great majority of theodolite traverses carried out for engineering purposes, but, because of the comparatively small distances involved, such traverses may now be classed as secondary

The same general methods are applied in primary traverse, but greater precaution is necessary to limit the propagation of error in the longer distances covered The angular work may be efficiently checked at intervals by observations for azimuth, but it is difficult to eliminate cumulative errors of linear measurement The best check is afforded when the traverse runs between stations of the main triangulation. If this is impracticable, as much of the work as possible should be arranged to form closed polygons Otherwise, recourse must be had to determinations of astronomical latitudes or telegraphic longitudes as checks These are, however, unsatisfactory because of the possibility of abnormal deviations of the plumb line affecting the astronomical results The astronomical stations must be sufficiently wide apart to afford room for the development in the traverse of as great an error in their latitude or longitude difference as that due to the uncertainties of the astronomical observations and the unknown effect of local deflections of the vertical In careful traversing they need not be at closer intervals than about 100 miles (*cf.* *Ex* 10, page 190).

Preliminary Field Work of Primary Traverse.—The route is selected with a view to (1) providing points suitably placed for the control of the detail surveys to follow, (2) avoiding difficult ground for the linear measurements, (3) making the courses as long as possible to reduce errors arising from defective centering of the instrument and inaccurate bisection of the signals The last requirement materially influences the accuracy of the angular work, and an endeavour should be made to avoid introducing courses less than 1000 ft. long This usually necessitates the employment of an advance party to clear through forest or bush and to prepare the ground for the linear measurement

Traverse stations should be marked in a permanent manner. If it is impracticable to provide mark stones, a stout peg should be driven flush with the ground, a tack marking the exact point, and a cairn of stones should be built over it On long courses intermediate points likely to prove serviceable to the topographers are also marked.

Linear Measurement of Primary Traverse.—The linear measurement is performed by means of a steel or invar tape, 100 ft. to 300 ft. long, standardised periodically against a reserve tape. Owing to the quality of the angular work and the necessity at times of measuring over rough ground, the refinements of base measurement are not warranted. The tape is either held horizontally or the slopes are measured by clinometer, alignment being given from the theodolite. A constant tension is applied by means of a spring balance, and the temperature is recorded at every few tape lengths. On elevated ground the measurements should be reduced to sea level. Mistakes are best guarded against by double measurement with tapes of different lengths, or a check may be applied by tacheometry, by pacing,

or by more than one member of the party keeping a tally of the number of tape lengths. Closure errors vary greatly according to the nature of the ground, the range extending between about 1/2000 and 1/50,000. A speed of 5 or 6 miles a day should be maintained in country of average difficulty, but in dense forest the rate of progress may be as low as 1 mile a day.

The field book, besides recording the measured directions and lengths of the traverse lines, with particulars required for the correction of the latter, should contain a record of the chainage of marks left between stations and of streams, roads, etc crossed A full description of the sites of stations should be given by means of sketches and notes

Angle Measurement of Primary Traverse.—The theodolite should be a 5-in or 6-in. instrument with micrometer reading When the traverse proceeds from a triangulation station, the geodetic position of the station and the azimuths of the lines from it to the adjoining trigonometrical points are extracted from the survey records The azimuth of the first course of the traverse is observed by reference to one or more of those adjacent stations When the traverse is isolated, astronomical observations are made for the first azimuth and for the latitude and longitude of the initial station

The measurement of the angles between the courses should be performed face right and face left on the direction or the repetition system (page 148), in preference to single observation methods of carrying forward the bearing The refinement of measurement should accord with the quality of the linear work, and practically it is seldom necessary to use more than two zeros, since errors of pointing, particularly on short courses, are likely to exceed instrumental errors The vertical angles between the stations are also measured

Azimuth is checked astronomically every clear night or at intervals not exceeding ten miles The difference between an observed azimuth and the bearing carried forward from the previous determination is caused by the error developed between them, errors of astronomical observation, deviations of the vertical, and the convergence of meridians. It is eliminated by applying corrections to each course on the assumption that each has contributed equally to the discrepancy. By thus distributing the convergence the adjusted directions are expressed as azimuths, and, to correspond with the convention adopted in geodetic surveying, are reckoned from south by west.

Alternatively, the directions of all traverse lines may be expressed in terms of their bearings from the meridian through the initial station. The bearing of a course from any station not in the meridian of the origin differs from its azimuth by the amount of convergence, and the convergence correction must be applied when checking bearings by astronomical azimuths. The difference in the station

positions computed on the two systems is negligible except in the most extended traverses.

Computation of Primary Traverse.—The discrepancies between observed and computed azimuths having been distributed, and the linear measurements corrected for standard, temperature, etc., the calculation of latitudes and departures is performed in the usual manner. Closed figures are adjusted as in Vol. I, page 161, *Ex* 2, weights being assigned with reference chiefly to the difficulties of linear measurement.

Starting with the known position of the initial point, the geodetic position of each traverse station can be obtained from that of the preceding station by using the adjusted azimuth and distance between them (page 183). Usually the positions of the azimuth stations only are required, and the distance and azimuth of the line between them is first calculated. If the distances are comparatively short, positions are obtained with sufficient accuracy by reference to a table* giving the linear values of one second of arc of meridian and of parallel for different latitudes. The algebraic sum of the latitude co-ordinates between the stations divided by the length of one second of meridian arc at their mean latitude gives the latitude difference in seconds. Similarly, the algebraic sum of the departures divided by the length of one second of arc of parallel at the mean latitude gives the longitude difference.

When the traverse is carried on to a station the position of which has been determined by triangulation, or when the traverse is sufficiently long that it may be checked astronomically, a discrepancy will usually appear between the known or observed value of the position and that computed through the traverse. The computed latitude and longitude of any intermediate station are probably in error by amounts respectively proportional to its latitude and longitude distances from the origin, and the intermediate positions are adjusted on this assumption. If the traverse varies considerably in its liability to error, these distances are weighted by estimation.

Rapid Triangulation.—In applying theodolite triangulation to the control of exploratory and reconnaissance surveys, the required rate of progress can be maintained only by considerable deviation from the methods of rigid triangulation. Time is saved not only by omitting refinements of measurement, but in greater degree by the adoption of devices for continuing the chain of triangles which would not be allowable in deliberate work. Less attention is paid to the proportions of the triangles, and greater dependence is placed upon stations fixed by intersection, resection, or astronomical observations.

Methods comprised under rapid triangulation vary greatly

* See *Tables for Determining Geodetic Positions*, published by the Admiralty, or Sir C. F. Close, *Text Book of Topographical and Geographical Surveying*.

according to the speed required, the strength of the party, and the equipment carried. Under favourable circumstances they merge into those of geographical surveying, and the results are then much superior to those which can be expected from an explorer using the most portable of instruments under conditions demanding the utmost speed.

Triangulation may proceed simultaneously with the sketching of topography. Otherwise the triangulation is executed by a party working in advance of the detail surveyors. In the latter case, the geographical co-ordinates of the stations fixed are worked out as soon as possible, and this information, along with a description of the sites, is sent back at frequent intervals for insertion on the plane table sheets. Preliminary reconnaissance and beaconing of stations before occupation are, as a rule, impracticable, except perhaps for the base net. The surveyor will have only a general idea of the direction in which the triangulation will develop, so that success depends greatly upon his skill in selecting and using elevated features which come into view as the party advances.

Rapid Base Measurement.—When the survey is isolated from existing systems, a base, at least half a mile long, is measured, but it may be a difficult matter to find a suitable site unless time is spent in reconnoitring. Since a refined measurement is not attempted, a very level site is not required, and the inexactness of the measurement makes it more than ever necessary to have a good extension net. If the base net is well-conditioned, the error of a rough base affects only the scale of the survey, and can be corrected by the measurement of a check base when a suitable site is reached.

Whenever possible, the base should be measured by tape. Usually the best that can be done is a double measurement by steel tape with the slopes measured or the tape held horizontally. When a suitable site for a taped base cannot be obtained, the measurement may be made tacheometrically with an improvised subtense bar. Otherwise, an astronomical base, lying roughly north and south, may be computed in the manner of a latitude and azimuth traverse (page 228) from the observed latitudes of its end stations and the observed or computed azimuth of the line joining them. Because of the uncertainties arising from local deviation, the greater the distance between astronomical base stations the better, and this method should be regarded only as a temporary expedient until a base can be measured.

Angle Measurement of Rapid Triangulation.—In some classes of rapid triangulation a 5-in. micrometer theodolite is used, but when weight must be reduced to a minimum a light reconnaissance or mountain transit is all that can be carried. Remarkably good work can be done with modern 3-in. and 4-in. theodolites. These may be obtained with vernier reading to 1′ for the 3-in. instrument and to 1′, 30″, or 20″ in the case of the 4-in., the latter having a weight as

low as 14 lbs , including tripod, cases, and accessories. A compass, preferably of trough form, is essential.

The azimuth of the base and the latitude of one end must be observed If possible, a telegraphic longitude will be obtained: otherwise a value is assumed for the longitude. The error of the assumed value affects all the positions equally, and may be corrected out when the longitude of any station can be obtained telegraphically or by an absolute measurement.

In measuring the angles of the base net, only very definite marks must be used for observing upon because of the comparative shortness of the sights. Whenever possible, base extension stations should be beaconed. It is then justifiable to observe on two zeros, but otherwise it is sufficient to take face right and face left readings from one zero.

From the stations of the base net the surveyor must observe to all prominent natural features which may prove suitable for theodolite stations. Directions are also measured for the future fixing by intersection of such additional points as are likely to serve either as plane table stations or as guides to the sketching. Face right and face left rounds should be taken more to guard against mistakes of reading and booking than to eliminate instrumental error. In judging as to which of the points before him will serve as future theodolite stations, the surveyor is usually faced with the difficulty that he will be unable to visit points unless near the route to be followed The future course of the journey is probably not definitely known, and must be estimated from information obtained as to easy routes of travel and positions of camping grounds and water. It is therefore advisable to observe as many prominent objects as possible in the time available, as it frequently becomes necessary to occupy a station which was intended only as an intersected point

Having completed the observations at a station and descended to the route of the march, the surveyor will find that many of the points to which sights have been taken are no longer visible, and their changing aspect cannot be watched as the journey proceeds. When the next station is occupied, it will be found no easy matter to recognise them from the new point of view

Identification of Points.—To avoid a breakdown in the continuity of the triangulation through failure to identify stations, the triangle sides must be kept shorter than would be necessary if the stations were regularly beaconed. The lines of the chain should seldom exceed twenty miles in length, although longer sights are allowable to intersect points outside it. In observing, a good general rule is to sight only to the highest points of hills, but if the top is rather flat and presents no definite mark for sighting, the observation is made on an isolated tree or prominent rock near the summit. Such marks, however, are apt to prove especially troublesome to recognise from different standpoints.

Additional precautions affecting the routine of observation must be regularly adopted. At each station the theodolite should be oriented by compass, so that the observed directions are magnetic bearings. On proceeding to occupy a point, reference to the back bearing of a preceding station affords a valuable check against gross mistakes of position The record at each station should include notes and sketches of the appearance of the marks bisected and of the outline of the surrounding ground as seen through the telescope.

As an aid to the identification of points, a plane table proves of great service, and, if at all possible, the triangulator should carry one of light pattern, even although he may not be required to take topography. By its means a small scale diagram of the triangulation is kept up by drawing a ray to every point observed with the theodolite. The diagram, besides affording a ready means of identifying points, also exhibits the state of the work at any time, and enables the surveyor to judge the quality of intersections. Points which have been observed for the first time are approximately located on the sheet by the intersection of rays drawn from the theodolite station and from a subsidiary plane table station fixed by resection two or three miles distant in a direction transverse to the triangulation chain. The forward points should be easily recognised, as their appearance will not have changed greatly in the limited distance between the two positions of the plane table. During the journey, when opportunities occur of obtaining good views of the forward points, the plane table is set up, and its position is plotted by resecting on back stations or intersected points Then by pointing the alidade through the plotted points representing either intended theodolite stations or points to be further intersected their changed appearance may be examined as an aid to their future identification. At the same time, points to which only one ray has previously been drawn are approximately located, and a new set of points may be plotted during the journey for use in resecting the surveyor's position when finding his way towards a station. Plane table resections provide a check against setting up the theodolite on the wrong peak, but the identification of the actual spot to be occupied must be based upon the sketches of the site made at back stations.

Use of Intersected and Resected Stations in Rapid Triangulation.—When it is impracticable to observe all three angles of each triangle, the system can be carried forward by the aid of intersected points. In Fig. 79, A, B, C, D, and E are stations lying along the route followed, and only these are occupied Points *a*, *b*, *c*, *d*, and *e* are among those intersected, and, because of their favourable situation and natural marking, are selected for use in continuing the chain. The length of side AB having been computed through the preceding figures, triangle AB*a* is solved for B*a*, and the result is used in solving BC*a* for BC. Distance CD may now be obtained by solving BC*b* for C*b*, and CD*b* for CD, but it may also be derived from the results

of the observations to c by solving BCc for Cc, and CDc for CD The mean of the two values for CD is to be adopted for continuing the computation. Whenever possible, a pair of intersected points should be thus used: these may lie on the same side of the line of occupied stations or on opposite sides, as at d and e

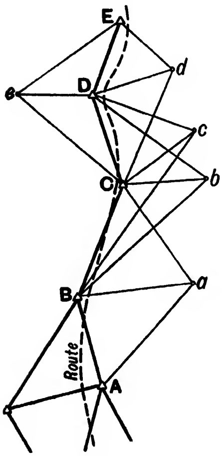


FIG 79

It is sometimes necessary to occupy a point to which no forward sights have been taken. Observations must then be made upon previously located points to enable the position of the instrument station to be computed by the method of trigonometrical interpolation or resection (Vol I, page 389). A sufficiently good result may be secured if attention is paid to the quality of the fix, and this can first be tested on the plane table The accuracy of the method is much improved by observation of an astronomical azimuth to one of the back stations

It may be found impracticable to maintain a continuous system of triangulation, particularly when the route passes through belts of wooded country Triangulation is then confined to the more favourable ground, each section having its own base line determined by latitude and azimuth or measured roughly along the ground The intervening gaps may be bridged by latitude and azimuth traverse, or, if the route lies nearly east and west, longitude may be determined by wireless, by transport of chronometer watches, or by an absolute method

Graphic Triangulation.—Triangulation by means of a small plane table is sometimes employed for the main control of reconnaissance surveys The general features of the field work are similar to those of rapid triangulation by theodolite The method is best adapted for running subsidiary chains in topographical and geographical surveys, the sketching of detail proceeding simultaneously The conduct of the work and its adjustment between known positions are described in Vol I, Chap V.

Latitude and Azimuth Traverse.—If the route lies approximately north and south, the method of latitude and azimuth traverse is available for the rapid location of a system of rather widely separated but intervisible points. If A and B are two such points, a latitude determination is made at each, and the azimuth of the line between them is observed at A or B or from both stations From these data the length of AB and the longitude difference are computed.

The accuracy of the method is controlled not only by that with which the latitude and azimuth determinations are made, but also by the value of the azimuth. Errors of observation have least in-

fluence when the course lies north and south, and their effect increases as the direction deviates from the meridian The method is impossible of application in the case of a course lying east and west, and should not be used for azimuths within 45° of that direction. Since errors of astronomical observation and the unknown effects of local deviation are independent of the distance between the stations, the latter should be as great as possible compatible with good visibility

Although not capable of a high grade of accuracy, the method is useful for rapid work in rough country The circumstance that a forward sight is not essential proves an advantage when it is found necessary to depart from the intended route of march When, after conducting a latitude and azimuth traverse so far, the general direction of the route turns towards the east or west so that the azimuths cannot be kept within 45° of the meridian, triangulation or ordinary traverse must be substituted until the method can be resumed

The method of calculating distances and differences of longitude is described on page 187

Control by Astronomical Positions.—In this method the latitudes and longitudes of rather widely spaced points are obtained, the subsidiary work consisting of rough traverse The method is suitable for the rapid survey of flat country, in which latitude and azimuth traversing cannot be employed The application of wireless telegraphy to longitude determination has removed the difficulty of obtaining satisfactory longitudes, and with its aid the method has been found particularly useful in regions barren of detail In such circumstances the route traverse need not be continuous.

VERTICAL CONTROL

The methods of precise and trigonometrical levelling have been described in Chap V The former should be employed as the basis for the vertical work of important or extended topographical surveys It may, however, be impracticable to run more than a few lines of precise levels, from which trigonometrical levelling may proceed, and very often the latter is exclusively used. Barometric levelling, although much less suitable for control than for detail work, is not infrequently applied to the entire levelling of reconnaissance surveys

Barometric Levelling.—In barometric levelling the relative altitudes of points are determined by ascertaining the difference of their depths below the upper surface of the atmosphere, an operation analogous to sounding in hydrographical work The measurement is performed by observing the atmospheric pressure at the several points and deducing the corresponding relative elevations. The pressures are measured by (1) mercurial barometer, (2) aneroid

barometer, (3) boiling-point thermometer. The mercurial barometer exhibits the height of a column of mercury which balances the column of air above the instrument. The aneroid records the pressure exerted against a hermetically sealed elastic box. The temperature of boiling water as ascertained by the boiling-point thermometer affords an indirect method of barometry, since water boils when the elastic force of the vapour equals the external pressure on the water surface.

Scope.—The great merit of barometric levelling lies in the rapidity with which measurement may be made of the difference of elevation of points at considerable horizontal or vertical distances apart. On this account it is greatly used in preliminary reconnaissance and exploratory surveys, but the degree of accuracy to be expected is much inferior to that of spirit levelling

Mercurial Barometers.—The mercurial barometer consists essentially of a vertical glass tube about 33 in long, closed at the top and having the bottom placed in a bath of mercury The tube being exhausted of air, and the bath exposed to atmospheric pressure, mercury will stand in the tube at a height sufficient to balance the pressure of the atmosphere on the free surface of the mercury in the reservoir.

The arrangement of a vertical tube standing in a bath of mercury is that adopted in the cistern barometer. In the syphon barometer the tube is bent up at the lower end to form a short vertical branch, the mercury in which is exposed to atmospheric pressure by the admission of air through a small opening near the top. The cistern barometer is the better type of mercurial instrument

The Cistern Barometer.—Figs 80 and 81 show the arrangement of a cistern barometer on the Fortin system, the particular feature of which consists in having the bottom of the cistern formed of fine-grained leather. The leather is permeable to air, and the mercury is subject to the action of the atmosphere without danger of leakage. The leather bag is attached to the bottom of the cistern, and is supported by the screw *5*, by means of which the level of the free surface of the mercury can be adjusted. At the top of the cistern there is fitted a glass sighting cylinder held against packings at top and bottom by four screws. The glass barometer tube enters the cistern through a collar where a secure joint is formed by leather or silk and glue. This collar carries the ivory gauge peg *7*.

The barometer tube is enclosed in a brass casing, the upper part of which is slotted on both sides to expose the mercurial column (Fig. 81). Graduations to ·05 in. are marked on one side. A tube, fitting the inside of the external casing, carries a vernier index, and by means of a rack actuated by a pinion through the milled head shown the lower edge of the vernier is brought to coincidence with the top of the mercury, and readings can be carried to ·002 in.

About midway up the barometer tube a thermometer is attached inside the external casing and close to the glass tube. For levelling work the instrument is mounted on a tripod, the head of which is arranged to ensure that the instrument maintains a vertical position

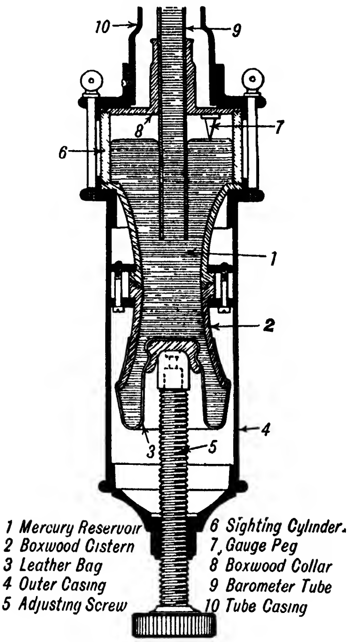


FIG. 80.—FORTIN BAROMETER.

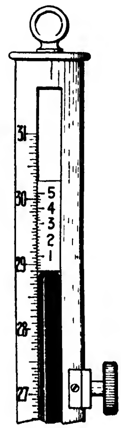


FIG 81.
READING
ARRANGEMENT.

To read the barometer, the surface of the mercury in the cistern is viewed through the glass *6*, and is adjusted by the screw *5* until it just touches the point of the gauge peg To effect this with the necessary precision, the eye should be placed level with the surface of the mercury and the cistern tapped gently to overcome adhesion between the mercury surface and the glass. The vernier index must then be set to the top of the mercurial column. The glass tube is first tapped near this point, and the index is racked until its front and back lower edges appear tangential to the convex surface of the mercury. The reading can then be observed. At each reading of the barometer it is necessary to record the temperature registered by the attached thermometer, and this reading should be taken first.

Other forms* of mercurial barometer are also used in levelling. Some of these are designed to afford maximum portability with minimum risk of breakage.

* See *Hints to Travellers*, Vols. I and II.

The Aneroid Barometer.—The features essential to all aneroids are (1) a thin metal box exhausted of air and hermetically sealed, (2) some mechanical or optical means whereby the small displacements of the surface of the box due to changes of atmospheric pressure may be magnified and read.

The form almost exclusively used for levelling purposes is of the Vidi type, in which the movement is multiplied by delicate levers and is shown by a pointer travelling over the graduations on a dial of from 2 in to 6 in. diameter (Fig. 82). The interior arrangement is shown diagrammatically in Fig. 83 The vacuum box is

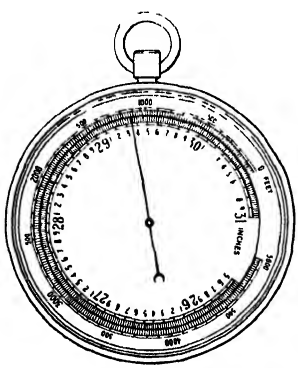
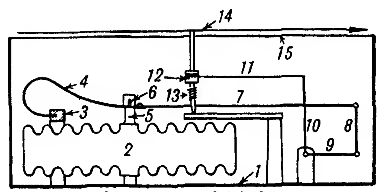


Fig 82
Aneroid Barometer



1 Base Plate 7,8,9 10 Levers
2 Vacuum Box 11 Chain
3 Bridge Piece 12 Drum
4 Spring 13 Spiral Hair Spring
5 Vacuum Box Axis 14 Pointer
6 Cotter or Knife Edge 15 Graduated Face

Fig 83 —Diagrammatic Section of Aneroid Barometer

made in the form of a flat cylinder, and is corrugated at top and bottom to increase its sensitiveness The bottom of the box is fixed at the centre to the base plate, and the post, or axis, *5* is attached to the top and communicates the tension of the spring *4*, which is as broad as the casing Increase of atmospheric pressure causes the knife-edge *6* to descend and *vice versa*, and this motion is transferred by the levers *7*, *8*, *9*, and *10* through the watch chain *11* to the pointer. The hair spring *13* keeps the chain taut by pulling against it with a force just sufficient to overcome axial friction.

The graduations on the dial represent inches of mercury, the spacing being obtained by subjecting the aneroid to known pressures under an air pump or by comparison with a standard mercurial barometer By means of an adjusting screw at the back of the instrument the pointer may be moved over the scale, and the instrument thereby set to agree at any time with a standard barometer under the pressure then prevailing. A scale of altitudes is usually provided, the divisions being spaced by conversion from the scale of inches by the use of a formula, but the indications of the altitude scale cannot be accepted for the best work unless a

correction for the temperature of the air is applied. The altitude scale is commonly engraved upon a movable ring so that the graduation representing the known elevation of a station occupied can be set under the pointer The absolute elevations of other points to which the instrument is carried can then be read directly. This is a convenient arrangement for rough observations, but introduces an additional source of error, since rate of increase of altitude is not equal to rate of decrease of pressure.

In reading the aneroid, it should always be held in the same position, vertically or horizontally, and the case tapped lightly to overcome any friction in the recording mechanism In estimating the position of the pointer, care must be exercised to avoid error due to parallax

On account of its portability and the greater speed with which observations may be made, the aneroid is preferred to the mercurial barometer in taking topography With the necessary precautions, results may be obtained with sufficient accuracy for the plotting of 25 ft contours on small scales, but in point of precision the aneroid ranks considerably below the mercurial instrument (see page 236)

Observations.—The barometric determination of elevations would be comparatively simple and accurate if the atmospheric pressures were always equal at points of equal elevation In reality, since the air is continually in motion, this is far from being the case, and, although the assumption of static equilibrium of the atmosphere forms the basis of the barometric levelling formulæ in practical use, the observations should be made in a manner designed to eliminate as far as possible errors due to variation in pressure throughout level layers of the atmosphere

Changes of atmospheric conditions may be classified as those due to (1) gradient, (2) temperature, (3) humidity.

Gradient —If the atmosphere were in static equilibrium, surfaces passing through all points at which the pressure is the same would be level surfaces. Actually, surfaces of equal pressure are irregular, and are subject to continual change of form Under normal conditions such a surface may within a limited area be regarded as a plane, the inclination of which to the horizontal is termed the barometric gradient. The direction and amount of the barometric gradient are subject to continual changes, which may be analysed as periodic, due to diurnal* and annual fluctuations in temperature, and non-periodic, arising from local conditions as to weather and topography.

Temperature.—The weight of the column of air of depth equal to the difference of level between two stations being compared depends upon its temperature. Observations for temperature

* See "Diurnal Atmospheric Variation in the Tropics, and Surveying with the Aneroid," by T. G. Gribble. *Min. Proc. Inst. C.E.*, Vol. CLXXI.

should be taken at both stations, but the mean of these may be considerably different from the mean temperature of the air column, since the thermometer is read in the stratum of air at the surface of the earth, which is warmer than the air column throughout the day and cooler at night. The consequent uncertainty with regard to the mean temperature of the air column forms an important source of error.

Humidity.—The proportion of water vapour present in the atmosphere is continually changing, and, as its density is much less than that of dry air, change of humidity is a second factor in varying the weight of the air column. In the best work, observations or humidity are taken by means of the wet bulb hygrometer, but as these are necessarily made near the ground, where the proportion of vapour is greatest and most variable, estimation of the mean humidity of the air column is as uncertain as that of its temperature.

Field Work.—A good determination of the difference of level between distant points can be obtained barometrically only from an extended series of observations at both stations by means of mercurial barometers. The greater the distance between the stations, the greater is the number of observations required for averaging. In the ordinary operation of taking topography by aneroid the usual problem, however, is to find the elevations of points at moderate distances from a station, the elevation of which has been otherwise ascertained. The determinations are then based upon either (1) single observations, (2) simultaneous observations.

Single Observations.—This is the commoner but less accurate method. The instrument is read at the reference station, and is then carried from point to point, and a reading is taken at each. When speed is of greater importance than accuracy, use is made of the altitude scale, which is first set at the base station to the known elevation of that point, and the elevations of the other points are read directly. The results of single observations may be very considerably in error if no attempt is made to eliminate errors arising from variations in atmospheric conditions.

The accuracy of the method may, however, be improved without much extra trouble. Change of gradient may be roughly ascertained by arranging the route that a return to the reference station is made at intervals. By observing the aneroid on such occasions, the rate of change of pressure is ascertained, and intermediate readings can be roughly corrected by time intervals on the supposition that the variation at the base represents that throughout the surrounding area. Otherwise, the rate of change may be roughly obtained during a circuit by remaining at one point for half an hour or so and noting any variation which may occur. If the route extends between two stations of known elevation, any change of pressure which has occurred during the journey will be evidenced

at the second station by a discrepancy in the difference of level obtained, and can be roughly distributed to the intermediate points The atmospheric temperatures are taken at the several points for correction of the barometer readings.

Simultaneous Observations.—In this method two instruments, which have previously been compared, are employed. One is kept at the base station, and is read at regular intervals throughout the day: the second is carried to the various points to be levelled, and each reading is taken simultaneously with an observation at the base. Otherwise, the transported instrument is read at irregular intervals as required, and the simultaneous pressure at the base is deduced by interpolation from the values observed there

The method is designed to reduce the effects of atmospheric changes to a greater degree than is possible by single observations, and is to be preferred, particularly where reference stations are widely separated.

Reduction of Observations.—From the nature of the case, a simple formula for difference of elevation is not available. The surveyor uses tabular values prepared from one or other of the various formulæ which have been proposed.

Theoretical investigation of the subject was first made by Laplace, who showed that

Difference of elevation $= C(\log H_1 - \log H') \times a \times b \times c$,

where H_1, H' = barometer readings in inches at lower and upper stations respectively, both reduced to 32° F.,

C = a constant, variously estimated as 60,159 to 60,384 for ft. units,

a = a factor allowing for the mean temperature of the air column and an average amount of moisture $= \left(1 + \frac{t_1 + t' - 64}{900}\right)$, where

t_1, t' = air temperatures in degrees Fahr. at lower and upper stations respectively,

b = a factor allowing for the variation of gravity with latitude $= (1 + \cdot 0026 \cos 2\phi)$, where ϕ = mid-latitude of the stations,

c = a factor allowing for the diminution of gravity with altitude $= \left(1 + \frac{X + 52{,}252 + 2L_1}{R}\right)$, where X = difference of elevation as obtained without this factor, L_1 = elevation of lower station, and R = radius of earth in ft $= 20 \cdot 89 \times 10^6$.

The reduction of barometric readings to the temperature 32° F. requires a knowledge of the temperature, T, of the instrument as given by the attached thermometer. The reduction formula is

Reading reduced to 32° F. = Actual Reading $(1 - a(T - 32))$,

where a = differential coefficient of expansion of mercury and the metal (brass) of the scale = say $\cdot 00009$.

Readings of compensated aneroids are not subject to this reduction, and the term 52,252 in the c factor is omitted in their case

The Laplace formula has been tabulated in various ways Loomis' Tables are frequently used, and are published in the Indian Survey Auxiliary Tables, the Smithsonian Miscellaneous Collections, *Hints to Travellers*, etc

Several modifications of the formula have been made. That by Baily is

Difference of elevation in ft $= 60{,}346 (\log H_1 - \log H') \times d \times a \times b$,

where H_1, H' = unreduced barometer readings at lower and upper stations respectively,

d = temperature correction to barometer $= \dfrac{1}{1 + {\cdot}0001(T_1 - T')}$, where

T_1, T' = readings of the attached thermometers at lower and upper stations respectively,

a = as before,

b = latitude factor $= (1 + {\cdot}002695 \cos 2\phi)$, where ϕ = mid-latitude.

Baily's Tables are given in the Indian Auxiliary Tables, the Smithsonian Collection, Sir C F Close's *Text Book of Topographical and Geographical Surveying*, etc

Sources of Error.—These may be classed as

(1) Errors due to Natural Causes,
(2) Instrumental Errors,
(3) Errors of Observation,
(4) Errors of Reduction

(1) Errors due to atmospheric conditions are the most important The most difficult of elimination are those arising from the existence of permanent gradient and the impossibility of accurately estimating the temperature and humidity of the air column Observations must, of course, be suspended during storms, and should not be made in situations where wind eddies will cause abnormal readings, as may occur on the leeward side of obstructions

(2) Instrumental errors are of minor importance in the case of a good mercurial barometer if the necessary precautions are taken in using and transporting it The instrument with its attached thermometer must have been compared with a standard, and a table of corrections obtained.

The aneroid, on the other hand, cannot be regarded as a precise instrument. Accurate graduation is a matter of considerable difficulty, and the readings should be compared under various temperatures with those of a standardised mercurial instrument, and the corrections noted These corrections do not, however, remain constant with the lapse of time The variations arise from changing elasticity of the vacuum box and the mainspring, wear, and temperature effects (in "compensated" as well as uncom-

pensated instruments) Refined reading is impracticable. The fineness of reading possible depends upon the range of the instrument, as the graduation is, of course, more open in an instrument designed for reading elevations from 0 to 1000 ft than in one ranging from 0 to 10,000 ft For work covering large variations in altitude, aneroids of different ranges are carried, *e g* 0 to 5,000 ft , 4,000 to 10,000 ft., etc , overlaps being necessary since the indications are uncertain towards the extremes of the scale

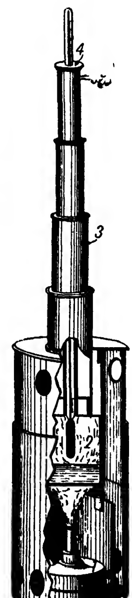
(3) In observing, precautions are necessary to avoid anomalous indications arising from unrepresentative temperature conditions and sluggishness or drag in the instrument Barometers should therefore be carefully shielded from the rays of the sun. The existence of drag is evident after a sudden change of elevation, as the instrument does not respond immediately After ascending or descending a steep slope, a stoppage should be made for a few minutes, and the barometer then gently tapped and read

Errors of parallax in reading the instruments are always present In the mercurial instrument they occur in inaccuracy of contact between the gauge peg and the surface of the mercury, in setting the vernier index, and in reading the thermometer. Parallax error in reading the aneroid is considerably greater, and depends upon the distance between the pointer and the dial Errors of reading are reduced to negligible limits by repetition of observations

(4) The formulæ by which differences of barometer readings are reduced to differences of elevation are not of an exact nature, but it may be taken that errors thus introduced are negligible in comparison with those due to the foregoing causes

Limits of Error.—Looking to the uncertainties inherent in barometric levelling as performed in taking topography, it is impossible to set a value on the degree of accuracy which may be expected Some remarkably close agreements with the results of spirit levelling have been obtained by prolonged observations with mercurial barometers, but such methods are impracticable in ordinary topographical surveying

The Boiling-point Thermometer.—Fig 84 illustrates the usual portable form of boiling-point thermometer or hypsometer Sensitiveness is an essential requirement in the thermometer, which is graduated from about 180° to 215° F. and divided to 0·2° The thermometer is held by the rubber washer *4*, so that the bulb is immersed in the steam in the boiler, and the stem is subjected to a current of steam in the telescopic jacket *3*.



1 Spirit Lamp
2 Boiler
3 Jacket
4 Rubber Washer

FIG. 84
BOILING-POINT THERMOMETER.

In using the instrument, the boiler is about one-third filled with rain water. The thermometer is introduced through the tube leading from the boiler, and is adjusted so that the bulb is just clear of the water. The spirit lamp is lit, and the thermometer is read when the mercury becomes stationary. When not in use, the thermometer is carried in a brass tube with a rubber lining

Observations and Reduction.—The general procedure in observing is the same as that with the barometer Simultaneous observations give the best results, but are often impracticable. In the method of single observations, if the surveyor is remote from the reference station, he must rely upon estimated values for the atmospheric pressure there at the times of his various observations. Mean pressures at different seasons are given in published charts.

Before reducing, the thermometer readings are corrected for index error, which should be ascertained periodically. Difference of elevation may be obtained from barometric tables if the barometer readings equivalent to the boiling-points are computed. Several tables of elevations corresponding to boiling-points are, however, available Examples will be found in *Hints to Travellers* and Sir C. F. Close's *Text Book of Topographical and Geographical Surveying*.

In the absence of a table, the following empirical formula may be used :

$$E = t\,(521 + \cdot 75\,t) \times a,$$

where E = elevation in ft above that at which water boils at 212°,
t = degrees Fahr of boiling-point below 212°,
a = air temperature factor of the barometric formulæ.

Only rough results are possible since the thermometer can be read only to the nearest ·01 in , which corresponds to over 50 ft. of elevation.

THE SURVEY OF DETAIL

With the exception of photographic surveying, the methods for the survey of detail tabulated on page 220 have already been dealt with in this volume and in Vol I, Chap III, IV, V, VI, and IX. Features of the work special to small scale mapping are given below, but the subjects of theodolite traverse and tacheometry are not treated further. Although excellent for close contouring, tacheometry, on other than the subtense bar system, is of minor utility for rapid work on small scales.

Plane Tabling.—In view of the great importance of plotting detail in the field, plane tabling is the most extensively used method for the survey of topography. It is particularly useful in open ground, but for surveying through tracts of bush the plane table is employed only to a limited extent in small scale mapping, and compass traversing is preferred. The methods of plane table

surveying have been treated in Vol I, Chap. V, and it remains to outline here the routine followed in the mapping of detail on topographical and geographical scales.

Field Sheets.—The sheets on which the field mapping is performed are called field sheets The scale adopted is commonly that on which the final map is to be made, but if it is intended to reproduce the survey on more than one scale, the largest must be used for plotting the field sheets. In country favourable for plane tabling the field sheets are completed on the plane tables. If, however, the survey includes compass traversing in parts, such work is usually better plotted on a larger scale in the first instance These subsidiary surveys are laid down on auxiliary sheets, and are subsequently incorporated in the field sheets in camp

Each field sheet is to contain a definite part of the survey, bounded by meridians and parallels, and commonly embraces a quarter of the area included in the final map sheets A degree of latitude and of longitude can be taken on the 1/250,000 scale, and proportionately less on larger scales. This leaves sufficient margin on a medium-sized board.

Preparation of Field Sheets.—Before proceeding to take topography, the surveyor must prepare his sheet, after it has been properly seasoned, by drawing the graticule and plotting the positions of the trigonometrical points by which his mapping will be controlled. The graticule, or network of meridians and parallels, for the area embraced by the field sheet is constructed as described on page 273 The graticule interval must be a simple fraction of a degree, the smaller the greater the scale It is generally made 15′ for mapping on geographical or topographical scales, and may be subdivided to 5′. The graticule must extend beyond the limits of the area to be surveyed to give a surrounding margin of sufficient breadth to contain trigonometrical points which, although outside the section, will be used to control detail near the edge.

As soon as the graticule is inked in, the positions of triangulation and traverse stations and of intersected points are plotted thereon as described on page 275. In addition to checking the plotting by scaling between the points, their plotted positions are verified in the field. The table is set up at one of them which occupies a commanding position, and is oriented by the ray to any other. The plotting of all stations within view is then checked by sighting them successively.

Field Work.—Plane table stations are almost exclusively located by resection from trigonometrical points. The stations are situated as far as possible on elevated ground, but occasional fixings are required on the lower ground for the survey of detail which cannot be obtained from the hill stations. It is economical to work from

high ground to low. The distance between plane table stations naturally depends upon the openness of the country, and is likely to be greater with an experienced than with an inexperienced topographer On the easiest ground four or five fixings per square mile are recognised as sufficient for work on the 1/62,500 scale. For smaller scales the number of points occupied may be considerably reduced.

Detail is surveyed by sketching between a few intersected points. Elevations are taken by vertical angles with the telescopic alidade or the Indian clinometer (Vol. I, page 219), by aneroid, or by a combination of vertical angles and aneroid readings The elevations of plane table stations are obtained by observing the vertical angles and scaling the distances to two trigonometrical stations, one result serving to check the other The elevations of intersected points are obtained from those of the plane table stations by similar measurements, and the contours or form lines are then sketched in by estimation. Barometric levelling is a valuable adjunct for determining spot heights around a station or on the route between stations As the scale decreases, fewer points are fixed for the control of sketching: the requirements of geographical mapping are met by sketching topography entirely from the plane table positions and elevations.

The art of sketching topographic form is one which demands considerable judgment The inexperienced surveyor usually errs by including unimportant detail too small to show clearly upon the finished map The topographer should possess a sound knowledge of land forms in order that he may select for representation the significant features which express the character of the region The ability to sketch these with the desired accuracy is acquired by experience The natural tendency to exaggerate the roughness of difficult country and the smoothness of flat ground should be recognised and avoided

The area mapped in each field sheet should extend beyond the strict boundary of the section by the inclusion of an additional strip, about half a mile to a mile wide, round it This overlap between adjoining sheets is necessary to ensure a good connection between them by the adjustment of small discrepancies.

Compass Traversing.—Recourse is had to compass traversing (Vol. I, page 151) between theodolite stations for the survey of detail in thickly wooded regions. The lengths of the courses are usually strictly limited by the nature of the country, and this adds to the other inaccuracies of the method When a wide region has to be surveyed by compass, the area is divided up by systems of deliberate compass traverse forming a framework from which the bulk of the detail is surveyed by rapid traverses of short length. For economy, the framework itself usually comprises traverses of different grades of precision, and in extended work may consist of primary, secondary, and tertiary systems.

Primary Compass Traverse.—Routes for primary traverse are selected over ground favourable for linear measurement, and so that as long courses as possible may be obtained with a minimum of clearing The instrument is either a circumferentor or a large size prismatic mounted on a tripod Bearings are read to the nearest 5′, both forward and back bearings being observed at each station The steel tape is stretched under a constant tension, and the slopes are measured by clinometer A double measurement is made as a check The chainages at which roads, streams, etc are crossed are entered in the field book, and important features of detail on either side are surveyed by intersection, bearing and distance, or rough offsets Clinometric or aneroid heights are observed along the traverse

When there is a considerable distance between control points, primary traverses should be arranged to intersect each other and form closed figures capable of adjustment Traverses must also fit between the control points on the theodolite framework A graphical adjustment is usually sufficient in small scale work

Secondary and Tertiary Compass Traverse.—Secondary traverses are run between stations of the main framework or of the primary compass traverses Because of the smaller distance between checks, rougher work is allowable, and greater speed is attained A 4-in prismatic compass on a tripod or staff is suitable, and forward and back bearings are read to the nearest 10′ or 15′ Ordinary steel taping is sufficient, the tape either being held horizontally or the inclinations measured Distances should be checked for mistakes by pacing

In tertiary traverse the forward and back bearings need only be read to the nearest degree with the prismatic held in the hand Steel taping is commonly employed, but the use of a light steel wire rope 200 ft or 300 ft long is sometimes preferred on account of its superior strength A chain is objectionable in dense undergrowth

It is usually sufficient to plot secondary and tertiary traverses by protractor and to make a graphical adjustment between control points

Route Traverse.—Under this heading may be classed all methods of traverse of lower grade than tertiary They are extensively used in reconnaissance and exploratory surveys, when speed is of greater importance than precision Such work is sometimes controlled only by astronomical determinations, and the resulting map of the route followed is necessarily of inferior accuracy On the other hand, rough traversing between frequent control points is a useful method of surveying thickly wooded belts in deliberate mapping Detail on either side of the route is surveyed by intersection of compass bearings, bearing and distance, and by sketching.

Bearings are observed as in tertiary traverse, but in thick bush

it is sometimes impracticable to obtain sights of more than a few yards, and bearings are taken towards the sound of a whistle at the invisible forward station. The linear measurements of the traverse legs and for the inclusion of adjacent detail are made by rough methods, such as by wheel, pacing, time, and sound.

Measurement by Wheel.—On level ground remarkably good results are obtained by running a wheel along the line and determining the distance as the product of the number of revolutions by the circumference of the wheel. The reliability of the method is reduced on rough ground owing both to the presence of slopes and the effects of jolts The revolutions may be counted by watching a piece of bunting attached to one of the spokes, or the count is performed automatically by odometer or by perambulator. The odometer is a device which is fixed to the wheel of a vehicle and exhibits on a dial either the number of revolutions made or the distance run The perambulator resembles the front wheel of a bicycle with fork and handle-bars . the wheel is usually of wood, and the tyre of hard brass The recording mechanism moves a pointer over a small dial graduated to read to yards.

Measurement by Pacing.—The ability to maintain a constant length of pace is acquired by practice For the same person the length of the pace increases with increase of speed, and decreases with increase of slope of the ground, whether going uphill or downhill. The surveyor should ascertain the length of his step by walking at average speed over a known distance on level ground. In measuring by pacing, he should keep to his natural step instead of trying to pace yards, but the stride should be lengthened on going up or down slopes as an attempt to cover a constant horizontal distance at each pace It is usual to count only every second step, *i.e* those of either the right or the left foot

The trouble of counting is eliminated by carrying a pedometer. Externally this instrument resembles a watch, and it is carried upright in the pocket The jolt given at each step actuates the mechanism, and the distance walked is recorded on the face, which is graduated to $\frac{1}{4}$ mile and is read by estimation to $\frac{1}{10}$ mile The instrument is adjustable to suit the length of pace of the user. In a very similar instrument, called the passometer, the readings represent the number of paces only, so that no adjustment is required for its use by different observers.

If the surveyor is mounted, distances may be obtained by counting the paces of the animal, but the accuracy is rather less than that of human pacing because of the effect of slopes and varying speed. The value of a pace should be ascertained by riding a measured distance both at a walking gait and a trot, the number of strides of one of the forelegs being counted. A pedometer may be carried and calibrated in this manner for use on horseback.

Measurement by Time.—In this method distances are estimated from the time taken in travelling Note is made of the times at which the beginning and end of each traverse course is reached as well as of the estimated rate of march between. The method is useful for work on horseback, since the speed of a particular animal is fairly constant when either walking or trotting It is frequently applied in running traverse on rivers Experienced oarsmen can maintain a nearly uniform stroke, and the speed of the boat can be determined over a known distance when rowing with and against the stream If the traverse is made with the boat drifting down-stream, the speed of the current should be ascertained at intervals by timing the boat over a measured distance When a power launch is used, its speed should be measured for various rates of running the engine.

Measurement by Sound.—This consists in firing a gun at one end of the line and noting at the other end the time which elapses between seeing the discharge and hearing the report Two or three repetitions are made, and the results are averaged It may be taken that sound travels at the rate of 1,090 ft per second in still air at a temperature of 32° F and that the speed increases by 1·1 ft per second for every degree rise in temperature The accelerating or retarding effect of wind is eliminated by firing and observing at each end of the line, the mean result giving the required measurement.

When the stations are not intervisible, two observers, A and B, take up positions at either end of the distance to be measured: each is provided with a revolver, and A has a stop-watch A fires, and observes the time. Immediately on hearing the report, B fires, and A notes the time at which the sound reaches him The interval between the two times observed by A is found to be greater than that required for sound to travel from A to B and back to A, chiefly on account of an inevitable delay by B between hearing A's shot and replying Huddart* finds that the subtractive correction to the time interval is practically independent of the distance, and with experienced observers is fairly constant at 0·7 to 0 8 sec.

Route Sketching.—It is frequently desirable to make a scale sketch of the topography while running a route traverse. To facilitate sketching, various types of sketch board have been designed in the form of a miniature plane table with an attached compass, whereby bearings to side objects, as well as those of the traverse, may be observed Fig 85 shows the Verner Cavalry Sketch Board. The instrument is strapped to the left forearm for use on horseback, but can also be mounted on a Jacob staff when required. The board measures about 9 in. by 7 in., and carries two rollers over

* "Sketch Mapping with Special Reference to Southern Nigeria." *Min Proc. Inst C E.*, Vol. CLXIX.

which a continuous roll of paper is passed Orientation is performed by reference to the small compass fitted on one side. The functions of an alidade are performed by a scale which is held in any required position by two rubber bands as shown

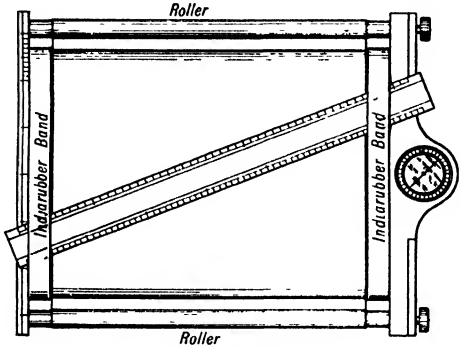


FIG 85 —CAVALRY SKETCH BOARD.

PHOTOGRAPHIC SURVEYING

Photographic surveying, or phototopography, is a method of surveying in which the detail is plotted entirely from photographs taken at suitable camera stations The method has been extensively employed throughout the Continent and in Canada It is adapted for the small scale mapping of open mountainous regions, but, unless the photographs are taken from the air, it is useless in flat or wooded country

In its general features, photographic surveying resembles plane tabling by intersection and resection, but, as the plotting is performed in the office, the accuracy attained in the representation of topographic detail is necessarily inferior to that of field sketching The principal merit of the system is the rapidity of the field work the time spent in the field may be put at about one-third that of plane tabling This proves a valuable feature under adverse climatic conditions In mountainous country there may occur only brief intervals during which the peaks are clear of mist, and the operations at a camera station can be completed with much less delay than would occur with any other system

General Principles.—In plotting the map, the distances and elevations required must be obtained from the perspective dimensions on the photographs. The process—termed iconometry—is therefore the reverse of perspective drawing.

In Fig. 86, O represents the optical centre of the camera lens. OP, its principal axis, intersects the vertical negative at P, the principal point of the picture, of which HH′ is the horizon line and VV′ the principal line. The image thrown on the plate is an inverted

perspective, but an upright perspective of the same dimensions can be imagined received on a glass plate parallel to the negative and placed, as shown by dotted lines, in front of O with its principal point on the axis of the lens and at a distance from it equal to OP Since the lens is focussed for distant objects, OP equals f, the focal length of the lens This perspective corresponds to the photographic print, which is therefore a perspective with view point O.

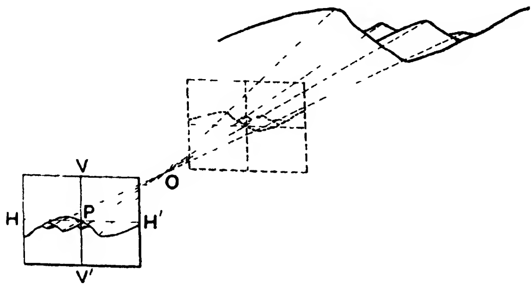


FIG 86

A single photograph does not furnish the complete data required for plotting the features it includes If, however, a second photograph of the same area is taken from another known station, and the orientation and elevation of the camera at the two stations are known, the points appearing in both photographs can be located both horizontally and vertically Thus, in Fig 87, let O and O′ be the plotted positions of the stations, then the data regarding the orientation of the camera will enable XX and YY, the picture traces, to be correctly placed on the plan at f in. in front of O and O′ respectively The abscissæ P_1a_1, P_1b_1, P_2a_2, P_2b_2, etc are transferred by dividers from the prints to the corresponding picture traces, and rays from O through the points a_1, b_1, etc. thus obtained will yield with corresponding rays from O′ the required intersections A, B, etc

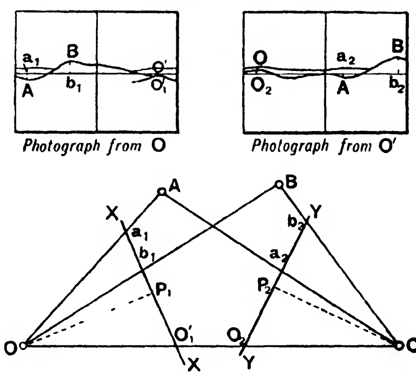


FIG 87

The elevations of intersected points can be ascertained from that of either camera station The effects of curvature and refraction may be neglected, so that the horizon line on a print intersects points whose elevation is that of the centre of the camera.

To obtain the elevation of any other point, such as A, the ordinate Aa_1 is measured on the print. By similar triangles, this represents an actual distance of

$$h = \frac{Aa_1}{Oa_1} \cdot OA,$$

and the elevation of the distant point = the elevation of the camera station + the height of the camera above the ground $\pm h$.

The Surveying Camera.—An ordinary camera may be utilised for surveying if it is fitted with spirit levels so that the photographic plate may be set accurately vertical. The instrument should have a ground glass screen through which the view embraced by each photograph may be examined to ensure that desired features near the sides are included. The lens should have a large field free from distortion. A knowledge of the distance from the optical centre of the lens to the sensitised surface of the plate is required in plotting, and it is therefore a great advantage to have a camera of the fixed focus type.

The labour of plotting is much reduced, and the camera is adapted for survey work, if means are provided for exhibiting on the negatives the trace of the horizontal plane passing through the centre of the lens, as well as the position of the principal point, in which the optical axis meets the plate. Their positions may be marked by four needles or notches in the frame against which the plate rests, and these are photographed upon the negative at each exposure. Two of the marks define the horizon line and the other two the principal line, and the intersection of lines drawn through them on the print or negative gives the principal point Alternatively, these lines are reproduced on the negative by having the frame fitted with hairs which are stretched just in front of the plate.

If the camera is provided with no other attachment, it must be used in conjunction with a theodolite, horizontal angles being taken to fix the positions of camera stations and to orient the photographs, and vertical angles to control elevations To economise weight, the same tripod and base plate may be used both for the theodolite and the camera, as in Canadian practice. In Europe the surveying camera has been combined with the theodolite to form the photo-theodolite

The Photo-Theodolite.—Several combinations of theodolite and camera have been devised and used.* The instrument designed by Mr. Bridges-Lee, and made by Messrs C F. Casella and Co, embodies some interesting features, and is illustrated in Fig. 88.

The camera is of the fixed focus type for 5 in. by 4 in. plates, and is mounted on an axis in the same manner as the upper plate of a theodolite. It carries a vernier by which the horizontal circle

* See Flemer, *Phototopographic Methods and Instruments.*

is read to single minutes. Upper and lower clamps and tangent screws are fitted as in the theodolite. A telescope with a vertical arc is mounted on the top of the camera box, and cannot be moved in azimuth relatively to the camera, the line of sight being in the same vertical plane as the optical axis of the camera lens.

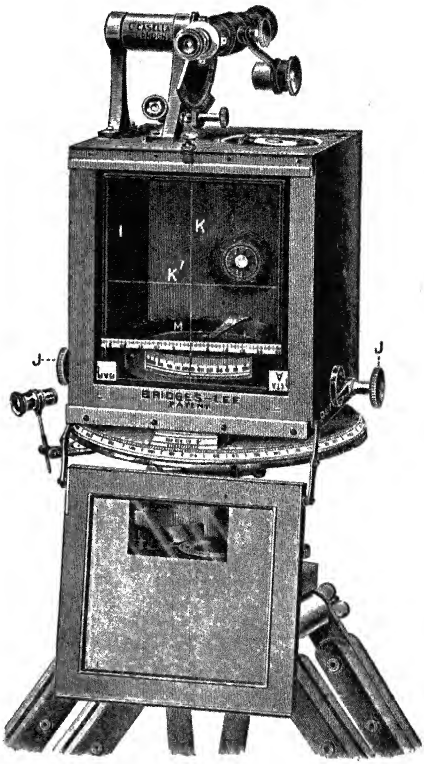


FIG. 88.
BRIDGES-LEE PHOTO-THEODOLITE.

Inside the camera box is a vertical frame I, carrying a vertical and a horizontal hair K, K′, situated in the same vertical and horizontal planes respectively as the optical axis. Attached to the same frame there is a horizontal transparent scale of angular distances, the graduations of which serve to show to the nearest 5′ the angular distances from the vertical line of points on the picture. The frame I is rigidly connected to a base plate which supports a circular magnetic compass M, the needle of which carries a vertical cylindrical transparent scale divided to half-degrees.

When the photographic plate is in position and the shutter of the dark slide is drawn out, the internal frame can be racked back by the screws J until the cross-hairs just touch the plate. The scale of angular distances is at the same time brought close up to the plate, and both it and the hairs are reproduced on the negative. In addition, the action of racking back places the compass needle on its pivot, and brings the compass scale sufficiently close to the surface of the plate that the graduations in the neighbourhood of the vertical hair are also distinctly shadowgraphed. The reading of the compass scale at which it is intersected by the vertical line on the photograph represents the magnetic bearing of the pointing, so that the orientation of each view is automatically recorded. The bearing to any other point is obtained by applying to that magnetic bearing the angular distance of the point from the principal plane as shown on the scale of angles.

Determination of Focal Length of Camera Lens.—If the focal length f of the lens is unknown, it should be ascertained at the outset The method given by Deville is as follows

The horizontal angle c (Fig 89) subtended at the camera station O by two distant points A and B is measured by theodolite A photograph is taken to include these points, and their distances Pa and Pb from the principal line are measured on the negative Then

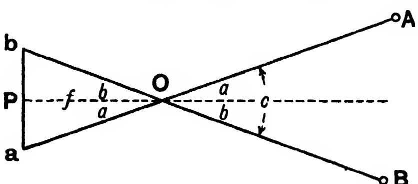


FIG. 89.

$$\frac{\mathrm{Pa}}{f} = \tan a, \text{ and } \frac{\mathrm{Pb}}{f} = \tan b,$$

$$\text{but } a+b = c,$$

$$\text{whence } \tan c = \frac{\dfrac{\mathrm{Pa}}{f} + \dfrac{\mathrm{Pb}}{f}}{1 - \dfrac{\mathrm{Pa}\cdot\mathrm{Pb}}{f^2}}, \text{ a quadratic for } f.$$

Determination of Principal and Horizon Lines.—The positions of these lines must be obtained and marked in the case of a surveying camera out of adjustment or in using an ordinary camera for surveying A photograph of a suspended plumb line gives the directions of the lines By including in the photograph three points subtending two known angles at the instrument, the position of the principal line is obtained as follows

In Fig 90, let A, B, and C be known points in plan or points subtending two observed angles at the camera station O On the photograph draw any line perpendicular to the image of the plumb line, and project the images of A, B, and C upon it Transfer the points a, b, and c so obtained to a paper straight-edge, and move the strip over the plot until a, b, and c simultaneously fall upon their respective rays OA, OB, and OC The edge of the strip now coincides with the oriented picture trace, and a perpendicular OP upon it defines the position of P relatively to a, b, and c The position of the principal line is therefore obtained, and OP should measure f

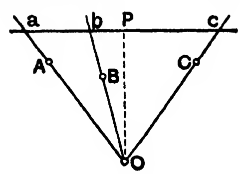


FIG 90

To find the position of the horizon line, the vertical angle a from the camera station, or the distance and relative elevation, of at least one pictured point A must be known The ordinate h of the image of that point from the horizon line is computed from

$$h = \mathrm{Oa} \tan a,$$

and a point on the horizon line is obtained by setting off from the

image of A the distance h parallel to the principal line. As a check, other points are obtained in the same way

Alternatively, the horizon line may be located by sweeping a horizontal plane through the camera station with a level and noting definite points cut by it These are subsequently identified and marked on the photograph

Field Work.—The entire control of a photographic survey may proceed simultaneously with the taking of the photographs, and the survey is then similar in principle to graphic triangulation A photographic survey is, however, generally based upon a previously executed system of triangulation Photographs are taken from such of the survey stations as are suitable for the purpose Many other camera stations will be found necessary in order to obtain suitable views of the whole region, and these are located by subsidiary triangulation, the angles of which are observed either by the photo-theodolite or by a separate theodolite It is frequently convenient to locate camera stations by resection or interpolation from observations on three triangulation stations

The selection of camera stations is an important part of the field work, involving consideration of the requirements of the subsidiary triangulation as well as the location of detail All features to be plotted must be photographed from at least two stations so situated that the lines joining them to the point give a definite intersection there. The more important points should appear on three or more photographs

For the orientation of the views, each should contain at least one point of known position, such as a triangulation station or a point whose direction is measured from the camera station This is desirable even when the photo-theodolite is used and the orientation of the picture is read on the circle as well as being automatically recorded in terms of the magnetic bearing As an additional check in plotting, adjacent views should overlap by an amount sufficient to make an easily recognised point appear in both The elevation of each camera station is obtained by trigonometrical levelling, usually by observation of the vertical angles to two triangulation stations. In cases where no station appears on a photograph, the vertical angle to at least one conspicuous point in it should be observed

In arranging the work at the camera stations, consideration should be given to the time of day at which each should be occupied so that the sun may be favourably situated with respect to the camera and the view A certain amount of shadow is useful, but areas totally in shadow should not be photographed, and, as a rule, the best results are obtained during the middle part of the day.

In addition to keeping an angle book, the surveyor should make a sketch of the view embraced by each photograph On these sketches are shown the approximate positions of triangulation

stations included, the points to which angular observations have been taken, and the names of peaks, rivers, roads, etc.

Preparation of Photographs for Plotting.—Needless to say, the negatives must be as sharp in detail and with as much contrast as can be secured Rather slow isochromatic plates should be used, and the exposure made with a small aperture. The plotting may be performed either from the negatives or from prints. The direct use of negatives is the less convenient but more accurate method, since prints rarely correspond in size with the negatives. It is usual to work with enlargements of two to four times the linear dimensions of the originals in order to reduce errors arising in taking dimensions from the photographs. Great care in enlarging is necessary to avoid distortion, and the enlargements should be tested for perceptible distortion. The use of enlargements on glass has been recommended for the plotting of important features

Orientation of Picture Traces.—The accuracy of the plotting depends upon the correct placing of the picture traces upon the plan just as a plane table plot is dependent upon the orientation of the table. The conditions that the principal point P should be at the focal distance from O, the plotted position of the station, and that the picture trace should be perpendicular to OP, define the trace as tangent to a circle with centre O and radius f The laying down of the picture traces is independent of the scale of plotting, dimensions from the photographs and the focal length being drawn full size. When enlargements are used, the enlarged focal length is laid down.

The method of placing the picture traces on the plan depends upon the data available.

(1) When the photo-theodolite is used, and the horizontal circle is read at each exposure, the bearing of the principal plane is known, and it is only necessary to set off this bearing, mark the point P at a distance f from O, and draw the trace perpendicular to that of the principal plane. The result should, however, be checked by one of the following methods.

(2) When the photograph includes a point of known position already plotted on the plan or one of known direction from the camera station, the orientation may be performed with respect to it In Fig 91, let A be the position of a known station, and let its distance from the principal plane on the photograph be Pa. On OA set off $Oa_1 = f$, and from a_1 erect a_1a_2 perpendicular to OA, and such that $a_1a_2 = Pa$. Oa_2 is evidently the trace of the principal plane, and, on marking off $OP = f$, a perpendicular through P represents the picture trace. Alternatively, the length Oa is calculated from $\sqrt{f^2 + Pa^2}$, and triangle OPa is constructed upon it.

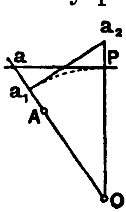


FIG. 91

(3) When the photograph includes two or more points of known position or direction, the orientation may be performed mechanically by means of a paper straight-edge on which are marked P, a, b, etc. The method for three or more points is given on page 248 and Fig. 90, the orientation being checked by the perpendicularity of the trace to OP and the length of OP In the case of two known points A and B, the point P on the paper straight-edge must be kept upon the circumference of an arc with centre O and radius f, and the strip adjusted until a and b simultaneously fall upon the rays OA and OB.

Plotting.—Before plotting detail, the prints are carefully studied, and every salient point to be plotted is marked with a fine dot and numbered The same points must be identified and similarly numbered on two or more photographs To locate the marked points on the plan, a convenient method is to mark off their distances from the principal plane on the straight edge of a strip of paper, which is then fixed on the drawing with its edge along the appropriate picture trace A similar strip is prepared from each photograph, and the lines through the station points and the points on the traces intersect at the required positions

The elevations of the plotted points may be computed as on page 246 or may be determined graphically Thus, in Fig 92, to obtain the elevations of A and B relatively to that of the camera, their ordinates h_1 and h_2 from the horizon line are taken from the photograph and set off at a and b on the plan as perpendiculars to Oa and Ob respectively The angles α and β subtended at O are the true vertical angles to the points, and the intercepts H_1 and H_2 perpendicular to the rays from the plotted positions of A and B represent to the scale of the map the required differences of elevation The results should be checked by the same construction with reference to the other stations from which the points are intersected

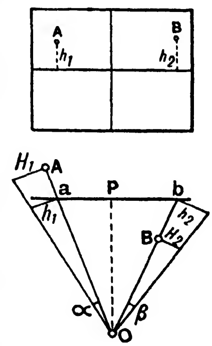


FIG 92

Contours are plotted by interpolation between located points the elevations of which have been determined as above The sketching is facilitated by the circumstance that the horizon line passes though all points on the photograph having the same elevation as the camera. For contouring it is therefore useful to have several photographs of the same area from different elevations. It is to be observed that a line drawn parallel to the horizon line does not mark a contour, nor do points intersected by such a line have a uniform angle of elevation or depression from the camera.

Figs. 93, 94, and 95 show part of a trial photographic survey, and illustrate the foregoing principles.

Stereo - photographic Surveying.—This comparatively recent development of photographic surveying enables the office work to be overtaken in considerably less time than is possible with the older systems The method consists in taking photographs in pairs after

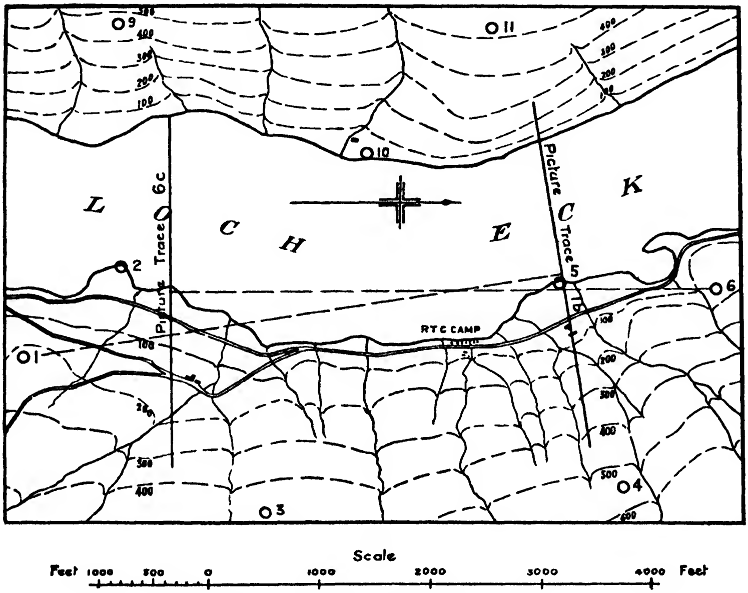
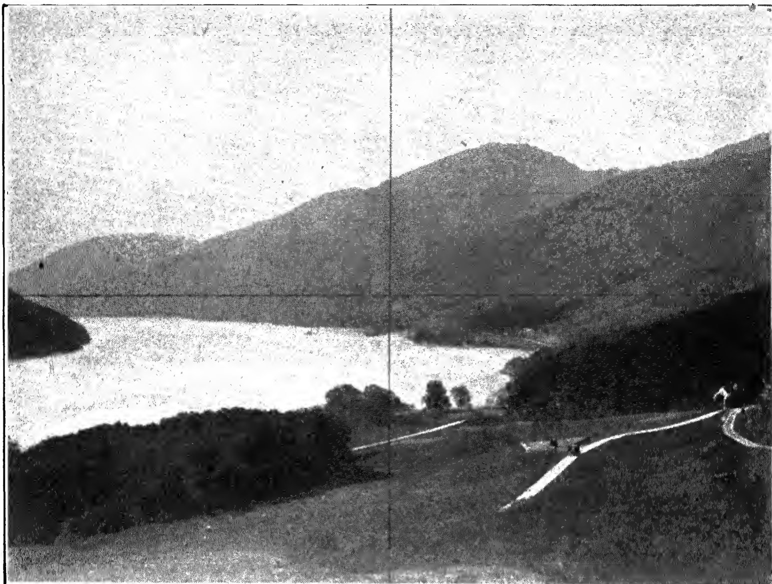


FIG 93 —PHOTOGRAPHIC SURVEY

the manner of ordinary stereoscopic photography The two exposures are made with the plates in the same vertical plane, but not necessarily at the same elevation The stereoscopic base line, or horizontal distance between the parallel principal planes, usually lies between 100 and 400 ft , the necessary length being proportional to the square of the distance to the points being located and the accuracy required and inversely proportional to the focal length The base is measured tacheometrically or by taping The photographs forming a pair when viewed through a special stereoscope show very bold relief because of the much greater distance between the camera stations than between the human eyes The plotting is performed with the aid of such a stereoscope

Field Work.—The instrument specially designed for stereo-photographic survey is the Zeiss photo-theodolite (Figs. 96 and 97). The method of locating camera stations by triangulation or resection is similar to that employed in the older systems The difference in the field work arises in the photographing of every point twice



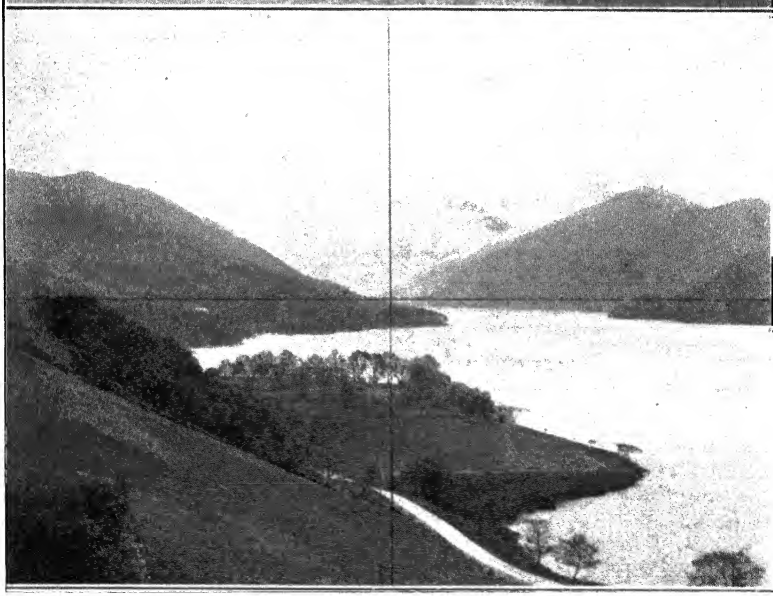


FIG. 94.—PHOTOGRAPH 1 b.
FIG. 95.—PHOTOGRAPH 6 c.

from parallel, instead of intersecting, orientations. When the camera has been set to include any required view at a camera station, the stereoscopic base is set off at right angles to the principal

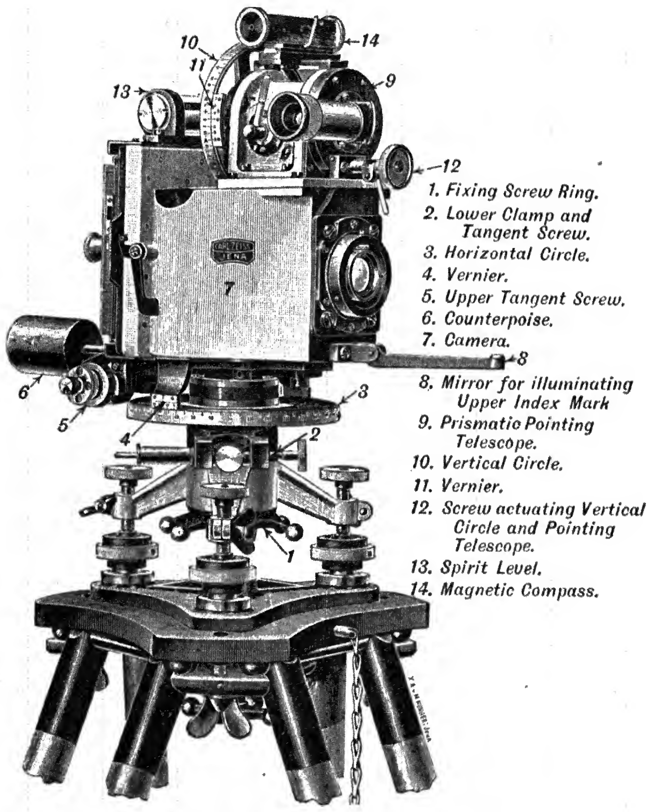


FIG. 96.—ZEISS PHOTO-THEODOLITE.

plane by means of a transverse telescope attached to the camera. The exposure is made, the base measured, and the camera transferred to the other end of the base and set parallel to its previous position by a backsight through the telescope to the point first occupied.

Plotting.—The principle underlying the method of plotting is illustrated in Fig. 98. The points O_1, O_2 represent the optical centre of the lens, and P_1, P_2, the principal point on the negative when the camera is at either end of the stereoscopic base of length *B*. A point A is represented on the negatives P_1a_1, P_2a_2 at a_1 and

a_2 respectively. Let AC = D be the perpendicular distance of A from the stereoscopic base.

$$\frac{D}{CO_2+B}=\frac{f}{P_1a_1}, \text{ and } \frac{D}{CO_2}=\frac{f}{P_2a_2},$$

$$\therefore \frac{D}{B}=\frac{f}{P_1a_1-P_2a_2},$$

$$\text{or } D=\frac{Bf}{P_1a_1-P_2a_2}.$$

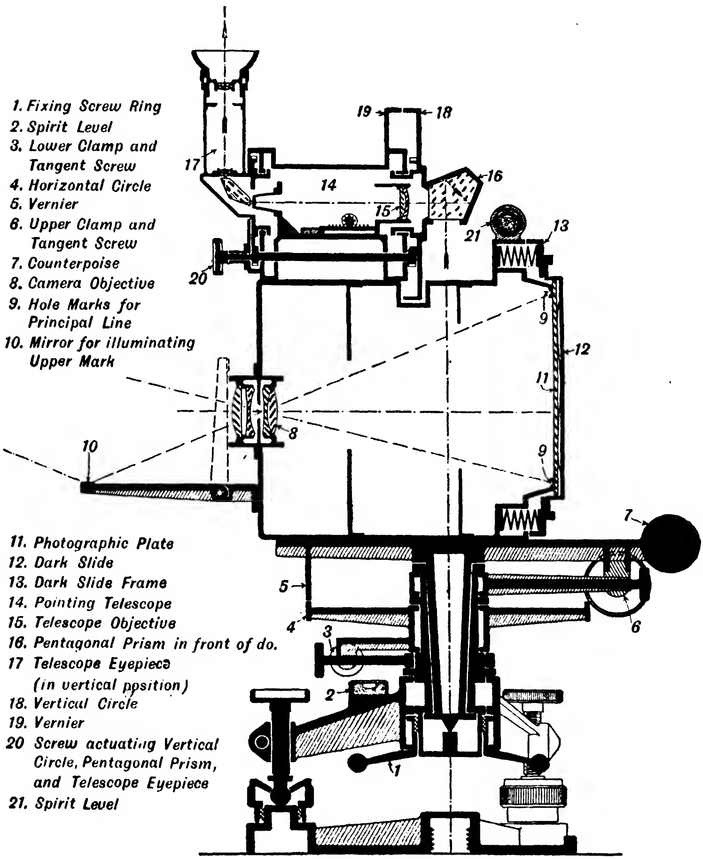


FIG. 97.—ZEISS PHOTO-THEODOLITE—DIAGRAMMATIC SECTION.

The quantity $(P_1a_1-P_2a_2)$ is called the parallax of the point A. If A is a point of known position, the above expression can evidently first be used for the evaluation of B, and from the value so obtained

those of D for unknown points can be derived. To fix the position of A in plan, its direction relatively to that of the principal plane is required, and is given by $\theta = \tan^{-1}\frac{P_1a_1}{f}$ or $\phi = \tan^{-1}\frac{P_2a_2}{f}$. The elevation of A relatively to that of either camera station is obtainable from the length h of its ordinate from the horizon line on the photograph, and has the value $\frac{Dh}{f}$.

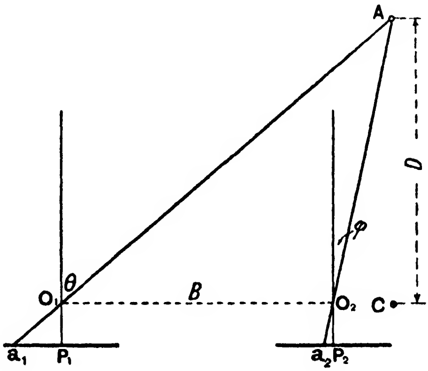


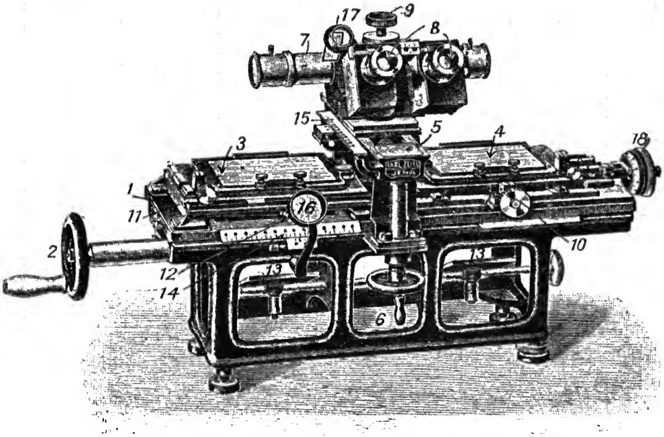
FIG 98

The practical value of the stereo-photographic method has been enhanced by virtue of the means designed for rapidly obtaining the data from the photographs This is accomplished by Pulfrich's stereo-comparator (Fig 99), which consists of a special form of stereoscope in which magnified images of the two photographs are simultaneously examined. Both eyepieces are fitted with exactly similar indices, which can be made to combine stereoscopically with the point to be plotted by adjusting the distance between the two views. Stereoscopic combination occurs when the index appears to be as distant as the point, and the parallax is then given on a scale The movements necessary to bring the point and the index together are also recorded on two scales, which are read for vertical angle and azimuth respectively. The plotting then proceeds in the ordinary manner, always with reference to the left-hand camera station, orientation being effected with reference to one or more known points.

Plotting may be performed much more rapidly by mechanical means. The stereoautograph,* invented by von Orel of Vienna, has proved very successful. It makes the plotting almost entirely automatic, and is capable of tracing the contour lines upon the map.

* See *Engineering News*, Vol. 69, No. 13, 1913, and *The Geographical Journal*, Vol. 38, No. 4, 1911, Vol. 41, No. 4, 1913. and Vol. 59, No. 4, 1922.

A somewhat similar instrument, called the stereo-plotter,* was designed at the Chatham School of Military Engineering, and has been employed on the Survey of India, etc.



1. Table carrying Plates
2. Handwheel for moving Table lengthwise
3, 4. Plates under examination
5. Table carrying Stereo-microscope
6. Handwheel operating Upper Table
7. Stereo-microscope
8. Microscope Eyepieces
9. Microscope Fixing Screw
10. Compensating Screw for difference of Elevation between the two Camera stations
11. Adjusting Screws for Plates
12. Clamp fixing Slide carrying Plate 3.
13. Illuminating Mirrors
14. Abscissa Scale
15. Ordinate Scale
16. Reader for Abscissa Scale
17. Mirror and Lens for reading Ordinate Scale
18. Micrometer for measuring parallax

FIG. 99.—STEREO-COMPARATOR.

Aerial Photographic Surveying.—Photographs taken from aeroplanes were extensively employed during the war for mapping on relatively large scales. The method is very suitable for small scale work, particularly in flat or densely wooded regions, and is well adapted for the rapid revision of existing maps. Aerial surveying is likely to develop extensively in the near future, and it is already being employed in different parts of the world. The use of

* See *The Geographical Journal*, Vol. 31, No. 5, 1908.

aeroplane photographs for the determination of relative elevations is not yet perfected, and the data obtained by photography must be supplemented by trigonometrical levelling.

In general, the positions from which photographs are taken and the direction of the camera axis at the instant of exposure are unknown As presently employed, aerial surveying is dependent upon a system of horizontal control, which, in other than revision work, should consist of a trigonometrical framework. The stations must be so marked that they can be identified on the photographs, which are used for the filling in of the intervening detail.

Plotting.—Plotting is simplified if the only photographs employed are those which have been taken with the camera axis vertical or nearly so. The scale of a photograph is, of course, not uniform, but if three or more points of known position can be identified on it, the detail can be transferred from the photograph to the map by geometrical or optical means. In the case where only three known points are available, the solution is troublesome and involves a knowledge of the focal length of the camera lens or its elevation above the points. In plotting from at least four known points the solution is independent of other data, and is performed as follows

(1) Straight lines on the ground form straight lines on the photograph. Corresponding lines are established by drawing, both on the map and the photograph, the sides and diagonals of the quadrilateral formed by the four fixed points By producing the sides to

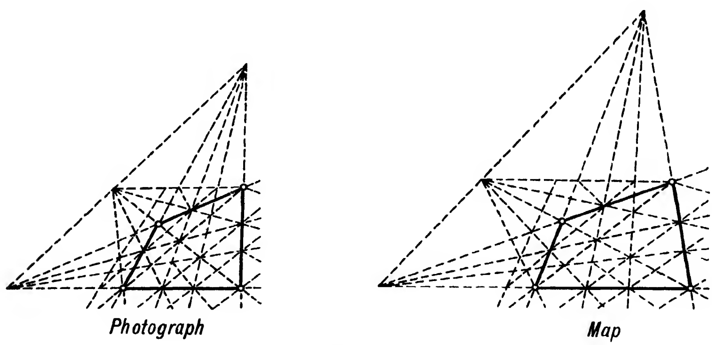


FIG. 100.

intersection and drawing successive diagonals, any number of corresponding points and lines are obtained (Fig. 100) By reference to these, the detail is transferred to the map either by estimation or by the use of proportional dividers

(2) By means of a camera lucida, whereby an image of the photograph is projected upon an adjustable board carrying a tracing of the survey framework. Trigonometrical points on the photograph

are made to coincide with the corresponding points on the map, and the detail is then drawn by going over the lines of the image.

(3) By an enlarging lantern. A tracing of the framework is pinned upon a movable board, and an image of the negative is projected on to it. By adjustment of the board the image is made to coincide with the control points of the tracing. The latter is now replaced by photographic paper, and a print is obtained sensibly true to scale. Finally, the required detail is inked up on the print so that it may be traced. This last has proved the quickest and most accurate method.*

Future developments of aerial photography lie in the direction of making the survey increasingly independent of work on the ground For the survey of relief it is practicable to employ stereo-photography in conjunction with the aneroid heights of the camera

REFERENCES ON TOPOGRAPHICAL AND RECONNAISSANCE SURVEYING

"Anglo-German-Belgian Boundary Commission, 1911 Report on the Work of the British Section" Colonial Office, 1913

BAGLEY, J W "The Use of the Panoramic Camera in Topographic Surveying, with Notes on the Application of Photogrammetry to Aerial Surveys." U S Geological Survey, Bulletin No 657, 1917.

BAKER, I. O *Engineers' Surveying Instruments* (Chapter XII) New York, 1909.

"Bridges-Lee Photo-Theodolite, The" *Engineering*, Vol 64, 1897

BURCHALL, P R. "Elementary Survey of the Present Position of Aerial Photogrammetry" *British Journal of Photography*, Nov 28, 1919

CLOSE, SIR C F. *Text Book of Topographical and Geographical Surveying.* London, 1913.

DEVILLE, E *Photographic Surveying* Ottawa, 1895

DIXON, S. M. "Surveying with a Camera" *Min Proc Inst C E*, Vol. CLXXXIV.

FINCH, J K *Topographic Maps and Sketch Mapping* New York, 1920

FLEMER, J. A "Phototopography as practised in Italy and in the Dominion of Canada, with a brief Historical Review of other Photographic Surveys and Publications on the Subject" U S Coast and Geodetic Survey, Report for 1893, Appendix No 3.

—"Phototopographic Methods and Instruments" U S Coast and Geodetic Survey, Report for 1897, Appendix No 10

— *An Elementary Treatise on Phototopographic Methods and Instruments.* New York, 1906

GANNETT, S S. "Geographic Tables and Formulas" U S Geological Survey, Bulletin No 650, 1916

"General Instructions for Precise and Secondary Traverse" U.S. Coast and Geodetic Survey, Special Publication No 58, 1919

GRIBBLE, T. G "Diurnal Atmospheric Variation in the Tropics and Survey ing with the Aneroid." *Min. Proc. Inst. C.E.*, Vol. CLXXI

* "Mapping from Air Photographs," by Lieut -Col MacLeod, *The Geographical Journal*, Vol. 53, No. 6, 1919.

GUGGISBERG, F. G. *Handbook of the Southern Nigeria Survey.* London, 1911.

Handbook of Professional Instructions for the Topographical Branch, Survey of India, 1905.

HINKS, A R *Maps and Survey* Cambridge, 1913.

— "Stereographic Survey the Stereoautograph" *The Geographical Journal*, Vol 59, No 4, 1922.

HOLDEN, E. H "A New Precise Measurement Method with Special Line Clinometers and Tape Stretchers" *Eng News*, Vol 60, No. 26, 1908.

— "Primary Traverse Work of the Bronx Topographical Bureau, New York City" *Eng News*, Vol 63, No 20, 1910

HOLDITCH, SIR H T. "Geographical Surveying in India" Prof. Papers of the Corps of Royal Engineers, Vol 24, No 11, Chatham, 1898.

HUDDART, L. H L "Sketch-Mapping with Special Reference to Southern Nigeria" *Min Proc Inst. C E*, Vol CLXIX.

HUNTER, J DE G. "Notes on the Use of Thermometers, Barometers, and Hypsometers with Tables for the Computation of Heights" Survey of India, Miscellaneous Publication No 14, 1911.

IVES, H E. *Airplane Photography* Philadelphia, 1920

JONES, E. L "Surveying and Mapping from Airplanes" (Second Pan-American Aeronautic Convention). *Scientific American Supplement*, June 21, 1919, and *Engineering and Contracting*, Sept. 17, 1919.

— "Surveying from the Air" Journal of the Franklin Institute, Vol 193, No. 4, 1922.

LAUSSEDAT, A *Recherches sur les Instruments, les Méthodes, et le Dessin Topographiques* Paris, Pt I, 1901 ; Pt 2, 1903

LEMBERGER, O "Stereophotographic Surveying" *Eng News*, Vol 69, No 13, 1913

LEWIS, C G, and SALMOND, H G "Experiments in Aeroplane Photo-Surveying" Survey of India, Prof Paper No. 19, 1921

McCAW, G. T "Resection in Survey" *The Geographical Journal*, Vol. 52, No. 2, 1918

MACLEOD, M N "Mapping from Air Photographs" *The Geographical Journal*, Vol 53, No 6, 1919

— "Stereographic Survey the Autocartograph" *The Geographical Journal*, Vol 59, No 4, 1922

MERTIE, Jr, J B "Present Status of Photographic Mapping from the Air." *Eng News-Record*, Vol 82, No 21, 1919

MITCHELL, H C "The Topographic Survey of Cincinnati" *Eng News*, Vol 69, No 14, 1913

MOFFIT, F H. "Methods Used in Aero-Photographic Mapping" *Eng News-Record*, Vol 82, No 21, 1919

MULLER, G. A G "The Stereoscopic Method of Surveying, and a First Trial of its Application to a Railway Survey in China" *Trans. Am. Soc. C.E*, Vol LXXIX.

NELLES, D. H "Photogrammetry for Taking Topography of Watershed." *Eng News*, Vol 76, No 19, 1916

NEWCOMBE, S. F. "The Practical Limits of Aeroplane Photography for Mapping." *The Geographical Journal*, Vol. 56, No. 3, 1920.

PERROTT, S. W. "Photographic Surveying." *Trans. Liverpool Eng. Soc*, Vol. 31, 1910.

"Phototopography on the Alaskan Boundary Survey." *Eng Record*, Vol. 60, No. 13, 1909.

PLYMPTON, G. W *The Aneroid Barometer* New York, 1917.

RAYNER, W. H "Allowable Use of Small Angles in Surveying" *Eng. Record*, Vol 68, No 16, 1913

REEVES, E A "Surveying, Past and Present" *Journal of the Royal Society of Arts*, Vol LXIV, Nos 3330–32.

— (ed) *Hints to Travellers*, Vol I Royal Geographical Society, London, 1906

"Report on the Topographical Survey of the Orange Free State, 1905–11." London, 1913

THOMAS, W N "Sources of Error in Photographic Surveying" *Min. Proc. Inst C E*, Vol CCX

THOMPSON, F V "Stereo-Photo Surveying" *The Geographical Journal*, Vol 31, No 5, 1908

"Uganda-Congo Boundary Commission, 1907–8 Official Report of the British Section" Colonial Office, 1909

VERNER, W *Rapid Field Sketching and Reconnaissance*. London, 1889

WAINWRIGHT, D B "A Plane Table Manual" U S Coast and Geodetic Survey, Report for 1905, Appendix No 7 (revised 1916)

WHEELER, A O "Photographic Methods employed by the Canadian Topographical Survey" Department of the Interior, Ottawa, 1905

WHEELER, G M "Facts regarding the Principal Government Surveys of the World" U S Corps of Engineers, 1885

WILSON, H M *Topographic Surveying* New York, 1910

WINTERBOTHAM, H S L "The Economic Limits of Aeroplane Photography for Mapping and its Applicability to Cadastral Plans" *The Geographical Journal*, Vol. 56, No 6, 1920

CHAPTER VII

MAP CONSTRUCTION

In this chapter are considered the usual methods adopted for the plotting of large surveys and the preparation of the final map or series of maps Since the plotting of trigonometrical points, which control the detail, is itself controlled by the projection selected for the map, we shall first deal briefly with the subject of map projections

MAP PROJECTIONS

In the preparation of a map from the final results of the field work the first difficulty presented is that of representing on the plane surface of the paper the relative positions of features existing on the curved surface of the earth It is impossible to do so correctly, and the map must necessarily exhibit a certain amount of distortion or variation of scale, the amount of which rapidly increases the greater the area represented. The difficulty does not arise in the mapping of small areas, in which errors of scale are negligible when the plotting is performed on the assumption of a plane earth or by using computed spherical distances as rectangular co-ordinates When, however, the map embraces a large area, the control points are plotted from their latitudes and longitudes with reference to a number of lines drawn to represent meridians and parallels The system on which these lines are drawn is called a *Map Projection*, the network of lines representing the meridians and parallels being termed the *Graticule*

Classification of Projections.—Since it is impossible to represent correctly any considerable portion of the earth's surface on a plane, there can be no perfect projection The perfect projection would show the meridians as equally spaced straight lines converging correctly towards the poles. The parallels would intersect them at right angles at the correct intervals, and would be parallel to each other. The scale would, in consequence, be constant over the map, and we should have a correct representation throughout the map of distances and directions, and therefore of outlines and their contained areas.

Although constancy of scale is unattainable, many projections have been devised to make the resulting map correct in certain particulars. Thus, the scale may be constant along certain lines ;

directions from one point to any other point on the map may be correctly represented , or outlines, although distorted, may contain their correct areas. Other projections do not preserve any property of the spherical surface exactly, but are designed to give a minimum of distortion over the map or at least a fair general representation

In the manner of their construction, map projections range from geometrical projections in the ordinary sense of the word to purely conventional systems of representation. They may be classified as :

(1) *Perspective*, in which the portion of the earth's surface is drawn as it would be seen from a definite point

(2) *Conical*, in which the meridians and parallels may be supposed to be drawn on the surface of a cone, which is then developed.

(3) *Cylindrical*, in which a cylinder takes the place of a cone

(4) *Zenithal*, in which the lines are drawn on the same system as in the preceding two classes, but upon a plane.

(5) *Miscellaneous*

Projections may also be classed according to special properties they possess as

(1) *Azimuthal* or *Zenithal*, in which the azimuths from the centre of the map to all other points on it are correctly shown. This property is peculiar to the perspective and zenithal projections

(2) *Orthomorphic* or *Conformal*, in which the scale, although varying throughout the map, is the same in all directions at any point, so that very small areas are represented of correct shape. This property cannot be extended to large areas, the shapes of which are sometimes best represented by a projection which has not the orthomorphic property.

(3) *Equal-Area* or *Equivalent*, in which equal areas on the ground are represented by equal areas on the map This property cannot be possessed by an orthomorphic projection, as the representation would then be perfect

The names given to projections are descriptive of the method of construction and most important property, *e g.* Conical Equal-Area, Cylindrical Orthomorphic, etc. Many projections are, however, better known by the name of their inventor.

In the following pages a brief description is given of a sufficient number of the more common projections to indicate the features of the different classes. The factors involved in the selection of a projection suited to a particular case can then be considered.

PERSPECTIVE PROJECTIONS

Projections of this class are formed by ordinary geometrical projection upon a plane. As the name implies, the meridians and parallels are represented as they would be seen from a particular view-point, the distance of which from the centre of the earth controls the properties of the projection. The plane of projection

is normal to the line joining the view-point and the centre of the earth, its position on that line affecting only the scale of the resulting graticule According to the situation of the view-point, the projection plane may be parallel to the plane of a meridian, and the result is a meridian perspective · if the plane is parallel to the equator, we have a polar perspective, and in any other position, a horizontal perspective

Perspective projections are unsuitable for the mapping of comparatively small areas, but are of some importance for the representation of a hemisphere or more.

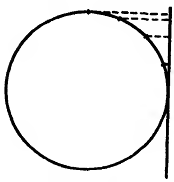


FIG 101

Orthographic Projection.—In this case the view-point is infinitely distant (Fig 101), and the projection is that employed in geometrical drawing The regions near the margin of the map are so greatly contracted as to make the projection of little use

Stereographic Projection.—The view-point is situated on the surface of the earth opposite the hemisphere to be mapped (Fig 102) Features towards the edge of the map are expanded, but the projection is orthomorphic.

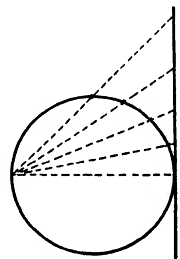


FIG 102

Gnomonic or Central Projection.—The view-point is situated at the centre of the earth (Fig 103), and the projection shows an enormous exaggeration remote from the centre of the map The projection has the special property that, assuming a spherical earth, great circles are represented by straight lines, and, in consequence, this projection has been applied in the preparation of navigational charts for great circle sailing.

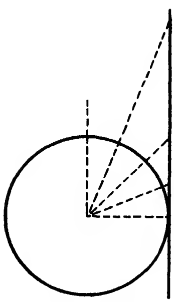


FIG. 103.

La Hire's Projection.—Since the orthographic projection contracts areas near the margin and the stereographic expands them, these effects may be neutralised by the selection of an intermediate view-point In La Hire's projection its distance from the centre of the earth is 1·71 times the radius

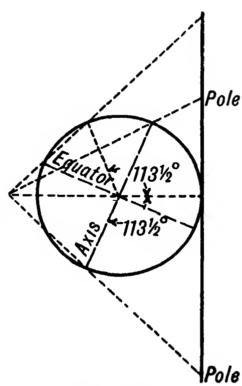


FIG. 104.

Clarke's Minimum Error Projection.—Col Clarke investigated the position of the view-point to give a minimum of total misrepresentation. In his minimum error projection

the point is so placed that the sum of the squares of the errors of scale in two directions at right angles to each other at all points on the map is a minimum. Its distance from the centre of the earth depends upon the extent of surface to be mapped, and has the values 1·625 R for an area contained by a small circle of 40° radius, 1·47 R for the hemisphere, and 1·367 R for a radius of $113\frac{1}{2}$°. This last (Fig. 104) enables about two-thirds of the earth's surface to be represented, and is known as Sir Henry James' projection, for which he originally proposed a distance of the view-point from the centre of the earth of 1·5 R.

CONICAL AND MODIFIED CONICAL PROJECTIONS

These form an important group of projections, and are greatly used in topographical and atlas maps, except for very high latitudes. They are of no value for the mapping of a complete hemisphere.

To follow the construction of the conical projection in its simplest form, let it be supposed that a cone, the axis of which coincides with that of the earth, touches the earth along a parallel of latitude ϕ (Fig. 105). The meridians are projected from the centre of the earth upon the surface of the cone, and, on developing the cone

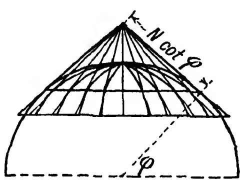


FIG 105.

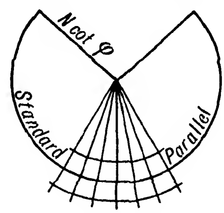


FIG 106.

they appear as equally spaced straight lines radiating from a point (Fig. 106) The parallel of contact, or standard parallel, is represented of correct length by a circular arc, the radius of which is R cot ϕ for a sphere, or N cot ϕ for the spheroid, where N is the length of the normal. The parallels on either side of it are constructed on the development.

Simple Conical Projection with One Standard Parallel.—The standard parallel is one passing through the middle of the area to be mapped. Its radius is computed from N cot ϕ, and the arc is drawn either by compasses or by locating a number of points on it by co-ordinates. The arc is divided off into equal parts representing to scale the linear equivalents of degrees or parts of a degree of longitude, the values of which for the parallel in question are extracted from tables. The meridians are drawn through these points towards the centre, and along any one of them the linear values of arcs of the meridian are set out. Through the points so

obtained the parallels on either side of the standard parallel are drawn as circular arcs concentric with the standard.

Features reproduced correctly in the projection are the scale along every meridian and along the standard parallel, as well as the perpendicularity of the meridians and parallels The scale along the parallels on either side of the standard is too great, the error becoming greater as the distance from the standard parallel increases. The projection is much used, and is suitable for the mapping of a region of any extent in longitude but narrow in latitude.

Conical Projection with Two Standard Parallels.—In this projection, sometimes known as the modified secant conical, the errors of the simple conical projection at a distance from the standard parallel are reduced by the adoption of two standard parallels, one towards the top and the other towards the bottom of the map These are constructed as concentric circular arcs of correct length and at their true distance apart Each is divided to scale in the same manner as the single standard parallel in the last case The meridians are straight lines passing through these points of division, and therefore radiate from the centre of curvature. The radii of the arcs representing the standard parallels are readily deduced from these data, for, in Fig 107, if

m = the true distance between the standard parallels,
p_1, p_2 = the lengths of, say, 1° on each,
r_1, r_2 = their radii,

then $\dfrac{r_1}{p_1} = \dfrac{r_2}{p_2} = \dfrac{m}{p_2 - p_1}$,

or $r_1 = \dfrac{mp_1}{p_2 - p_1}$,

and $r_2 = \dfrac{mp_2}{p_2 - p_1}$.

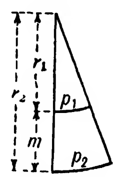


Fig. 107.

The concentric circular arcs representing the intermediate parallels are truly spaced as before

The use of two standard parallels makes the scale along the parallels too small between the standards and too great beyond them, but the error is kept within smaller limits than in the simple conical projection The limits of latitude embraced by the map being known, the standard parallels may be selected conventionally as the parallels situated at a fourth of the range of latitude from the top and bottom of the map respectively Otherwise they are selected so that the central and the extreme parallels may have the same absolute or proportional errors, or such that the mean length of all the parallels may be correct.

Neither of the above two projections is equal-area or orthomorphic, but, while retaining one or two standard parallels, they may be constructed either as equal-area or orthomorphic. For

maps having a small range of latitude there is little difference between such conical projections and those which have been described. They are therefore seldom constructed, more especially as the equal-area property may be obtained with less difficulty by means of certain of the modified conical projections.

Simple Polyconical Projection.—This is a modified conical projection in which each parallel has the same characteristics as the standard parallel of a simple conical projection. The radii of the parallels are given by $N \cot \phi$. The straight line representing the central meridian is intersected by the parallels at the correct intervals, and therefore the arcs representing the parallels are not concentric, but diverge from each other on either side of the central meridian. Each parallel is divided truly, and the meridians, other than the central, are curves through the points of division, and do not intersect the parallels at right angles

The scale along each parallel and along the central meridian is correct, but is too great along any other meridian, the error increasing with distance from the central meridian The projection is neither equal-area nor orthomorphic, but is suited for the mapping of countries having a small range of longitude. The ease with which it may be constructed from tables has led to its wide adoption, notably by the United States Coast and Geodetic Survey, for the graticules of topographical map sheets which need not fit exactly along the edges.

Rectangular Polyconical Projection.—This is a modification of the simple polyconical projection The parallels are constructed in the same way, but only one is divided truly The meridians are curves passing through these points and intersecting all the parallels at right angles The right angle intersections are therefore obtained by sacrificing the scale along all but one of the parallels. The projection is neither equal-area nor orthomorphic.

The International Map Projection.—A modified polyconical projection has been adopted for the International Map on the scale of 1/1,000,000. The sheets normally cover an area of 4° in latitude and 6° in longitude. The top and bottom parallels of each are constructed as in the simple polyconical projection, and are divided truly. The distance between them, however, is such that the meridians situated at 2° on either side of the centre are straight lines of correct length. The other meridians are straight lines joining corresponding points on the extreme parallels, and the intermediate parallels divide the meridians into equal parts.

Bonne's Projection.—This is a modified conical projection designed to represent areas correctly. One standard parallel is drawn as in the simple conical projection. The centre meridian is a straight line divided truly, and through the points of division the remaining parallels are drawn as concentric circular arcs with

the same centre as the standard. Each parallel is divided truly, and the meridians become curves passing through corresponding points on each.

The scale is correct along the central meridian and along each parallel, and the projection is evidently equal-area since the perpendicular distance between the parallels is correct. The scale along the meridians increases towards the sides of the map, and the projection is not suitable for a wide range of longitude. It has been used to a considerable extent on European surveys, and is that employed by the Ordnance Survey for the one-inch and smaller scale maps of Scotland and Ireland.

Sinusoidal Equal-Area or Sanson-Flamsteed Projection.—This is a special case of the last in which the equator is the standard parallel. The parallels become straight lines, correctly spaced and perpendicular to the straight central meridian The other meridians, obtained by dividing each parallel truly, would become sine curves for a spherical earth The projection has all the properties of Bonne's, and is used for the mapping of areas extending on either side of the equator.

CYLINDRICAL PROJECTIONS

These may be conceived as drawn on the development of a cylinder, and may be regarded as conical projections in the limiting case where the apical angle of the cone is zero. With the exception of Mercator's projection, they are of little or no practical value.

Simple Cylindrical or Square Projection.—The tangent cylinder touches the earth along the equator, which is therefore the standard parallel The meridians project into parallel straight lines perpendicular to the equator, and are spaced at their equatorial intervals The parallels on either side of the equator are parallel straight lines at their correct distances from it, so that the graticule consists of a series of rectangles which are practically squares. The exaggeration of scale along the parallels becomes very great away from the equator, and the projection could be applied only to the case of a very narrow strip along the equator By conventionally applying the system to the mapping of a narrow strip along any other parallel with that parallel as a standard, the rate of increase of distortion on either side is, of course, greater.

Cylindrical Projection with Two Standard Parallels.—The distortion of the simple cylindrical projection can be reduced by making the scale correct along two parallels, one on either side of the equator and equidistant from it Large errors of scale, however, occur along the other parallels

Cylindrical Equal-Area Projection.—The equal-area property may be obtained for the cylindrical projection by drawing the

parallels at $R \sin \phi$ from the equator, where R is the radius of the earth, assumed spherical, and ϕ is the latitude. The interval between the parallels therefore decreases north and south of the equator, and the projection could be usefully applied only in the case of a narrow strip along the equator.

Mercator's or Cylindrical Orthomorphic Projection.—To secure orthomorphism, the spacing of the parallels must be increased as we depart from the equator, in order to keep pace with the increase of scale along the parallels common to all cylindrical projections. For a sphere, this requires that the distance from the equator of a parallel in latitude ϕ shall be $= 2{\cdot}30259\ R \log \tan (45°+\frac{1}{2}\phi)$.

The projection shows an increasing exaggeration of areas away from the equator, and, as is evident from the formula, the representation of the poles is infinitely distant from the equator. The projection is, however, of great value for the construction of navigational charts, by virtue of the property that a line of constant bearing, known as a loxodrome or rhumb line, appears as a straight line on the chart. The rhumb line course between two points is longer than that along a great circle, but on account of its simplicity rhumb line sailing is preferred except for very long voyages To find the course to be followed by a ship sailing on a constant bearing between two points represented on a Mercator chart, it is only necessary to draw a straight line between them and protract the angle it makes with any meridian. The compass bearing is then obtained by applying the magnetic declination to the true bearing as protracted.

ZENITHAL PROJECTIONS

The essential feature of this class of projections is the correct representation of azimuths from the centre of the map The same property is possessed by the perspective projections, but is obtained in a different manner in those to be described, which are therefore classed by themselves. They may be regarded as the other limiting case of the conical projection. As the standard parallel of the simple conical projection increases in latitude, the tangent cone becomes flatter, and in the limit becomes a plane, tangent at the pole. A projection drawn on this plane by the same system as was used for the cone has the property of being azimuthal.

Zenithal Equidistant Projection.—In the case strictly analogous to the simple conical projection the pole forms the centre of the projection, and the meridians radiate from it at their true angles. The parallels become complete circles correctly spaced along the meridians and with the pole as their common centre. The resulting projection enables distances as well as azimuths from the pole to be shown correctly, and is suitable for the mapping of the polar regions, but the scale along the parallels increases away from the centre.

The same principle may be applied to the construction of a projection for any portion of the earth's surface by considering the projection as drawn upon a plane, tangent at the centre of the area to be included. Equally spaced rays from the centre would then represent great circles passing through the point of tangency and making equal angles with each other. The concentric circles equally spaced along the rays would represent equidistant circles on the earth. These lines are not required, and the meridians and parallels must be constructed. They are not simple curves, and are best drawn by plotting a number of their intersections by the use of tables or by computing their polar co-ordinates from the centre of the projection, as follows.

Let ϕ_c = the latitude of the selected centre of the projection,
ϕ = the latitude of the point to be plotted,
δL = the longitude difference between the centre and the point,
A = the azimuth of the ray from the centre to the point,
l = its length,
a = the angle it subtends at the centre of the earth

Then, for a sphere,

$$\cos a = \sin \phi_c \sin \phi + \cos \phi_c \cos \phi \cos \delta L$$

$$\sin A = \cos \phi \operatorname{cosec} a \sin \delta L$$

$$l = \frac{\pi R a^\circ}{180^\circ}.$$

The intersections are plotted by rectangular co-ordinates from the computed polar co-ordinates.

Zenithal Equal-Area Projection.—By sacrificing the equidistant property we may construct an equal-area projection by making $l = 2R \sin \frac{1}{2}a$. This projection, as well as the last, is useful for the mapping of large areas, and is much used for atlas maps.

Airy's Zenithal Projection.—This projection was designed by Sir George Airy to give a zenithal projection with a minimum of misrepresentation. It is that used by the Ordnance Survey for the ten miles to the inch map of England.

MISCELLANEOUS PROJECTIONS

The following conventional projections cannot properly be classed under any of the foregoing heads.

Globular Projection.—On account of its simplicity, this projection is frequently used for the representation of a hemisphere. The equator and the central meridian are equal straight lines at right angles, and each is divided into equal parts. The other meridians are circular arcs passing through the poles and the points of division of the equator. The meridian forming the circumference of the

map is equally divided, and the parallels are circular arcs through the points so obtained and the corresponding points of division of the central meridian.

Polyhedric Projection.—This projection, used for the topographical maps of several European surveys, may be regarded as a series of orthographic projections upon a number of different planes, one for each sheet. The plane of projection for a sheet is that passing through the four points on the earth's surface which form the corners of the sheet.

Projection by Rectangular Co-ordinates or Cassini's Projection.—A map is constructed on this projection when trigonometrical points are plotted by rectangular co-ordinates with reference to a central meridian and a fixed point upon it. The lengths of the perpendicular offsets from the reference meridian and their positions on it from the initial point are computed from the known latitudes and longitudes of the points The perpendicular from any point is an arc of a great circle at right angles to the meridian It is shorter than the arc of parallel intercepted between the point and the reference meridian, and intersects the latter at a different point. It was seen on page 189 that in a given distance along a great circle the divergence in latitude between the great circle and the parallel varies as the square of the distance and their difference in longitude is proportional to the distance.

To draw the meridians and parallels, a sufficient number of their intersections are laid down by rectangular co-ordinates, and in the resulting graticule the parallels and the meridians, other than the central one, are curved lines. The scale is correct along the central meridian, but is too great along the others, the error increasing towards the sides of the map The projection in unsuitable for a map covering a wide range of longitude, but is useful for the mapping of comparatively small areas. It was adopted by the Ordnance Survey for the maps of England on scales of from one to four miles to an inch and for the six-inch map of Great Britain and Ireland. Tables for its construction for the latitudes of Great Britain are published in the official " Account of the Methods and Processes adopted for the Production of the Maps of the Ordnance Survey of the United Kingdom "

Trapezoidal Projection.—In this purely conventional projection both meridians and parallels are straight lines The truly divided lines are the central meridian and two of the parallels, one at about a fourth of the range of latitude from the top of the sheet, and the other the same distance from the bottom. The projection is quite unsuitable for other than very small areas.

Field Sheet Projections.—These projections are used for the plotting of trigonometrical points controlling detail to be mapped in the field. Owing to the limited area embraced by each field

sheet, the projection may be of an approximate character designed for ease of drawing. Their construction is described on page 273.

The Choice of a Projection.—Several considerations have to be taken into account by the cartographer in selecting the projection best suited to a particular case, and definite rules cannot be laid down. The choice must be largely governed by: (1) the region to be mapped, particularly as regards its extent, general shape, and position on the earth ; (2) the purpose of the map , (3) whether the map is contained on (*a*) a single sheet or a series of sheets which can be fitted together, or (*b*) a series of independent sheets

(1) The choice of a projection for very large areas, such as a continent or a hemisphere, is a matter for the atlas maker. A fair general representation is all that can be secured For the largest areas, such as a hemisphere, a projection having the azimuthal property, such as the zenithal equidistant or equal-area, or Airy's and Clarke's minimum error projections are the most generally applicable, although the conventional globular is very commonly used.

As the area decreases, the choice becomes wider, and the projection may be selected to suit the shape of the area and its position on the earth's surface Zenithal projections are suited for roughly circular areas It has been seen that a simple conical projection is suitable for areas having a small range of latitude, while other projections, such as Bonne's, are better adapted for areas narrow in longitude.

Many projections, such as the conical, are ill adapted for the mapping of high latitudes, for which the zenithal projections are suitable For the mapping of equatorial regions the choice is wider, and for large areas the zenithal projections and Sanson-Flamsteed's are most commonly used

(2) The choice of a projection may be entirely controlled by the special purpose for which the map is intended to be used Thus, the deficiencies of the Mercator and gnomonic projections are overruled because of their suitability for navigational charts. In the case of maps made for statistical purposes the equal-area property is all important, while atlas maps made to exhibit trade routes from a centre should be constructed on the zenithal equidistant projection.

(3) The whole area surveyed may be represented on a series of sheets which may, or may not, be so projected that they can be exactly fitted together In the former case, there is one graticule, of which the individual maps contain different parts Some of the maps are, in consequence, more distorted than others, and this system should not be used if the series of maps embraces a very large area, since each sheet must be sensibly correct A one-projection series is, however, quite suitable for the topographical maps of a comparatively small country, provided the projection is well chosen. It has the merit that by fitting together neighbouring

sheets a smaller scale series may be reproduced by photography. The British Isles have been mapped on this system, the individual sheets being rectangular and not bounded by meridians and parallels It is, however, generally desirable that the edges of sheets should coincide with meridians and parallels for convenience in taking bearings from the map A conical projection with straight meridians is suitable, and by using two standard parallels the scale error is kept small The latter was originally proposed for the 1/1,000,000 maps of India before the International Map projection was adopted.*

When all the individual sheets are not meant to fit together exactly, the choice of a projection is not so difficult, since many projections are indistinguishable from each other within the small area embraced Ease of construction is then an important factor The polyconic projection, simple or rectangular, is that most frequently used, since the radii of the parallels depend only upon their latitudes and are tabulated once for all. Adjacent sheets will fit along the parallels the fit along the meridians, although not exact, is sufficiently good to enable field sheets to be joined.

MAP DRAWING

Plotting by Rectangular Co-ordinates.—In the case of comparatively small surveys the plotting of all control points is most simply performed by rectangular co-ordinates from a central meridian Their values may be worked out from the lengths of the triangle sides by plane trigonometry, the curvature of the earth being disregarded. Otherwise, the co-ordinates are computed from the geographical co-ordinates of the stations No graticule is required for the purpose of plotting, but it can be constructed on the final map

In large surveys all control points, both on the field sheets and the fair map, are plotted by geographical co-ordinates with reference to a graticule.

Construction of the Field Sheet Graticule.—The projections used for field sheets are approximately polyconical, but, to simplify their construction, each curve is drawn as a series of straight lines. The intersections of meridians and parallels are plotted either by rectangular co-ordinates or by the intersections of arcs The data required in their construction will be found in such publications as the British War Office Projection Tables, the Survey of India Auxiliary Tables, the United States Coast and Geodetic Survey Tables, and in the official *Text Book of Topographical and Geographical Surveying* by Col. Sir Charles F. Close.

* "On the Projection for a Map of India and Adjacent Countries on the Scale of 1/1,000,000," by Col St G C. Gore, Survey of India Professional Paper No. 1.

A graticule may be constructed without the aid of graticule tables by computing for the graticule interval the linear distance along the meridians and along each parallel and plotting each compartment of the graticule as a trapezium with the meridians equally inclined to the parallels. This system, known as the Survey of India, or Col. Blacker's, projection, is sufficiently accurate for limited areas, and is largely used for field sheets. Tables of the quantities used in its construction are given in the Indian Auxiliary Tables, Thuillier and Smyth's *Manual of Surveying for India*, and the Royal Geographical Society's *Hints to Travellers*, but the plotting may easily be performed with the aid of a table of linear values of arcs of latitude and longitude.

As an example, let it be required to construct a graticule of $30' \times 30'$ with $15'$ intervals between latitudes $40° 45'$ and $41° 15'$N.

Taking the straight line ABC (Fig 108) as the central meridian, mark off AB and BC to represent on the required scale the linear distances along the meridian from latitude $40° 45'$ to $41° 0'$ and from $41° 0'$ to $41° 15'$ respectively From the tables, these distances in miles are AB = 17·251 and BC = 17·252 With centre A and radius representing the linear value of $15'$ of arc of parallel in latitude $40° 45'$, *viz* 13 120 miles, describe short arcs at D and E. The corresponding radii for the latitudes of B and C are 13·071 and 13 021 miles respectively, and similar arcs are drawn at F and G from centre B and at H and J from centre C The corners of the trapezia are located by diagonals, the lengths of which are obtained from the tables or are calculated from

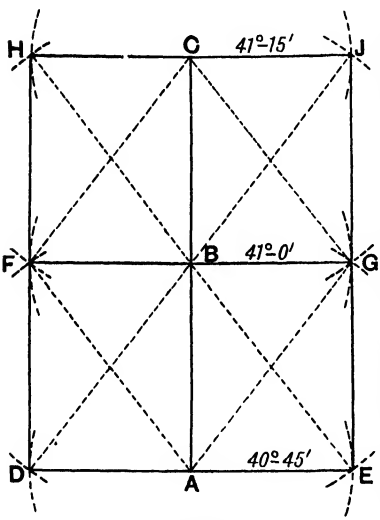


FIG 108

$$d = \sqrt{m^2 + p_1 p_2},$$

where m = the length of the meridian in the trapezium,

p_1, p_2 = the lengths of the parallels.

Having obtained the length of the diagonals in the two lower figures as 21·658 miles and that for the upper figures as 21·629 miles, points D, E, F, G, H, and J are fixed by intersecting the previously drawn arcs with arcs swept from centres A, B, and C. The points are then joined with straight lines. Each line is commonly divided into three equal parts, so that, by joining corresponding points, the graticule is divided into compartments of $5'$ of latitude and longitude.

The War Office tables are arranged for plotting as follows. The line ABC (Fig. 109) is drawn as before, and perpendiculars are erected through its points of division. Along these lines the abscissæ Ad, Ae, Bf, Bg, Ch, and Cj are set off equal to the tabulated values corresponding to the respective latitudes. The positions of the corner points d, e, h, and j are checked by scaling the diagonals dj and eh, which are also tabulated. From d, e, f, g, h, and j are erected perpendiculars to the construction lines, on the side next the pole, and on them are marked off the ordinates dD, eE, etc , the values of which are tabulated for the respective latitudes and distances from the central meridian. The points so obtained are the intersections of the required meridians and parallels

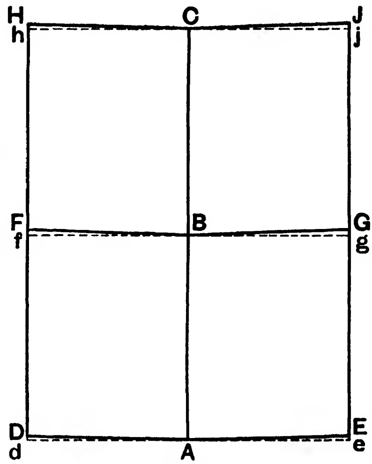


Fig 109

Plotting Trigonometrical Points.—The mapping of detail on the field sheets being based upon the station positions, the latter must first be plotted with reference to the meridians and parallels of the graticule by using the computed latitudes and longitudes of the stations. Each point is plotted from the nearest meridian and parallel by co-ordinates parallel to them The lengths of these co-ordinates represent the linear equivalents of the angular distances of the point from the graticule lines, and are computed by the use of tables giving the linear values of arcs of meridian and of parallel in different latitudes.

In place of scaling linear co-ordinates, the points may also be plotted by proportional division of the angular distances between the graticule lines. This is best done by diagonal scale Although less accurate than plotting by linear distances, this method is sufficiently so for small scale work, and when many points have to be plotted it proves the quicker method. It has the merit that points may be placed in their correct relative positions on a graticule which has suffered expansion or contraction through change of humidity.

Fig 110 illustrates the plotting of a station of latitude 1° 10′ 43″·75 N. and longitude 30° 11′ 14″·73 E. on a graticule which has been subdivided to 5′ sections.

The angular distances of the point from the nearest graticule lines are 43″·75 N. and 1′ 14″·73 E. To convert these to linear distances for the first method of plotting, we find from the tables

that in mid-latitude 1° 10′ 21″·87, say 1° 10′, the value of 1″ of arc of meridian is 100·7658 ft, and in latitude 1° 10′ 43″·75, say 1° 11′, the value of 1″ of arc of parallel is 101·4323 ft. Therefore the required co-ordinates are

$$43''\cdot75 \times 100\cdot77 = 4{,}409 \text{ ft. N. of the } 1^\circ\ 10' \text{ parallel,}$$
$$\text{and } 74''\cdot73 \times 101\cdot43 = 7{,}580 \text{ ft E of the } 30^\circ\ 10' \text{ meridian.}$$

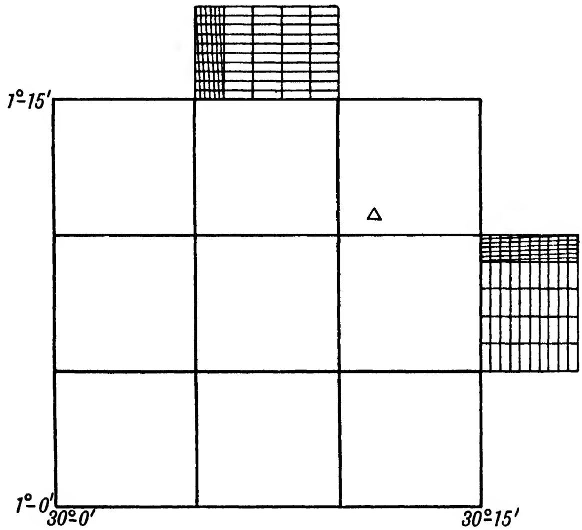


Fig. 110.

In applying the second method, two diagonal scales are constructed for the subdivision of a minute of meridian and of parallel respectively. The scales illustrated read to single seconds, but dimensions can be taken off to the nearest 0·1 sec. In high latitudes the value of 1 min. of parallel changes rapidly, and the same diagonal scale should not be used for all parallels: at least two, one near the top and the other near the bottom of the sheet, are required.

In either method the position of the plotted point should be checked by measurements from the farther sides of the graticule compartment.

Completion of Field Sheets.—In the case of a small topographical survey for a special purpose, the field sheets may form the final maps. When, as in the production of maps for publication, they are used as material from which the final map is compiled, they must still be completed with the same refinement, since their accuracy cannot be improved upon by the cartographer.

During the field work, information collected on auxiliary sheets is inserted in the field sheets as soon as possible, and the latter are inked up daily. It is important that the sheets should not be

overburdened with detail, particularly if the scale is larger than that of the final map. To promote clearness, it is a good practice to substitute numbers for names on the field sheets, the names being set out in a reference table or book. Before field sheets can be passed as complete, the edges should be compared with those of the adjoining sheets, and any discrepancies adjusted out or resurveyed.

Construction of the Final Map.—The data from which the fair map sheets are prepared are the survey records of the framework and the field sheets and field books The projection having been selected, the graticule is constructed by plotting a number of intersections of meridians and parallels by means of rectangular co-ordinates. Great care is necessary to ensure perpendicularity of the co-ordinates, and the operation is facilitated by the use of the Coradi co-ordinatograph If a table of co-ordinates is not available, the projection must be calculated, and, unless for the smallest scales, it is necessary to take account of the spheroidal form of the earth. A sufficient number of points should be plotted that they can be joined by straight lines without spoiling the appearance of the graticule, but a set of pearwood curves of large radius is useful The stations of the framework are plotted as on the field sheets and carefully checked.

The transfer of detail from the field sheets is best accomplished by squaring in. In this process a series of small squares is drawn to the appropriate scales on both the field sheet and fair map, their positions relatively to the control points being exactly the same on both sheets The detail is then plotted on the map with reference to the squares The method is particularly useful when the field scale has to be reduced. When a general topographical map is being constructed on a small scale, the cartographer must be competent to make a selection of the features to be included, so that the physical character of the country may be clearly represented.

Representation of Relief.—The manner in which the relief of the ground is shown contributes largely to the quality of the map. The principal methods of representation are by means of: (*a*) Contours or Form Lines, (*b*) Hachuring; (*c*) Shading. These are used singly or in combination, but a map is of minor utility unless contours, or at least form lines, are shown. Although the other methods are sometimes used by themselves, they are best regarded as aids to the contour line system

Contours.—Contour lines give by far the most precise delineation of relief, and are indispensable for engineering and other purposes for which the values of the elevations are required. They should be numbered, and the vertical interval should have a constant value throughout the map. They do not obscure the detail as much as other systems, but to avoid any tendency to confusion they are best shown in brown. In rugged country, where the contours run

close together, the nature of the relief may be presented without the necessity for close inspection of the map by drawing every fifth or tenth contour with a bolder line than the others. Form lines, or sketched contours, although of inferior precision, possess the merits of contours as a means of representing form, and are superior to hachuring or shading unaccompanied by contours.

To the inexperienced map user at least, the contour system alone does not impart to a map such a pictorial effect of relief that the general character of the country embraced by the sheet can always be understood at a glance. The difficulty may be overcome by the addition of hachures or hill shading but more easily by adopting the layer system of colouring the areas between adjacent contours. A range of graded tints is selected, each of which is applied between particular contours. There is no universally adopted colour scale. That on the Ordnance Survey half-inch maps embraces thirteen tints ranging, as the elevation increases, from green through buff to brown. Yellow and orange may be inserted between green and the paler browns, and the deepest browns sometimes merge into red, or are followed by blue greys and finally by white for the highest peaks The result is sometimes very effective, but it frequently happens that the deeper tints are made so dark that the detail is obscured If the map includes a wide range of elevations, it is difficult to obtain an entirely satisfactory system of colours for the higher ground, and it is then preferable to make one tint embrace several contours.

Hachuring.—Hachuring is a method of indicating relief in which short lines, called hachures, are drawn at right angles to the contours, *i e.* along the direction of steepest slope. On the assumption of vertical illumination of the features mapped, the flatter the ground the lighter does it appear, and the relative steepness of the slopes is so indicated in drawing the hachures On flat slopes the lines are rather widely spaced, and are drawn lightly so that the white intervening spaces predominate As the contours approach each other, the hachures are drawn closer together, and the lines are heavier, giving an increasingly dark tone as the slope increases. Definite scales of ratios between black and white for different angles of slope have been worked to, but, as they are difficult of application by those unaccustomed to their use, the variations are commonly made arbitrarily.

It is impossible to secure that every irregularity which can be shown by contours will be faithfully represented by hachuring. Hachures should therefore be accompanied by contours. The pictorial effect of relief may be improved by drawing the hachures in such a manner as to suggest oblique illumination. The light is arbitrarily assumed to come from the north-west or upper left-hand corner of the sheet and at 45° elevation. Slopes facing it are shown lighter, and those in shadow darker, than under vertical

illumination. The result gives an entirely false impression of the steepness of the ground, and the addition of contour lines is more than ever necessary.

The drawing of hachures is tedious, and requires to be very carefully performed in order to produce the required effect of relief. They must be exactly perpendicular to the contours, and in consequence are curved between non-parallel contours. The lines should preferably be slightly wavy, and should break joint on the

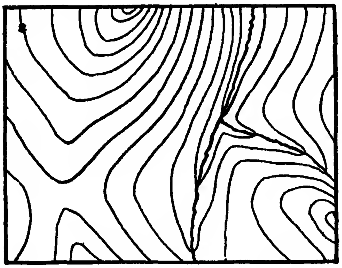


Fig. 111.—Contouring.

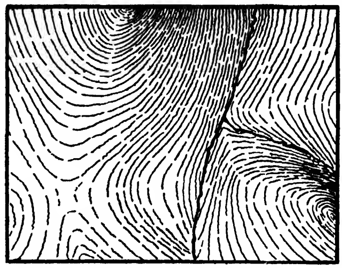


Fig 112 —Horizontal Hachuring

Fig. 113.—Horizontal Hachuring—Oblique Illumination

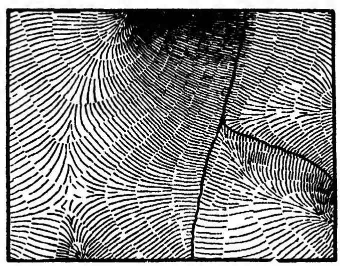


Fig 114.—Vertical Hachuring.

contours or, where the ground is very flat, on intermediate contours sketched in pencil.

Hachures possess the serious disadvantage that, unless perhaps when drawn in brown, they tend to obscure the other detail. In map reproduction, on other than the smallest scales, hachuring is now being susperseded by the layer system, but it is sometimes employed in combination with the latter. Examples of hachuring in black may be studied on the older one-inch Ordnance Survey maps.

The above system is called vertical hachuring to distinguish it from the horizontal system. In the latter, the strokes are drawn

parallel to the contours, and they therefore resemble broken lines. By introducing the same number between each pair of contours, the tone is darkened as the slope increases, and the effect of relief may be emphasised by drawing thicker lines on steep slopes Horizontal hachuring lends itself to oblique illumination, but it has not found the favour which has been accorded to the vertical system.

For the piece of ground represented by contours in Fig. 111, horizontal hachuring is shown with vertical illumination in Fig 112, and with oblique illumination in Fig 113 Fig 114 represents vertical hachuring with vertical illumination

Shading.—Shading differs from hachuring in that different intensities of slopes are indicated by shading applied with a stump or by colour wash instead of by lines. It has the merit of being less laborious than hachuring, but does not exhibit as much detail of form Shading is performed in grey or in brown, and the illumination may be vertical or oblique. The latter gives an effective result if the shading is not so heavy as to obscure the contours and other detail.

Except on atlas scales, shading is of little value unless the contours are drawn. In modern map reproduction the method is frequently used in combination with contours, the shading being printed in half tone. Hachures or layer tints may also be added, but in the latter case shading interferes with the colours, and, to minimise the confusion, very pale Indian ink should be used for the shade washes

Conventional Signs.—As the scale of the map decreases, it becomes increasingly difficult to show clearly all the information it is desired to convey A full code of symbols is necessary to avoid overcrowding of the map with written descriptions. In the case of official maps, a sheet of conventional signs is prepared, and these must be strictly adhered to by the draughtsmen, and should be understood by the user of the map

The civil engineer engaged in mapping should be guided in his selection of symbols by referring to official maps on a similar scale. It will usually be unnecessary to symbolise as much detailed information regarding existing artificial features as is required on general maps used for varied purposes Information relating to circumstances likely to influence the design and construction of the proposed works must, however, be shown as clearly as possible. The character of lines of communication is always important. Railways are usually shown in black, and distinction is made between double and single lines, and standard and narrow gauges. Roads, as well as railways, must be drawn of exaggerated width for clearness, and indication should be given of their quality, particularly with regard to suitability for mechanical transport Different classes are conventionally represented by differences in

breadth, or in the boldness of the lines, or by colouring the better roads All three conventions may be combined

Triangulation and traverse stations should be shown with their reference numbers, different symbols being used to indicate the system to which they belong Elevations shown should also be distinguished by different styles of figures according to the manner of their determination

All the symbols adopted should be illustrated in a list of conventional signs drawn in a convenient position on the map or on a separate sheet

Finishing the Map.—Unless for indicating relief, colour washes should be used sparingly, but water areas should be tinted blue Woods may be shown by a pale tint of green with or without tree signs, and boundaries are commonly emphasised by a band of colour

In lettering the map, all names should be placed to the right of the point to which they refer and parallel to the top of the sheet, except in the case of such items as rivers, ranges of hills, or railways Different kinds of features should have distinctive, but not elaborate, styles of lettering both for clearness and to avoid monotony

The border should be of simple form, and should be divided in latitude and longitude If the map extends over several sheets, it is an advantage in using the separate sheets if narrow overlaps are repeated on adjacent sheets

In the case of maps intended for photographic reproduction, it is usual to perform the plotting to twice the scale of the reproduction If the map is to be printed in more than one colour, drawings of the features to appear in each colour are prepared for photography *

Models.—The civil engineer may be called upon to prepare a model of the site of large works to illustrate the topographical conditions in the most expressive manner possible and one which readily appeals to persons unaccustomed to map reading

The model is constructed from the data furnished by a contoured map At the outset the vertical scale must be selected This will be the same as the horizontal scale for large scale models, but for scales smaller than about one inch to a mile the relief may be emphasised by exaggerating the vertical scale two or three times. For models on atlas scales greater ratios are adopted

The model is built up of sheets of paper, cardboard, or thin wood, each having a thickness proportional to the vertical interval between the contours The individual contours are traced on these sheets from the map by means of transfer paper, and each sheet is then neatly cut along the outline of its contour The shaped sheets are superimposed and glued together in their proper order and position, the correct placing of one sheet on another being facilitated if the

* See *Text Book of Topographical and Geographical Surveying*, edited by Col Sir C F Close and Col H St J L Winterbotham

upper contour is traced on the lower sheet The resulting model is terraced, but is of value in that it exhibits the positions of the contours For most purposes for which models are required, the steps should be filled in with wax or modelling clay, with due regard to topographical form The model is finally painted, detailed, lettered and varnished.

REFERENCES ON MAP CONSTRUCTION

"Account of the Methods and Processes adopted for the Production of the Maps of the Ordnance Survey of the United Kingdom" London, 1902

ADAMS, M *A Little Book on Map Projection* London, 1914

ADAMS, O S "General Theory of the Lambert Conformal Conic Projection" U S Coast and Geodetic Survey, Special Publication No 53, 1918

— "General Theory of Polyconic Projections" U S Coast and Geodetic Survey, Special Publication No 57, 1919

— "A Study of Map Projections in General" U S Coast and Geodetic Survey, Special Publication No 60, 1919

CLOSE, SIR C F *A Sketch on the Subject of Map Projections* London, 1901

— and WINTERBOTHAM, COL H ST J L *Text Book of Topographical and Geographical Surveying* London, 1925

— and CLARKE, A R "Map" *Encyclopædia Britannica*, 14th edition, 1929.

CRAIG, J I "Map Projections" Egyptian Survey Dept, Technical Lecture No 3, 1909

— "The Theory of Map Projections, with Special Reference to the Projections used in the Survey Department" Egyptian Survey Dept, Paper No 13, 1910

CRAIG, T "A Treatise on Map Projections" U S Coast and Geodetic Survey, 1882

DEETZ, C H "Lambert Conformal Conic Projection with Two Standard Parallels, including Comparison of Lambert Projection with Bonne and Polyconic Projections" U S Coast and Geodetic Survey, Special Publication No 47, 1918

DUCHESNE, C *Les Projections Cartographiques* Brussels, 1907

ECCLES, J A "On a Simplification of the Computations relating to Rectangular Co-ordinates" Survey of India, Miscellaneous Publication No 13, 1911

FINCH, J K *Topographic Maps and Sketch Mapping* New York, 1920

GERMAIN, A *Traité des Projections des Cartes Géographiques* Paris, *circ.* 1865

HINKS, A R *Maps and Survey* Cambridge, 1913

— *Map Projections* Cambridge, 1921

JURISCH, C H L M *A Treatise on Map Projections* Cape Town, 1890

MORRISON, G J *Maps, Their Uses and Construction* London, 1902

"On the Projection used for the General Maps of India" Survey of India. Miscellaneous Publication No 9, 1903

REEVES, E A *Maps and Map-making* London, 1910

Resolutions and Proceedings of the International Map Committee, Assembled in London, Nov 1909

ROYAL GEOGRAPHICAL SOCIETY *New Geodetic Tables for Clarke's Figure of* 1880, *with Transformation to Madrid* 1924 *and other Figures* London, 1927

SCHOTT, C A "A Comparison of the Relative Value of the Polyconic Projection used on the Coast and Geodetic Survey with some other Projections" U S Coast and Geodetic Survey, Report for 1880, Appendix No 15

SMITH, R S *Manual of Topographical Drawing* New York, 1894

STUART, E R *Topographical Drawing* New York, 1917

"Tables for Polyconic Projection of Maps, based upon Clarke's Reference Spheroid of 1866" U S Coast and Geodetic Survey, Special Publication No 5, 1917

TISSOT, M A *Mémoire sur la Représentation des Surfaces et les Projections des Cartes Géographiques* Paris, 1881

WAINWRIGHT, D B "A Plane Table Manual" U S Coast and Geodetic Survey, Report for 1905, Appendix No 7 (revised 1916)

WILSON, H M *Topographic Surveying* New York, 1910

YOUNG, A E "Some Investigations into the Theory of Map Projections" Royal Geographical Society, Technical Series, No 1, 1920

APPENDIX I

MECHANICAL COMPUTING

The highly developed calculating machines that are now available are frequently employed in surveying calculations, not only in the field, but also in cases where calculations can be concentrated in centralised offices A great variety of machines, both hand and electric, has been produced for commercial applications A machine for the particular needs of the surveyor should fulfil the following requirements —

(1) It should be a hand machine that can be easily transported, so that the same machine can be used either in the office or in the field

(2) It should not be subject to mechanical breakdowns, as it may frequently have to be used a long way from large towns where expert repair service is available

(3) It should be able to stand the abuse to which it may be subjected by junior or native clerks

(4) It should be an up-to-date machine embodying the latest and best developments of the calculating machine art

(5) It should serve the true purpose of a calculating machine, namely to lessen fatigue, increase accuracy and speed up the work Its operation should not be tedious or require great concentration or skill.

(6) The price should be reasonable

The above considerations rule out electric machines, as they require more upkeep and current is not usually available in the field , also keyboard machines on account of their weight and size, and also key-driven machines such as the Comptometer The choice therefore falls on barrel-type machines, such as the Nova-Brunsviga, Odhner and Facit Of these the Nova-Brunsviga (Fig. 115) has perhaps been more widely applied than any of the others, and will, therefore, be described in detail

A calculating machine is really simple, and may be learned in an hour, as no new arithmetical conceptions are required It will perform the four fundamental processes of arithmetic—addition, subtraction, multiplication and division—and since all arithmetic consists of combinations of these four processes, it may be said that it can be used for any arithmetical problem. Consider first addition.

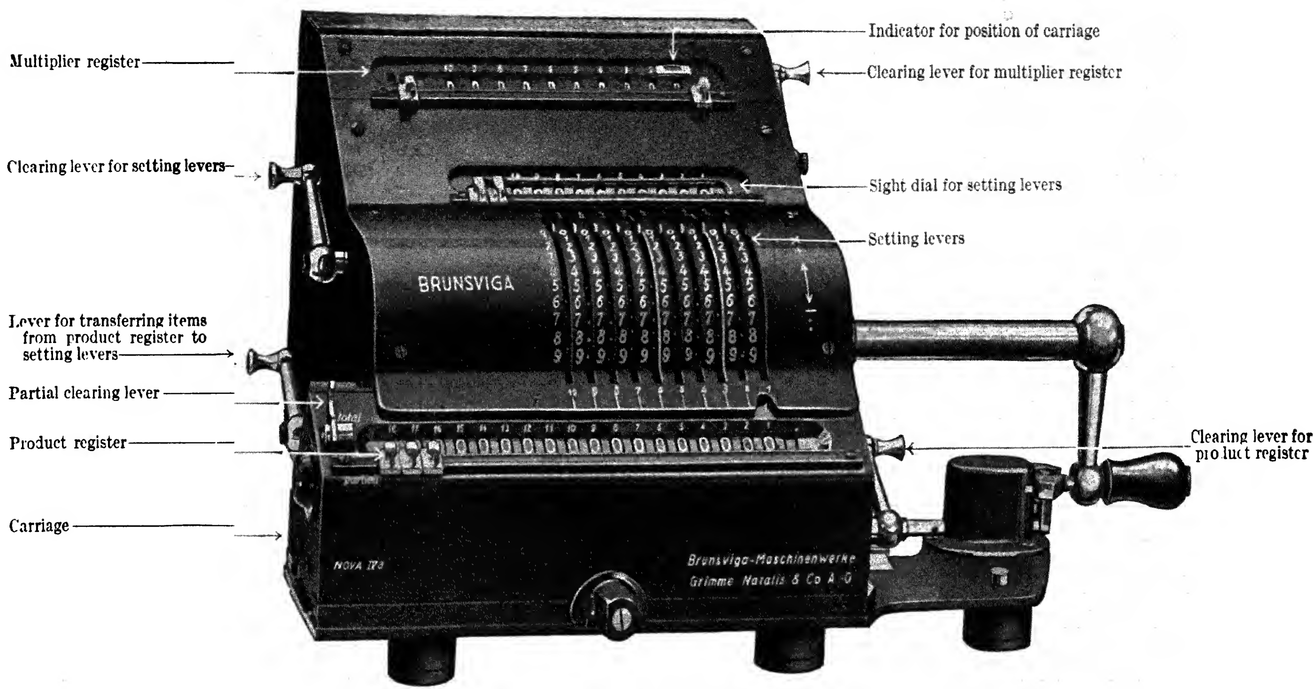


FIG. 115.—NOVA-BRUNSVIGA CALCULATING MACHINE, MODEL IVA.

It is necessary to have a means of conveying to the machine the number to be added, this is done by means of setting levers, each of which may be set in any one of the positions 0, 1 . 9 The right-hand lever represents units, the next tens, and so on The number set is shown in a sight dial, situated immediately above the levers, so that the setting is easily verified The next requirement is a register for showing the results of additions, this is situated in the front of the machine, and is known as the product register A forward turn of the operating handle adds whatever number is set on the levers to whatever number is already in the product register Needless to say, the product register can be cleared or zeroised, which is done by a lever on its right A lever on the left of the machine will clear the setting levers, or they may be altered at will by hand For subtraction the handle is turned backwards

Multiplication is really only continued addition If 6 be added seven times, the same result, namely 42, is obtained as by multiplying 6 by 7 Consider this principle applied to the multiplication of 789 by 456 The ordinary hand or schoolboy method results in a "sum" as shown If 789 be set on the levers of the machine, and the handle turned 6 times, the product 4734 will appear in the product register In order to eliminate the risk of error in counting the handle turns, a revolution counter or multiplier register is provided at the top of the machine We now require the product of 789 and 5, but wish the units of this product to be added in the tens wheel of the product register This is easily effected, because the product register is mounted in a movable carriage By means of the carriage spacer in front the carriage may be stepped one position to the right, so that its second or tens wheel is aligned with the units setting lever At the same time a moving indicator in the multiplier register has moved one position to the left, indicating that turns of the handle will now be effective in building up the tens figure of the multiplier The operator makes five turns, but the new product 3945 is not separately shown, it has been added to 4734, so that the product register now shows the product of 789 and 56, namely 44184 In this respect the machine is superior to the schoolboy, it does not write its separate "partial products," to be added at the end, but adds them as it goes along The problem proposed is completed by stepping the carriage once more to the right, and making four more turns The two factors and the product, namely 456, 789 and 359784, are now all visible, the two former may be verified, and the latter copied on to the computing sheet

```
   789
   456
  4734
 3945
3156
359784
```

A slight variation, known as short-cutting, is usually applied to the digits 6, 7, 8 or 9 in a multiplier Thus a multiplier 19 may be treated as 9 in the units position and 1 in the tens position, but two forward turns in the tens position and one backward or subtractive turn in the units position will yield exactly the same result, with a

saving of 7 turns, i.e. 3 instead of 10. This process soon becomes automatic and involves no mental effort; on the average it reduces the number of handle turns required by forty per cent, especially if the process is extended to include 5s that are preceded or followed by a larger digit.

Mechanical division is equally simple, and, like multiplication, merely follows the schoolboy theory. Suppose 54321 is to be divided by 23; the normal pen and paper method is shown alongside. The dividend 54321 is set on the setting levers, and the handle turned, so that the dividend appears in the product register (it is usually put in the left of this register). Having cleared setting levers and multiplier register, the divisor 23 is set on the levers, choosing levers in such a way that the 23 is aligned with the 54 of the dividend. As the first figure of the quotient is 2, we require to multiply the divisor by 2, and to subtract the product from the first two figures of the dividend. It has already been seen that multiplication by 2 is effected by making two turns, and that subtraction is effected by backward turns; hence the operation required is evidently two backward turns, after which the remainder 8 (together with the unused figures 321 of the dividend) appears in the product register. An apparent difficulty arises with the multiplier register, which now becomes the quotient register, counting, as in multiplication, the number of handle turns. It was observed that in short-cutting two backward turns produced an 8, whereas now a 2 must be recorded. Actually the multiplier register has two sets of figures on each wheel, and a sliding window exposes one set or the other. The multiplication set consists of white figures 0, 1 . . . 9, and the division set of red figures 9, 8 . . . 0. The machine senses whether the operator is multiplying or dividing, by virtue of the fact that the first turn of the handle is forward in one case and backward in the other, and automatically exposes the desired set of figures. At this stage there is a red 2 in the multiplier register. The next operation in the hand method is to bring down another figure; on the machine this is done by stepping the carriage one position to the left, so that instead of having 8 under 23, we now have 83. Three backward turns will produce 3 in the quotient, and leave the remainder 14 (together with the unused 21) in the product register. Step the carriage again. Instead of estimating that the next figure of the quotient will be 6, and making six backward turns, the operator merely turns backwards, making the remainder 119, 96, 73, 50, 27 and 4 with successive turns. As soon as the remainder is less than the divisor the process is naturally finished, and the correct figure (here 6) of the quotient appears in the multiplier register. The carriage is stepped, and one turn found to be sufficient in the next position. Should the operator inadvertently turn too many times,

```
23)54321(2361
   46
   --
    83
    69
    --
    142
    138
    ---
      41
      23
      --
      18
```

he is informed of the fact by the ringing of a bell, and corrects the over-turn by a forward turn.

On the Nova-Brunsviga a transfer lever on the left of the machine enables any number in the product register to be transferred to the setting levers while the product register is being cleared. This feature is useful when the product of three or more factors is required, or when the complement of a negative result in the product register is required ; in the latter case the negative result is transferred to the levers, and the handle turned backwards.

Simple as these processes are, both in theory and practice, many electrical machines, such as the Archimedes, Hamann Selecta, Madas, Marchant, Mercedes, Monroe and Rheinmetall, make them

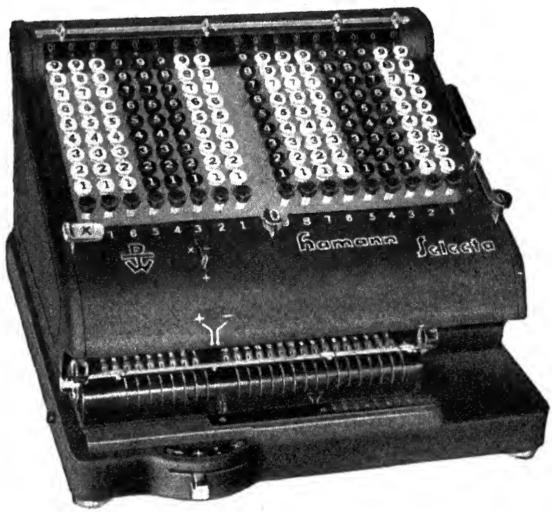


FIG. 116.—Hamann Selecta Calculating Machine.

automatic. The Hamann Selecta, illustrated in Fig. 116, has two keyboards (instead of setting levers), one for the multiplier and one for the multiplicand. The operator sets both factors and presses a button ; the machine then multiplies the two factors, with automatic short-cutting, at the rate of six revolutions a second. Thus two seven-figure numbers, once set, would be multiplied in about four seconds. Division also is fully automatic once the dividend and divisor are set, and takes about one second for each figure of the quotient. These machines, although unsuitable for field use, are of great value where computations can be concentrated.

The principal advantages offered by the use of calculating machines are :—

(1) They increase the speed with which computation may be done,

both directly, and also indirectly, by virtue of the fact that there are fewer errors to be removed than in the case of hand or logarithmic calculations

(2) They relieve the computer of much mental effort In particular they remove the *fear* of making errors, which is one of the most fatiguing elements in computing

(3) They will perform all four arithmetical processes, in any desired combination, e g $ab+cd$, whereas logarithms may be used for multiplications and divisions only

(4) They permit the employment of junior grades of labour for routine calculations

(5) They are used with natural values of the trigonometrical functions instead of with logarithmic values Every computer knows that the differences of the log sin or log tan of a small angle, or of the log cos or log tan of a large angle, are large and troublesome, whereas in a natural table the differences of sines or cosines are never troublesome, while the tangent is difficult only in the case of large angles, and even then trouble can usually be avoided by using the cotangent

In addition to easier interpolation, natural values of the trigonometrical functions offer a more equable sensitivity for inverse interpolation, as will be seen from the following table

DIFFERENCES FOR 1″ IN A 7-FIGURE TABLE

Angle	sin	cos	tan	cot	log sin	log cos	log tan	log cot
0°	48	0	48	∞	∞	0	∞	∞
15	47	13	52	724	78	6	84	84
30	42	24	65	194	37	12	49	49
45	34	34	97	97	21	21	42	42
60	24	42	194	65	12	37	49	49
75	13	47	724	52	6	78	84	84
90	0	48	∞	48	0	∞	∞	∞

For the cosines of small angles or the sines of large angles, the sensitivity of the natural table is more than twice that of the logarithmic table , in other words the differences for a small variation are more than twice as great, so that the uncertainty in an angle interpolated from an accurate function is only half as great In logarithmic work an angle less than 45° is often determined from its sine, and one greater than 45° from its cosine, in order to avoid insensitive parts of the table It will be seen that the natural sine of an angle of 60° is more sensitive than the logarithmic sine of an angle of 45° Again a tangent formula is frequently used in logarithmic work, especially with auxiliary angles whose magnitude cannot be controlled, to avoid insensitive regions, although this often leads to heavy interpolation near 0° and 90° Since a tangent is usually computed as the quotient of two numbers, the machine

user can always divide the smaller of his two numbers by the greater, thus obtaining a result between 0 and 1, whose sensitivity is always ample yet never embarrassingly large

Calculating machines are particularly useful in least square adjustments, in which they save a great deal of writing and adding, as quantities ab entering into a summation Σab are automatically added on the machine as each product is formed The Nova-Brunsviga Model IVA (Fig 115) is also capable of evaluating Σabc, which arises when unequal weights are assigned to observations The clearing of the 18-figure product register may be controlled by a partial clearing lever, whose effect is to prevent the left-hand nine figures from being cleared If a_1b_1 is formed in the usual way in the right of the product register, it may be transferred by means of the transfer lever to the setting levers, multiplication by c_1 gives $a_1b_1c_1$, which again may be transferred to the setting levers. The carriage is now moved to the extreme right, and the product turned into the left of the product register for storing The operator proceeds to form $a_2b_2c_2$ in the same way, the left-hand half of the product register, containing $a_1b_1c_1$, remaining uncleared, and showing $a_1b_1c_1 + a_2b_2c_2$ and finally Σabc

Another type of Brunsviga machine, specially developed for surveying calculations, is shown in Fig 117 Here two machines are mounted together, so that both can be turned by the single crank handle The machine on the left may be made to rotate in the same direction as that on the right, or in the reverse direction, or it may remain stationary. The obvious application of this machine is to cases where two numbers are multiplied by a common multiplier, as when the sine and cosine of an angle are multiplied by a length, when resolving into rectangular co-ordinates But, just as the logarithmic computer develops methods suited to the means at his disposal, so does the machine user develop a technique with the powerful means at his disposal. As an illustration suppose we wish to evaluate b from

$$b = \frac{a \sin B}{\sin A}$$

If we divide a by $\sin A$ on the right-hand machine, while at the same time $\sin B$ is set on the left-hand machine, and this machine is set to go in the opposite direction to the right-hand machine, the desired quantity b will be in the product register of the left-hand machine, and $a \div \sin A$ will be in the multiplier register In other words a single operation, namely a division, has evaluated the complete expression

The use of proportional parts in interpolation may be dispensed with if a calculating machine is available Thus to interpolate $\sin 12° \ 34' \ 56''{\cdot}78$ from

$$\begin{aligned} \sin 12° \ 34' \ 50'' &= 0{\cdot}217\ 8120 \\ & \qquad\qquad 473 \\ 12 \quad 34 \quad 60 \ &= 0{\cdot}217\ 8593 \end{aligned}$$

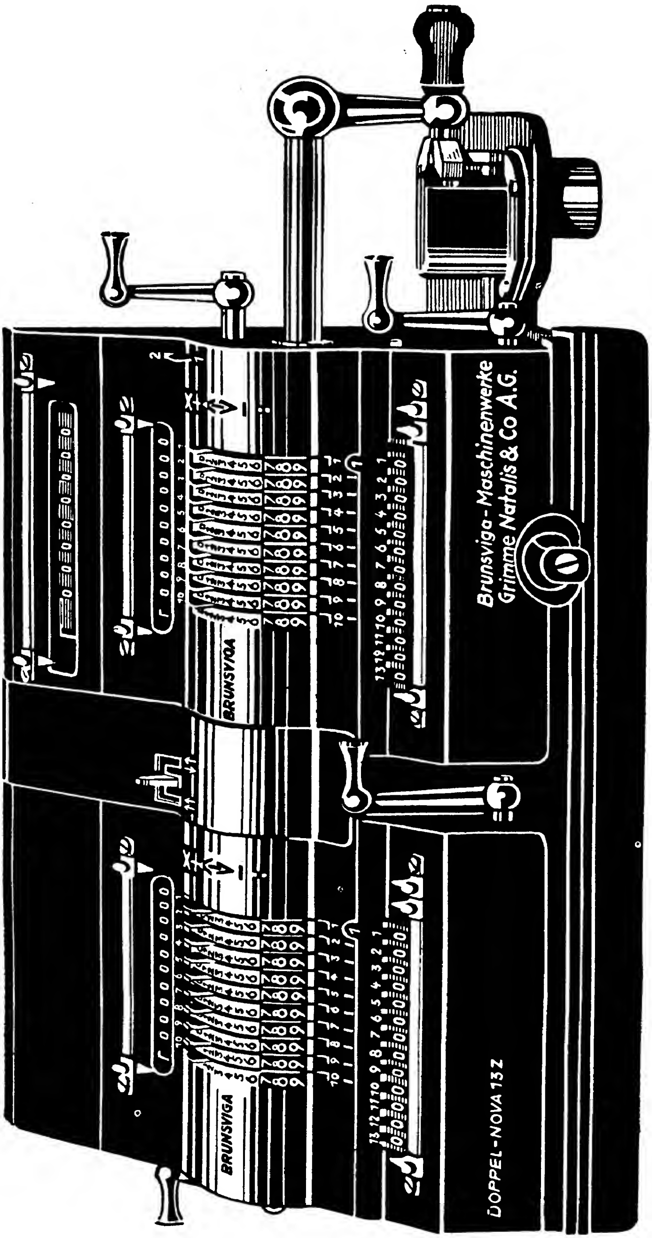


FIG. 117.—Brunsviga Calculating Machine, Model Double Nova 13z.

set 0·217 8120 on the setting levers, and multiply by 1·000 Clear the setting levers and multiplier register and set 473 , multiply by 1·000 and check both the settings by verifying that the product register shows 217 8593 followed, of course, by three ciphers Change the multiplier to 0 678, when the product register will show the required result, namely 0·217 8441, which can, if desired, be immediately transferred by means of the transfer feature to the setting levers for further multiplication The double or twin machine just described may be used for the simultaneous interpolation of the sine and cosine of an angle

The following list shows the tables that may be used for natural values of the trigonometrical functions The full titles are given in the references at the end of this appendix

No of Decimals	Interval of Argument	Author
4	1′	Milne-Thomson and Comrie
4	0°·1	Milne-Thomson and Comrie
4	10ˢ	Comrie
5	1′	Istituto Idrografico
5	0° 01	Lohse
5	0ᵍ 01	Steinbrenner
6	10″	Peters
6	10″	Brandenburg
6	0ᵍ 01	Peters
7	10″	Brandenburg
7	0°·001	Peters
8	1″	Gifford
8	0ᵍ·01	Roussilhe and Brandicourt
15	10″	Andoyer

The most generally useful of these tables is the 7-figure table for every 10″ by Brandenburg Similar 7-figure tables have been published by Ives and by Benson, but they are marred by errors Gifford's tables, which also contain a considerable number of errors, are, unfortunately, not arranged semi-quadrantally, so that two openings are necessary to find the sine and cosine of an angle It has been announced* that Peters and Comrie have prepared the manuscript of two 900-page volumes containing 7-figure and 8-figure tables of sines, cosines, tangents and cotangents, for every 1″, arranged semi-quadrantally These are now awaiting the necessary financial support for publication

It must be emphasised that the best result from a calculating machine cannot, in general, be obtained by simply taking the formulæ and methods developed for logarithmic work, and applying the machine to them. The expert computer will start *de novo*, and build up, from fundamental formulæ and first principles, methods

* *Monthly Notices of the Royal Astronomical Society*, Vol 92, p. 341

that utilise to the full the new facilities afforded by even the simple machines described One simple illustration suffices to illustrate the possible advantages of a machine. If we require a radius vector r from rectangular co-ordinates x and y, the usual logarithmic method is based on

$$\tan\theta = \frac{x}{y}$$
$$r = x \operatorname{cosec}\theta = y \sec\theta$$

involving four entries in tables The direct formula

$$r = \sqrt{x^2+y^2}$$

would involve five entries and much writing, but may be evaluated directly on a machine without any tabular entries or writing, although the use of Barlow's *Tables* will shorten the labour of finding the square root

REFERENCES TO TABLES

ANDOYER, H *Nouvelles Tables Trigonométriques Fondamentales (Valeurs Naturelles)* In three volumes Paris, Hermann et Fils, 1915, 1916 and 1918

BARLOW'S *Tables of Squares, Cubes, Square Roots, Cube Roots and Reciprocals of all Integer Numbers up to* 10,000 Third Edition, edited by L J Comrie London, Spon, 1930

BRANDENBURG, H *Sechsstellige trigonometrische Tafel fur Berechnungen mit der Rechenmaschine* Leipzig, Alfred Lorentz, 1932

BRANDENBURG, H *Siebenstellige trigonometrische Tafel fur Berechnungen mit der Rechenmaschine* Second edition Leipzig, Alfred Lorentz, 1931

COMRIE, L J *Four-Figure Tables of the Natural and Logarithmic Trigonometrical Functions with the Argument in Time* London, published by the author, 1931

GIFFORD, E *Natural Sines to every Second of Arc and Eight Places of Decimals*, second edition, 1926, also *Natural Tangents, etc*, 1927, and *Natural Secants, etc*, 1929 Manchester, Heywood

LOHSE, O *Tafeln fur numerisches Rechnen mit Maschinen* Leipzig, Wilhelm Engelmann, 1909

MILNE-THOMSON, L M, and COMRIE, L J *Standard Four-Figure Mathematical Tables* London, Macmillan, 1931 Edition A, with positive characteristics in the logarithms, Edition B, with negative characteristics in the logarithms

PETERS, J *Sechsstellige Tafel der trigonometrischen Funktionen, enthaltend die Werte der sechs trigonometrischen Funktionen von zehn zu zehn Bogensekunden des in 90° geteilten Quadranten und die Werte der Kotangente und Kosekante fur jede Bogensekunde von 0° 0′ bis 1° 20′* Berlin, Dummler, 1929

PETERS, J *Sechsstellige trigonometrische Tafel fur neue Teilung* Berlin, Wichmann, 1930

PETERS, J *Siebenstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades* Berlin-Friedenau, Verlag der Optischen Anstalt C P Goerz Akt-Ges, 1918

ROUSSILHE and BRANDICOURT *Tables à 8 Décimales des valeurs naturelles des sinus, cosinus et tangentes dans le système décimal, de centigrade en centigrade* Section de Géodésie de l'Union Géodésique et Géophysique Internationale Publication Spéciale No 1 Paris, Dorel, 1925

STEINBRENNER, G *Funfstellige trigonometrische Tafeln neuer Teilung zum Maschinenrechnen* Braunschweig, Grimme Natalis and Co , 1914

Tavole Logaritmiche a cinque cifre decimali Terza edizione Raccolte e Pubblicate per cura dell 'Istituto Idrografico della R Marina Genova, 1922

APPENDIX II

Calculation of Geodetic Positions.—In addition to Clarke's and Puissant's formulæ which are given on pages 184 and 185, mention should be made of the "Mid-Latitude" formulæ These are simpler to use than either Clarke's or Puissant's, but do not give such accurate results They may however be used for lines not exceeding 25 miles in length, in latitudes less than 60°, without introducing errors greater than 0″·01

They are —

$$\tan \tfrac{1}{2}dA = \tan \tfrac{1}{2}dL \sin(\phi + \tfrac{1}{2}d\phi) \sec \tfrac{1}{2}d\phi \qquad (1)$$

$$d\phi = \frac{l}{R_m \sin 1''}\cos(A + \tfrac{1}{2}dA) \qquad (2)$$

$$dL = \frac{l}{N_m \sin 1''}\,\frac{\sin(A + \tfrac{1}{2}dA)}{\cos(\phi + \tfrac{1}{2}d\phi)} \qquad (3)$$

The notation is that shown on page 184, with the addition of the symbol N_m, which is the normal at the mean latitude ϕ_m

In using these formulæ it is first necessary to find approximate values for $d\phi$ and dL to use in equation (1) An approximate value for $d\phi$ may be found from equation (2) by substituting R for R_m, and $\cos A$ for $\cos(A + \frac{1}{2}dA)$ The approximate value for $d\phi$ is then used in equation (3) and $\sin A$ is used for $\sin(A + \frac{1}{2}dA)$. In this way an approximate value is found for dL

To determine the algebraic signs of the quantities dA, $d\phi$ and dL a diagram should be drawn with the points A and B in their relative positions, and the meridians through each converging towards the pole The signs may then be obtained by inspection of the diagram.

If it is required to determine the quantities l, A and A' from the given quantitues ϕ, ϕ', L and L', the mid-latitude formulæ provide a direct solution of the problem The value of dA is obtained from equation (1) Equation (3) divided by equation (2) gives

$$\tan(A + \tfrac{1}{2}dA) = \frac{N_m}{R_m}\frac{dL}{d\phi}\cos(\phi + \tfrac{1}{2}d\phi) \qquad (4)$$

When the values of A and dA have been found the value of l can be obtained from equation (2) or (3)

If ϕ, ϕ', L and A are known and l, A' and L' are required, the mid-latitude formulæ may be used as follows Obtain an approximate value for dL by substituting A for $A + \frac{1}{2}dA$ in equation (4) By inserting the value of dL so found in equation (1) the value of dA is found with sufficient accuracy The values of l and dL can then be obtained from equations (2) and (3) respectively

APPENDIX III

Transverse Mercator Map Projection.—The tangent cylinder touches the earth along a central meridian To secure orthomorphism, the spacing of the meridians must be increased as we depart from the central meridian The projection shows an increasing exaggeration of areas away from the central meridian The Transverse Mercator has been adopted for the topographical maps of Great Britain, Egypt and some of the British African Colonies

APPENDIX IV

Aerial Photographic Surveying.—Photographs taken from the air may be used for the revision of existing maps, or for new surveys For the revision of existing maps, it is sufficient to obtain photographs covering the whole area For new surveys, each part of the area should appear on at least two photographs, so that stereoscopic views may be obtained of the whole ground In addition, the positions and heights of a certain number of points in the area must be fixed by ground survey methods These points are called "ground control points" The most accurate results are obtained from vertical photographs, that is from photographs taken with the axis of the camera pointing vertically down But time and expense are saved by the use of oblique, as well as vertical photographs The modern tendency, therefore, is to use multiple-lens cameras, which take one vertical, and up to six oblique photographs at one exposure When taking photographs, the aeroplane should fly on a straight horizontal course, and keep on an even keel, otherwise some of the area will not be covered by the overlap of the photographs, and the tilt of the camera will produce corresponding variations of scale in each photograph An experienced pilot can generally avoid tilts greater than two degrees

Plotting.—If the photographs are not seriously tilted, and the country is not very hilly, it may be assumed that the bearings of all points from the principal point of the photograph are virtually true It is then possible by graphical methods to plot a framework of points to the mean scale of the photographs, each point being

fixed by the intersection of two or more rays from principal points. Any ground control points that lie in the area are first plotted in their true positions, by means of their rectangular co-ordinates, and the framework of intersected points is adjusted to fit the control points The detail of the map is then traced from the photographs, and adjusted to fit into the framework of intersected points The contouring of the map involves the use of a topographical stereoscope fitted with movable parallactic grids If two ground control points, of which the heights are known, appear in the overlap which is under examination, then the heights of any other points in the overlap can be measured by use of the grid When the heights of a sufficient number of points have been determined in this way, the draughtsman draws in the contours on one of the photographs of the pair The stereoscope enables him to see the country in relief, and so to determine the shape of the contours, the heights already fixed control the positions of the contours The contours are traced from the photographs in the same way as the detail Topographical maps of fair accuracy, on scales not larger than 1/20,000, may be compiled in this way from air photographs

For larger scales, and more accurate surveys, it is necessary to take account of the height and tilt of the camera at each exposure. To determine the relative heights and tilts of the camera from a series of photographs a compound stereoscope must be employed A number of these instruments have been made, to a variety of designs The simpler form of the instrument provides angular measurements, from which the relative positions and tilts of the camera at each exposure can be computed If three ground control points appear in the series of photographs, the absolute co-ordinates and height of any point of detail in the series can be calculated In this way a rigid framework of points is provided for the map, but the computations are long and tedious In the more elaborate instruments, computations are avoided by the provision of mechanism for automatic plotting When a pair of photographs have been set in their correct relative positions in the instrument, the operator sees the ground in its true relief, and also a mark which appears to float in space By manipulating the controls he can bring the floating mark down into contact with the ground at any desired point The pencil of the automatic plotter then registers the position of this point on the map By making the floating mark follow the line of a road, or a river, or any other detail, the feature is automatically drawn on the map Contours may be drawn by fixing the floating mark at the required height, and moving it over the area, while keeping it in apparent contact with the surface of the ground

REFERENCES

GRUBER, O VON "Photogrammetry" London, 1932.
HOTINE, M "Surveying from Air Photographs" London, 1931.

TABLE OF CONSTANTS

	Number	Logarithm
π	3 14159 26536	0 49714 98727
$1/\pi$	0 31830 98862	9 50285 01273
e, base of natural logarithms	2 71828 18285	0 43429 44819
$\log_{10} e = M =$ modulus	0 43429 44819	9 63778 43113
$\log_e 10 = 1/M$	2 30258 50930	0 36221 56887
1 radian, in degrees	57 2957 79513	1 75812 26324
1 radian, in minutes	3437 74 67708	3 53627 38828
1 radian, in seconds	206264 80625	5 31442 51332
1 degree, in radians	0 01745 32925	8 24187 73676
1 minute, in radians	0 00029 08882	6 46372 61172
1 second, in radians	0 00000 48481	4 68557 48668
1 metre	39 37011 3 inches	1 59516 666
1 metre	3 28084 27 feet	0 51598 542
1 kilometre	0 62137 173 miles	9 79335 149
1 inch	25 39997 8 millimetres	1 40483 334
1 foot	0 30479 973 metres	9 48401 458
1 mile	1 60934 26 kilometres	0 20664 851
1 French legal metre	3 28086 93 feet	0 51598 893
1 United States legal metre	3 28083 33 feet	0 51598 417
1 kilogramme	2 20462 23 pounds	0 34333 420
1 pound	0 45359 243 kilogrammes	9 65666 580

Logarithms of numbers less than unity have been increased by 10

ANSWERS TO EXAMPLES

CHAPTER I (page 33)

1 −12° 48′ 20″·3
2 55° 17′ 55″
3 12^{h} 36^{m} 10^{s}·6
4 10^{h} 08^{m} 33^{s}·7
5 21^{h} 14^{m} 57^{s}·0
6 22^{h} 41^{m} 46^{s}
7 12^{h} 06^{m} 00^{s}·7
8 22^{h} 46^{m} 01^{s}, eastern
9 22^{h} 13^{m} 06^{s}·7 and 3^{h} 06^{m} 14^{s}·3, 55° 30′ 41″
10 12^{h} 10^{m} 28^{s}·2

CHAPTER II (page 96)

1 4^{m} 11^{s} fast, 5^{m} 06^{s} slow
2 1^{m} 15^{s}·5 fast
3 38^{s}·5 fast
4 6^{m} 12^{s}·3 slow
5 4^{m} 20^{s}·4 fast
6 90° 17′ 28″, 71° 57′ 28″ west of south
7 64° 32′ 42″ west of south
8 0° 55′ 52″ west of south
9 132° 26′ 50″, 21^{h} 44^{m} 43^{s}
10 55° 56′ 03″ N
11 51° 30′ 12″ N
12 2° 22′ 40″·8 N
13 49° 01′ 36″ N
14. 23° 10′ 59″·0 E
15 17^{s}·34 gaining
16 30′ 19″·5, 49′ 34″·5, 1° 13′ 43″·5, 53′ 10″·5 west of A
17 6^{s}·55 losing, 21° 03′ 36″·8
18 13′ 39″ W or 88° 42′ 36″ W

CHAPTER III (page 157)

1 58 ft
2 812 ft
3 Line of sight fails to clear C by 64 ft
4 −3·0046 ft
5 31,348·544 ft
6 1724·45 ft
7 16,532·37644 m
8 +0·00586 ft
9 97° 08′ 39″·88
10 −1″·32
11 +3″·9
12 71° 49′ 46″·22

CHAPTER IV (page 189)

Answers to questions on angle adjustment are given in terms of the seconds only

1 *a*, 11″·97, *b*, 50″·17, *c*, 36″·08, *a* + *b* + *c*, 38″·22
2 *a*, 30″·60, *b*, 13″·11, *c*, 3″·31, *d*, 49″·47, *e*, 23″·51
3 AOB, 32″·74, BOC, 21″·88, COD, 2″·54, DOE, 17″·57, EOA, 45″·27
4 *a*, 40″·26, *b*, 15″·28, *c*, 32″·46
5 *a*, 10″·99, *b*, 30″·42, *c*, 29″·20
6 *a*, 22″·6, *b*, 44″·9, *c*, 52″·5
7 ±0″·51, 1″·44
8 ±0·039 ft
9 1 1·28, 6″·00
10 32 miles
11 41″·01
12 ±0·01803 ft

13 A, 27″·19, B_1, 35″ 19, C_1, 57″·62, B_2, 2″·82, C_2, 14″·36, D, 42″·82

14 *a*, 3″·28, *b*, 34″·48, *c*, 14″·81, *d*, 46″ 50, *e*, 24″·21, *f*, 16″·73, *g*, 32″·56, *h*, 7″ 43

15 *a*, 3″ 35, *b*, 34″·38, *c*, 14″·69, *d*, 46″·43, *e*, 24″ 13, *f*, 16″ 85, *g*, 32″·66, *h*, 7″ 52

16 AB, 45,367·24 m , BC, 37,700·48 m

17 6,382,000 m

18 50° 25′ 11″·49 N

19 25,866·4 m , 178° 30′ 56″·4

20 1″ meridian, equator, 30·7153 m , 50°, 30·8977 m , 1″ parallel, equator, 30 9229 m , 50°, 19 9161 m

INDEX

D

E

F